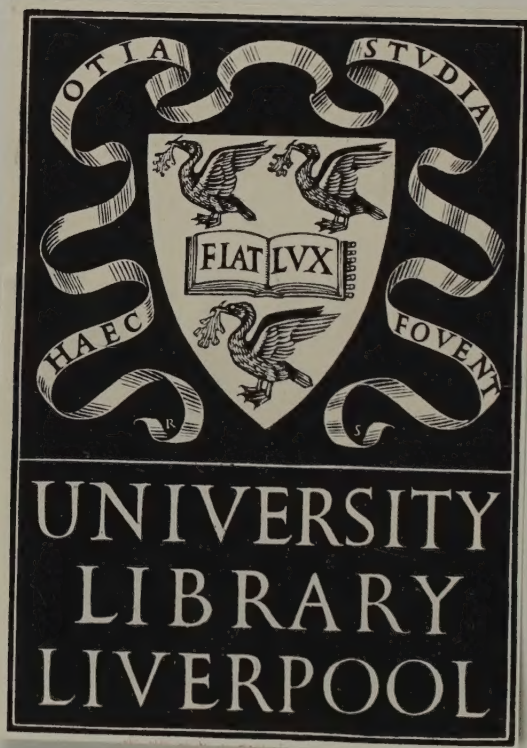
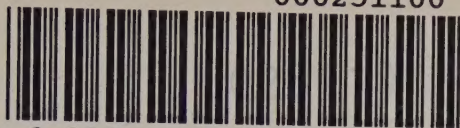


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THE MEASUREMENT OF AIR FLOW

THE MEASUREMENT OF AIR FLOW

by

E. OWER

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REVISED



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PREFACE TO THE THIRD EDITION

THE reception accorded to the first two editions of this book gives me reason to believe that it has had some success in fulfilling the purpose for which it was written. As I explained in the preface to the first edition, that purpose is to provide a text-book both for students and for engineers engaged on work such as ventilation and fan engineering, which involves the measurement of the speeds and pressures of moving air. It deals with both theory and technique, but I have not included more theory than I consider necessary for an adequate grasp of the subject. The practical side is emphasised throughout; and the last chapter, which I added in the second edition, consists entirely of fully-worked examples of problems of air-flow measurement taken from practice, showing in detail how some of the methods and principles described in earlier chapters are applied.

But for the war of 1939-45, and the conditions it left in its train, this third edition would have appeared several years ago. I have used the enforced delay to revise the earlier text more critically than I might otherwise have done; and I believe that in at least one respect this edition is better than those that preceded it. The first edition was written more than twenty years ago, when I was comparatively young, and I have long since recognised that when I wrote it I was, perhaps like other young men, unjustifiably impressed with the importance of my own work, which I quoted in far too great detail and with too much emphasis. This shortcoming was still apparent in the second edition, which I wrote some six years later. In this third edition, I have tried to preserve a proper balance between my own work and that of others; I have quoted myself only when the complete treatment of the subject seemed to require it, and then only as one of the many to whose work I have thought it necessary to refer.

I am grateful to the reviewers of the earlier editions: all of them generously refrained from mentioning this fault.

The general scope of the book remains much as before, but the text has been completely revised, extensively altered, and brought up to date; and there are numerous additions. The chapter on the flow in pipes is entirely new. Since the second edition appeared, great advances have been made, partly in our knowledge of pipe flow, but more in our recognition and treatment of it as a particular example, and a very important one, of boundary-layer flow. I have developed the new chapter from this point of view, and have added a section on the flow in curved pipes and the reduction of the losses in bends by the use of guide vanes. The advantages to be gained from these devices do not seem yet to be fully appreciated by engineers.

Finally, as regards this chapter, I have made it Chapter III instead of Chapter IV, since it now contains much fundamental matter which is used in most of the other parts of the book ; all the later chapters are more specialised and less general.

Chapter VI on orifices and other constrictions has once more been largely re-written, as it was in the second edition. Again, as then, the main source of my information has been German work ; many admirable German publications on this subject appeared between 1932 and the outbreak of war.

In the chapter on the vane anemometer, it has afforded me some satisfaction to be able to include an account of some work that has established the validity of an assumption I made in 1927, to enable me to solve a differential equation arising in the consideration of the behaviour of a vane anemometer in a fluctuating air current. Until I obtained this experimental verification in 1937, I was always rather uncomfortable about this assumption, for which there was no rigorous mathematical foundation.

The remaining additions consist mainly of descriptions of several useful instruments and methods of measurement, some of them new (since 1932), and some omitted from the earlier editions, perhaps without good reason. Among the latter are the rotameter and the ordinary gas meter.

My thanks are due to the owners of "Mine and Quarry Engineering" and to the Institution of Chemical Engineers for permission to reproduce diagrams and textual matter from some articles I contributed to the former and a lecture I gave to the latter. I also wish to thank the following :

The Gas Light & Coke Company, for providing me with a diagram and a description of the working of the wet gas meter.

Alexander Wright & Co. Ltd., for permission to reproduce Fig. 72.

The Metropolitan-Vickers Electrical Co. Ltd., for lending me the block for Fig. 70.

The Rotameter Manufacturing Co. Ltd., for photographs from which Figs. 68 and 69 were prepared.

E. OWER

Hampton, October, 1948

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CHAPTER I

INTRODUCTORY

ALTHOUGH this book deals primarily with the measurement of air flow, the methods to be described will in general be applicable to the flow of other gases with very little, if any, modification, except as regards the numerical values of the various physical properties occurring in the equations. Further, much of the theoretical matter will apply to the flow of liquids as well as gases, although, for practical reasons that will usually be obvious, the methods of measurement may not always be applicable.

When physical measurements have to be undertaken, a choice of methods is often available, and the experimenter must then decide which of these is best suited to his particular purpose. His selection will be governed by considerations of simplicity, directness, and the degree of accuracy he requires ; he should always avoid a complicated method when a simpler one will equally serve his ends. Let us therefore consider what means are available for the problem with which we are here concerned, namely, the measurement of the speeds of streams of liquids or gases, with particular reference to the motion of air along pipes or ducts. A method that immediately commends itself from the standpoint of simplicity is the introduction into the fluid of some indicator, such as a small light body, a puff of smoke, or a drop of colouring matter, which will be carried along by the stream and may be timed over a measured distance. Little consideration is however needed to show that the difficulties and disadvantages of this method are such as to preclude its practical application, except, perhaps, in a few very special instances. The only other direct method that immediately suggests itself is to collect and weigh the quantity of fluid delivered in a given time, whence the mean speed past any section of known area can be calculated if the density of the fluid is known. This method, which is often employed for the measurement of water flow, is simple and convenient for an incompressible fluid whose

density is greater than that of the medium surrounding the pipe or channel in which flow is taking place ; for gases, the experimental difficulties are serious, and for air flowing along pipes situated in the atmosphere, so great as to render the method virtually impracticable.

Thus, direct methods of measurement are not suited to our purpose, and it is therefore necessary to resort to the measurement of some physical effect arising from the motion. Three such effects have been found by experience to be suitable: namely, pressure changes associated with the motion ; mechanical effects, such as the rate of rotation induced in light vanes suitably mounted in the stream ; and, lastly, the rate of cooling of a hot body, such as an electrically heated wire, introduced into the air current. Of these, the first is of the greatest importance, since, as we shall see later, a properly designed instrument, suitably inserted in the stream, experiences a pressure, commonly called the "velocity head," which is entirely characteristic of the motion, and can be measured with a suitable pressure gauge. If such an instrument is constructed in accordance with certain well-established principles, which are explained in Chapters II and IV, it may be used without calibration as a standard for the measurement of air speed. This is not true of anemometers * that depend for their action either on mechanical or on electrical effects ; instruments of both these types are subject to individual variations, and require calibration against a standard instrument of the pressure-measuring type.

Instruments that involve the measurement of pressure can be subdivided into two distinct groups, depending on whether there is or is not a flow of air through them when they are in use. If there is no flow through, we have what we shall term a pressure-tube anemometer, the characteristic feature of which is two separate and differently-shaped tubes, each containing, in the end exposed to the air current, an orifice or a group of orifices at which a pressure is set up by the motion of the stream. These tubes are connected at their other ends to opposite sides of a differential pressure gauge, which prevents all flow through the anemometer and measures the difference between the pressures

* The term "anemometer," as its derivation implies, includes all types of instruments used for the measurement of air speed.

at the two groups of orifices. It is this pressure difference that is characteristic of the air speed. The remarks made in the previous paragraph about the possibility of constructing anemometers that can be used to measure air speed without calibration apply to this type of instrument above all others ; and it is by virtue of this property that one variety (the pitot-static tube, see Chapter IV), which is particularly easy to construct so that it will measure velocity head accurately, has been adopted universally as the standard against which practically all other types of anemometer are ultimately checked.

Pressure anemometers through which there is a flow involve the measurement of a pressure difference which is not the velocity head, but depends on the dimensions of the instruments themselves as well as on the motion of the air through them. Moreover, their calibration is more sensitive to small differences in shape of the instruments than is the case for a good pressure-tube anemometer. Nevertheless, within their limitations, such instruments have many useful applications. Commonly-used examples are the plate orifice, the nozzle, and, to a smaller extent, the venturi tube. For accurate work they should all be calibrated before use.

Mechanical anemometers comprise three main types—cup, swinging-plate, and vane anemometers. The first two of these are not of importance to engineers, and no attention beyond the following brief description will be devoted to them here. On the other hand, the vane anemometer, which is fully discussed in Chapter VII, is a convenient instrument for practical use and has a comparatively wide field of application.

In its original form, the cup anemometer consists of four hemispherical cups carried, with their bases vertical, at the outer ends of four light arms, which are symmetrically disposed in a horizontal plane and are attached to a central sleeve free to rotate about a vertical axis. The cups are arranged in pairs so that the concave side of one member of a pair is presented to the air current at the same time as the convex side of the diametrically opposite cup. At any instant, then, the air on one side of the median plane through the vertical axis of the instrument will be blowing into the interior of one or two cups and, on the other side of this plane, on the exterior of the opposite cup or cups. Hence, since the aerodynamic force on a cup with its

concave side presented to the wind is greater than when the wind is blowing on its convex face, rotation will ensue at a rate depending on the air speed. Instruments of this type are bulky and not easily portable ; they are chiefly used in meteorological work.

In 1926, Patterson ¹ claimed certain advantages for a three-cup type which was adopted to some extent in Canada. More recently, Sheppard ² has developed a much improved three-cup instrument with conical instead of hemispherical cups. Extreme lightness of construction and the use of an elegant indicating mechanism with very little friction have resulted in a very sensitive instrument—it responds to an air speed as low as forty feet per minute—and one that behaves satisfactorily in a fluctuating wind,³ in which the older types tend to over-estimate the mean speed considerably. Sheppard's instrument has been used in investigations of atmospheric turbulence, and the Meteorological Office have adopted the three-cup type with conical cups as standard for their cup anemometers. These are, however, larger and heavier than the Sheppard instrument, as they are intended for use in stronger winds.

The swinging-plate anemometer, now rarely used, consists of a vertical plate, suspended on knife edges, which deflects under the action of an air current to such an angle that the restoring torque due to the weight of the plate is equal to the air torque. The angle of inclination may therefore be taken as a measure of the air speed. Alternatively, the plate may be mounted on a torsion wire passing through an asymmetric vertical or horizontal axis ; in this case, there will be a relation between the air speed and the amount of twist of the wire necessary to bring the plate into normal presentation to the current.*

Electrical or hot-wire anemometers have not hitherto been extensively used, although certain types have been developed for industrial application. The chief factor restricting their wider use is probably the more elaborate apparatus and manipulation that they entail. As laboratory instruments, they may be made to give excellent results ; and they are particularly

* An anemometer of this kind has been designed for low-speed work by J. P. Rees. It indicates air speeds ranging from 10 to 180 feet per minute. (See also page 151.)

well suited to the measurement of low air speeds, a purpose for which they may be designed to have a considerably more open scale than is possible with pressure-tube anemometers.

REFERENCES

1. Patterson, *Trans. Roy. Soc. Canada*, **20** (1926), p. 1.
2. Sheppard, *Jour. Sci. Insts.*, **17** (1940), p. 218.
3. Scrase and Sheppard, *Jour. Sci. Insts.*, **21** (1944), p. 160.

CHAPTER II

GENERAL PRINCIPLES OF THE PRESSURE-TUBE ANEMOMETER

A. PRACTICAL CONSIDERATIONS

PROBABLY the most fruitful method of elucidating physical problems, particularly in the domain of engineering, is an attack in which theory and experiment play joint and complementary rôles ; and we often find it illuminating to consider the conditions of a problem from a practical standpoint before undertaking a theoretical treatment. This is true as regards an understanding of the broad principles upon which the action of the pressure-tube anemometer depends, and we shall devote the first part of the present chapter to a consideration of these principles from a practical point of view. It must not be supposed, however, that the following remarks are intended to convey anything more than a rough mental picture of what occurs. Certain material considerations, which must eventually be included, have been temporarily neglected, the more theoretical treatment being reserved for a later stage.

The natural starting-point is the consideration of the pressures that exist in a moving fluid, and it is in the conception of these that a good deal of confusion exists amongst practical engineers. It is well known that at any point within a stationary fluid there is a hydrostatic pressure which acts equally in all directions. If now the fluid is set in uniform, unaccelerated motion, the hydrostatic pressure will still persist, although its magnitude may be changed ; and if we imagine a very small, thin, flat disc placed at any point in the fluid, and moving with it, this " static " pressure, as it is called, will act on the two faces of the disc. Let us assume that there is a continuous stream of the fluid in movement, and that the motion is uniform and does not vary with time, so that at any section perpendicular to the current the conditions are constant.

If, instead of moving with the fluid, the small disc remains stationary at a point in the section under consideration, and if, moreover, the plane of the disc is parallel to the direction of motion, it is clear that the pressure acting on the two faces will be the static pressure existing at that point in the section of the fluid, provided that the presence of the disc does not disturb the conditions of flow.

Let us now consider what occurs if the disc is rotated so that the stream impinges normally on one face ; when the disc moves with the stream, it will again be acted upon by the static pressure only, but if it is held stationary at a given point, its front face will experience the static pressure and an additional pressure arising from the impact of the moving stream. It is important to observe that this additional impact pressure is entirely characteristic of the motion, depending, as it does, for any particular disc, on the speed and density of the fluid only, so that it can be used as a measure of the speed.

We see therefore that if the disc is stationary and normal to the stream, so that the forward motion of the fluid impinging on it is arrested, the pressure exerted will be the sum of the static and the impact pressures at the point in the fluid at which the disc is situated ; whereas if the disc is placed so that the fluid streams past it without disturbance the static pressure only is experienced. Suppose now that the imaginary disc is replaced by an open-ended tube of small bore bent to face the stream, and that the other end of the tube is connected to one limb of a simple U-tube pressure gauge (see Fig. 1), the other limb of which is open to the atmosphere. The forward motion of the fluid impinging on the open end of the tube will again be arrested, and the pressure gauge will record the sum of the impact and static pressures, or the " total head " as it is commonly called. It is found by experiment that the impact pressure measured by a facing tube is, within wide limits, unaffected by the shape and size of the tube, and is, in fact, equal to the " velocity head " or " velocity pressure " to which reference has already been made in Chapter I. The relation that exists between this pressure and the density and velocity of the fluid will be derived in the second part of the present chapter ; at this stage it is sufficient to observe that the velocity head is a function of the motion only.

Thus, whereas the velocity head is uniquely determined by the speed * of the fluid, the static pressure is not. It follows that the total head in the same fluid may, under two different sets of conditions, be different, although the speed in each case may be the same ; so that, in order to determine the speed, it is

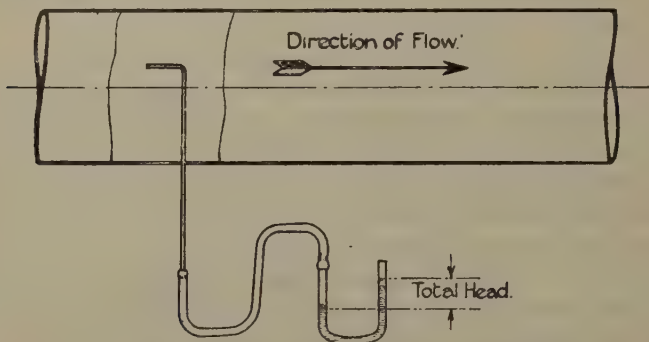


FIG. 1.—Measurement of total head.

necessary to measure both total head and static pressure, since the velocity head cannot be observed separately.

As an example of two instances in which a fluid moves at the same speed but has different values of the total head, we may consider the case of air that is being moved along a given pipe by means of a fan. It is clear that the air may be made to travel at the same speed whether the fan is blowing or sucking it down the pipe, and the velocity head will be the same in the two cases. On the other hand, the static pressure in the pipe will be higher than atmospheric when the fan is blowing and lower when it is sucking, so that the total head in the former case will be higher than in the latter.

Reverting now to a consideration of the open-ended tube facing an air stream, we see that the pressure gauge to which it is connected will indicate the sum of the velocity and static pressures of the air at the mouth of the tube. If, therefore, the other side of the gauge is connected to a tube or other device which in some manner measures the static pressure, the reading of the gauge will be equal to the difference between the

* We are, for simplicity, neglecting in this discussion the effect of variations in the density of the fluid.

pressures acting on the two sides, i.e. to the velocity pressure, and will thus serve directly to determine the air speed.

It is upon this principle that all pressure-tube anemometers depend. For the measurement of the total head, any form of open-ended tube facing upstream will be reliable, but some care is necessary in the design of the device for measuring the static pressure. The important condition to be observed is that, in the region where the static pressure is measured, the air must not have a velocity component perpendicular to its original direction of motion before the tube was inserted ; in other words, any device for measuring static pressure will give erroneous results if it deflects the air to any appreciable extent. The design of static-pressure tubes is fully discussed in Chapter IV ; at this stage it is sufficient to note that it is, in fact, possible to make a static tube that measures the static pressure to a degree of accuracy well within practical requirements. Such a tube, suitably used in conjunction with a facing or total-head tube, constitutes a complete pressure-tube anemometer which does not require calibration, save in exceptional cases, and therefore serves normally as a fundamental standard instrument for the measurement of air speed.

Anemometers of this nature are extensively used in practice. The only disadvantage inherent in this method of measurement is that the velocity pressures corresponding to low air speeds are small : a speed of 20 feet per second, for example, produces a pressure equivalent to only about 0.09 inch head of water. It follows, therefore, that sensitive pressure gauges or "manometers" are necessary for the measurement of low speeds. In fact, for air speeds below about 15 feet per second it is usually advisable to use some other measuring device, such as the vane anemometer, the orifice plate, or the venturi tube, or, for very low speeds, the hot-wire anemometer.

Before proceeding further, we shall find it necessary to understand the conventions used in the measurement of pressures. For convenience, the pressure of the undisturbed atmosphere—the barometric pressure prevailing at the time and place of measurement—is generally taken as the datum or zero static pressure ; and all static pressures are expressed in terms of the number of units (usually, in England, inches of water column) by which they exceed or fall short of this arbitrary datum. It

follows that static pressures less than atmospheric will be recorded as negative pressures, of which it is actually not possible to form a conception. But no confusion will arise if it is remembered that such pressures are really positive when referred to the true zero of pressure, namely, the vacuum. The zero for velocity head, of course, is the static pressure in the moving fluid ; in this case, the atmospheric pressure is not to be added on.

B. ELEMENTARY THEORETICAL CONSIDERATIONS

Attempts to visualise conditions of flow are aided by the introduction of the conception of " stream lines." In any mass of moving fluid, we assume the field to be mapped out by a system of lines such that the direction of any one of these lines at any point coincides with the direction of motion of the particle of fluid situated at the chosen point at the instant under consideration. The stream lines will thus give an instantaneous picture of the flow at any moment. If the flow is unsteady, so that it varies with time, this picture will change from instant to instant ; it may, in the case of periodic motion, undergo a series of changes which recur at regular intervals. For steady flow, on the other hand, the stream lines will preserve their configuration unchanged at all instants, and will show the actual paths of individual particles of the fluid. This type of motion is known as " streamline " motion, the unsteady variety being termed " turbulent " flow. As we shall see later, both types of flow are possible in pipes, although in practice the latter type occurs by far the more frequently.

In general, the rigorous theoretical treatment of problems of fluid motion is difficult, demanding mathematical analysis of a character considerably beyond the scope of this book. By the introduction of simplifying assumptions, however, it is sometimes possible to obtain solutions of certain types of problems, which serve a useful purpose by indicating to us how the various properties of the fluid may be expected to govern its motion. The value of solutions thus derived is increased when, as not infrequently happens, experimental results agree more or less closely with the theoretical predictions. We must, however, avoid the danger of attaching to such solutions a significance

that their nature does not warrant : it is essential to remember that they are only approximate and mainly of use as guides to thought.

The viscosity of a fluid is the property that is largely responsible for the difficulty mathematicians find in solving problems of fluid motion ; and it is only by neglecting viscosity altogether, or by regarding its effects as localised within a comparatively narrow zone adjacent to solid boundaries, that most of the progress in the theoretical treatment has been made. Comparison with observation has shown that a certain law, which is of fundamental importance in the measurement of flow by pressure-tube anemometers, may be simply deduced on the former hypothesis, i.e., by assuming that no frictional forces occur between adjacent particles of the fluid, or between the fluid and any solid boundaries with which it is in contact. A fluid of this type is called an inviscid or perfect fluid, and the law in question is named after Bernoulli, by whom it was first stated in 1738. It may be derived in the following manner.

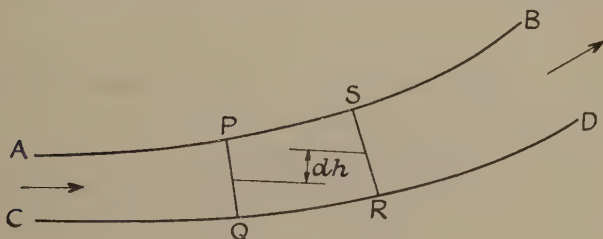


FIG. 2.

Let ABCD (Fig. 2) represent a longitudinal median section through a tube of fluid bounded by stream lines. Such a tube is termed a stream tube, and AB and CD are two of the bounding stream lines. For steady motion, the boundaries of the stream tube will preserve a constant shape, although the actual fluid in the tube will be constantly changing.

Consider a small element PQRS of the tube of length ds , which is so small that the elements PS and QR of the stream lines may be considered as straight lines. If v and p are respectively the speed and pressure across the section PQ, we may, since ds is small, write $v + dv$ and $p + dp$ for the corresponding quantities across section RS, where dv and dp are small.

Let a = area of section PQ,

Δ = weight of unit volume of liquid,

g = acceleration due to gravity,

dh = difference of level between the centres of area of PQ and RS.

If second order quantities are neglected, the resultant force along the axis of the tube due to the pressure on the faces of the element can be shown to be equal to adp , acting in the direction opposite to that of the motion. To this must be added the force acting in the same direction due to the weight of the fluid in the element PQRS. The latter force is equal to Δadh acting on PQ and is zero on RS, the resultant being Δadh . The sum of these forces is equal to the rate of change of momentum through the element. The mass of fluid entering per second across PQ is given by $\frac{\Delta}{g} av$, and, since PS and QR are stream lines, the mass leaving at RS is also $\frac{\Delta}{g} av$, so that the change of momentum per second is $\frac{\Delta}{g} avdv$, in the direction of motion.

We therefore have the equation

$$\frac{\Delta}{g} avdv = -adp - \Delta adh,$$

$$\text{or} \quad \frac{\Delta}{g} vdv + dp + \Delta dh = 0. \quad . \quad . \quad (1)$$

The quantity Δ/g is usually denoted by ρ , so that (1) may be written

$$vdv + \frac{dp}{\rho} + gdh = 0.$$

On integration along the stream tube, this equation becomes

$$\frac{v^2}{2} + gh + \int \frac{dp}{\rho} = \text{constant} = C, \quad . \quad . \quad (2)$$

which applies to all sections of a given stream tube. The quantity h may be taken as the height of the section measured above any arbitrary datum. An alteration in the datum level

will merely involve an alteration in the value of C for the stream tube. When the stream tube becomes infinitely narrow it may be considered as forming a stream line, and we may therefore say that equation (2) gives the relation between pressure and velocity along a stream line.

If the flow is horizontal, the datum level may be taken as being that of the stream line itself, and h becomes zero. In practice, when dealing with the flow of air we almost invariably find that the quantity gh is negligibly small in comparison with the sum of the other two terms of equation (2); so that the simplified form of this equation, namely,

$$\frac{v^2}{2} + \int \frac{dp}{\rho} = C, \quad \dots \quad (3)$$

which is strictly only applicable to the case of horizontal flow, may generally be used without appreciable error.* In the following treatment we shall therefore not consider differences of level.

A further simplification can generally be made, namely, to treat the fluid as incompressible, so that ρ is constant and (3) becomes

$$\frac{1}{2}\rho v^2 + p = C, \quad \dots \quad (4)$$

which is Bernouilli's equation in the form most commonly used in practical work. For air, the effect of compressibility can be neglected up to speeds of about 200 feet per second (see page 16), so that (4) holds for all conditions ordinarily encountered.

It will be seen that both terms occurring on the left-hand side of (4) have the dimensions of force per unit area, i.e., of pressure; the first term is the quantity we have called the velocity head, the second is the static pressure, and the sum of the two is the total head. Bernouilli's equation is therefore equivalent to the statement that the total head along a stream line is constant. This equation has been derived on the assumption that the flow is frictionless, but experiments indicate that it can be applied with good accuracy in many cases of turbulent flow of real fluids.

* In special cases, however, when two points of measurement are at appreciably different levels, it may be necessary to allow for the difference in level. The actual difference at which this allowance becomes significant depends on the velocity head being measured. Approximately, the atmospheric pressure decreases by 0.1 inch of mercury per 100 feet of height.

We may now consider the behaviour of the facing tube in the light of Bernoulli's equation. If a stream line is brought to rest by causing it to impinge on a small obstacle introduced into the fluid, the pressure p_1 acting on the obstacle will, if the latter be assumed to be so small as to affect the one stream line only, be equal to the total head at that point in the free stream before the obstacle was introduced, i.e.,

$$p_1 = \frac{1}{2}\rho v_0^2 + p_0. \quad . \quad . \quad . \quad (5)$$

where v_0 and p_0 are respectively the velocity and static pressure at the point in the unobstructed stream.

This pressure p_1 will be the pressure acting at the mouth of an open-ended tube facing the stream. Theoretically, as will be clear from the preceding analysis, this is true only if (a) the tube is so narrow that one stream line only is brought to rest, (b) flow through the tube is prevented, as it is if the other end is connected to a manometer, (c) the effects of viscosity on the action of the tube can be neglected, and (d) the tube does not disturb the flow. Of these, condition (b) is the only one that can be definitely fulfilled, and the fact that a tube of finite size facing the air current does indeed measure the total head rests not upon a rational theoretical basis but upon a large amount of carefully obtained experimental evidence. Such experiments show beyond doubt that the total head is accurately measured by a facing tube even in turbulent flow, and further that, provided it has an orifice facing upstream, variations in the size and shape of the tube within wide limits are not accompanied by any errors in the readings. As regards the effect of viscosity, experiments by Miss Barker¹ have demonstrated that this can be neglected except for very small tubes working at very low speeds, outside the range ordinarily encountered in practice.* Thus the facing tube gives an accurate reading of p_1 under practical conditions, although theoretically this result could only be expected for the stream-line flow of a perfect inviscid fluid with an infinitely small tube.

Now we see from (5) that p_1 , the total head at the point occupied by the mouth of the facing tube, is equal to the sum

* For air at ordinary temperatures and pressures, viscosity effects on the total-head tube may be neglected, provided that the product vd is greater than 0.01, where v is the air speed in feet per second and d the diameter of the tube in feet.

of the velocity head $\frac{1}{2}\rho v_0^2$ and the static pressure p_0 in the undisturbed flow at that point. If therefore p_0 is measured by a properly designed static-pressure tube connected to the other side of the same manometer to which the total-head tube is connected, the resultant pressure P indicated by the manometer is given by

$$P = p_1 - p_0 = \frac{1}{2}\rho v_0^2, \quad . \quad . \quad . \quad (6)$$

from which v_0 can be calculated. Details of the method of calculation are given in Chapter V.

Reading of a Total-Head Tube at High Speeds.—The equation just derived for the pressure at the mouth of a total-head tube is strictly valid only when the fluid is incompressible, but we shall see immediately that it applies with sufficient accuracy for most practical cases of air flow. The general equation that takes account of compressibility, i.e., changes of density with pressure, may be obtained as follows.

Before the total-head tube is inserted into the stream the pressure and velocity are related according to equation (3), namely,

$$\frac{v_0^2}{2} + \int \frac{dp_0}{\rho_0} = C. \quad . \quad . \quad . \quad (7)$$

When the tube is inserted, we assume that the velocity at the mouth is reduced to zero and the pressure becomes p_1 , so that, by application of Bernoulli's theorem,

$$\int \frac{dp_1}{\rho_1} = C. \quad . \quad . \quad . \quad (8)$$

We may now equate the left-hand sides of (7) and (8) and thus obtain

$$\frac{v_0^2}{2} + \int \frac{dp_0}{\rho_0} = \int \frac{dp_1}{\rho_1}. \quad . \quad . \quad . \quad (9)$$

The assumption, which appears to be justified by experimental evidence, is now made that pressure changes are sufficiently rapid to be adiabatic, so that $p_1/\rho_1^\gamma = p_0/\rho_0^\gamma = K$, where γ for dry air has the value 1.408.

Hence, from (9),

$$\frac{v_0^2}{2} + K^{1/\gamma} \int \frac{dp_0}{p_0^{1/\gamma}} = K^{1/\gamma} \int \frac{dp_1}{p_1^{1/\gamma}},$$

which, on integration and substitution of (p_0/ρ_0^γ) for K , reduces to

$$\frac{p_1}{p_0} = \left[1 + \frac{\gamma - 1}{\gamma} \cdot \frac{\rho_0}{p_0} \cdot \frac{v_0^2}{2} \right]^{\frac{\gamma}{\gamma - 1}}.$$

Expanding by the binominal theorem, we obtain

$$\frac{p_1}{p_0} = 1 + \frac{\rho_0 v_0^2}{p_0} + \frac{1}{\gamma} \frac{\rho_0^2}{p_0^2} \cdot \frac{v_0^4}{8} + \frac{2 - \gamma}{\gamma^2} \cdot \frac{\rho_0^3 v_0^6}{p_0^3 48} + \dots,$$

or

$$p_1 = p_0 + \frac{\rho_0 v_0^2}{2} \left[1 + \frac{\rho_0}{\gamma p_0} \cdot \frac{v_0^2}{4} + \frac{(2 - \gamma) \rho_0^2}{\gamma^2 p_0^2} \cdot \frac{v_0^4}{24} + \dots \right]. \quad (10)$$

Now a_0 , the velocity of sound in air at pressure p_0 and density

ρ_0 , is equal to $\sqrt{\frac{\gamma p_0}{\rho_0}}$.

Hence (10) may be written

$$p_1 = p_0 + \frac{\rho_0 v_0^2}{2} \left[1 + \frac{1}{4} \frac{v_0^2}{a_0^2} + \frac{2 - \gamma}{24} \cdot \frac{v_0^4}{a_0^4} + \dots \right], \quad (11)$$

which is the general expression for the pressure at the mouth of a total-head tube.

For air at ordinary temperatures and pressures, a_0 may be taken as roughly 1,100 feet per second, and it is then easy to see from (11) that v_0 may be as high as 200 feet per second before the second term in the expansion becomes equal to 0.01. Hence for all speeds below this the second and subsequent terms may be neglected, and the ordinary form of (11), namely,

$$p_1 = p_0 + \frac{1}{2} \rho_0 v_0^2,$$

may be employed. For very high speeds, exceeding the velocity of sound, Rayleigh² and Stanton³ have shown that (11) requires modification on account of the effects of stationary waves, but this point is outside the scope of the present work.

REFERENCES

1. Barker, Miss, *Proc. Roy. Soc., A*, **101** (1922), p. 435.
2. Rayleigh, *Proc. Roy. Soc., A*, **84** (1910), p. 247.
3. Stanton, *Proc. Roy. Soc., A*, **111** (1926), p. 306.

CHAPTER III

THE FLOW OF AIR IN PIPES OF CIRCULAR CROSS-SECTION

MOST of the problems of air flow with which the engineer is concerned relate to the flow in pipes, and the present chapter will be devoted to consideration of the characteristics of that type of flow. This will be found to serve the additional useful purpose of disclosing many features of which account must be taken in all methods of measurement.

A large amount of work has been done in this important field, mainly in England, the United States, and Germany, with the result that the characteristics of pipe flow, at all events in circular pipes, are now well established. The flow in non-circular pipes has not been so extensively studied, but there is no reason to suppose that, in its main essentials, it differs much from the flow in circular pipes.

Viscosity.—In our study of this subject we can no longer neglect the property of viscosity, which we have ignored in the elementary discussions of the preceding pages. In Chapter II we considered the pressure acting perpendicularly to the face of a very thin, flat, stationary disc with its plane along the direction of a moving air stream. If we were to mount this disc on an apparatus capable of measuring forces acting along the direction of the stream, we should find that, in addition to the pressures normal to the two faces, there was a force acting along the direction of motion, as though the air were trying to stick to the two faces of the disc and drag the latter along with it. It is this property of air (and all fluids) that is called the viscosity; it produces tangential forces not only between the air and solid boundaries along which it may be flowing, but also between adjacent particles of the air itself.

In order to see how viscosity may be defined and measured, let us take the case of a viscous fluid moving past a stationary plane surface, the flow being everywhere parallel to the surface.

Experimental evidence suggests strongly that the fluid particles in immediate contact with the surface are at rest. As we proceed farther out from the surface into the fluid, the speed parallel to the surface increases, as the faster-moving layers of the fluid tend to accelerate the slower layers nearer the surface, while they themselves tend to be retarded by the latter. There is thus a tangential force on the surface in the direction of motion and a velocity gradient in the fluid in a direction normal to the surface. If v is the velocity parallel to the surface at a point distant y from it (Fig. 3), the velocity gradient at the surface (or the rate at which the velocity increases as the distance from the surface increases) is given by dv/dy * measured at the surface; and, if F is the total tangential force acting on an area S of the surface, the coefficient of viscosity (usually denoted by μ) of the fluid is defined as being such that

$$F = \mu S (dv/dy)_{\text{surface.}}$$

In general, instead of considering the tangential force on the solid boundary, we may replace the latter by the surface of

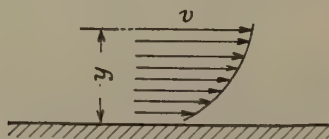


FIG. 3.

separation between any two contiguous layers of the fluid itself, and write

$$F = \mu S \cdot dv/dy,$$

where dv/dy is now the velocity gradient at the point in the fluid in question.

From this we see that μ may be defined as "the tangential force on unit area of either of two horizontal planes of indefinite extent at unit distance apart, one of which is fixed, while the other moves horizontally with unit velocity, the space between

* If the velocity is proportional to the distance from the surface, the gradient is, of course, simply v/y .

being filled with the viscous substance." This definition is due to Maxwell.

The Boundary Layer.—It is now generally accepted that, when a fluid flows along a stationary surface forming the boundary of a solid body, the fluid particles in the immediate vicinity of the surface are at rest. At a certain distance from the surface the full stream velocity is reached; and, for fluids of low viscosity, such as air or water, the rise to this velocity from the zero velocity at the surface occurs within a comparatively narrow layer of fluid known as the "boundary layer." The velocity gradient is rapid near the surface and then falls away, the free stream velocity being approached asymptotically.

This boundary layer is not a mere theoretical abstraction: its existence has been amply confirmed by experiment, and progress in the theoretical study of the motion of fluids has been advanced considerably by increasing knowledge of the properties and behaviour of the boundary layer. The flow in pipes is essentially a special case of boundary-layer flow, and it will therefore repay us to discuss the properties of the boundary layer in greater detail.

Take first the case of air flowing along a flat smooth plate which is so wide that, in the region we are considering, no disturbing transverse flow is introduced by the edges. If, in addition, there are no entry disturbances at the leading edge of the plate, the boundary layer will have zero thickness actually at the leading edge and will increase progressively in thickness as the flow proceeds along the plate. Also, in the forward part of the boundary layer the flow will be steady and "streamline," that is, everywhere parallel to the surface with no components of velocity perpendicular to the surface and no fluctuations of velocity along it.

This streamline boundary layer does not persist indefinitely: at a certain distance along the plate a different type of flow sets in. Its characteristic feature is that, superimposed on the average flow along the surface, there are rapid and small velocity fluctuations both parallel and perpendicular to the surface, somewhat resembling the way in which on a gusty day the wind rises and falls intermittently above its average

speed, with possibly up and down gusts as well. This turbulent boundary layer, as it is called, represents the final state of flow, and it thickens up indefinitely as the flow proceeds along the plane.

Now suppose the surface is rolled up to form a hollow cylinder with the air flowing along the inside. We then have a pipe, and the conditions of flow are similar in many ways to those for a surface. Near the inlet, when entry disturbances have been damped out, we have a length of streamline or laminar flow, which is succeeded by a breakdown to turbulent flow at a certain distance along the pipe.

If there is a carefully shaped ("bell-mouthed") inlet to the pipe, so that entry disturbances are reduced as far as possible, we shall find that, quite close to the inlet, the velocities are equal at all points in a transverse section practically right up to the walls—the boundary layer is just beginning to form and is very thin. At sections farther downstream, as wall friction makes its effect felt, the boundary layer becomes progressively thicker; and we find the velocity profile—that is, the curve showing the distribution of velocity across the section—becoming more and more peaked at the centre as the retarded layer spreads farther into the interior of the pipe (Fig. 4). In

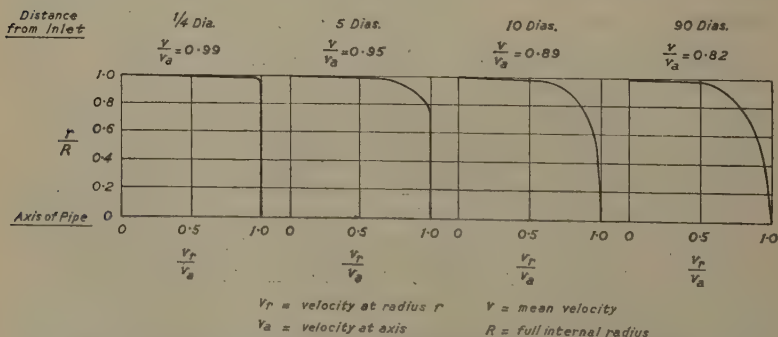


FIG. 4.—Velocity distribution at various sections in a straight pipe.

the case of the flat surface there is no limit to the thickness the boundary layer can develop, but in a pipe the limit is obviously half the diameter. When this thickness is reached, the whole of the flow across the section is boundary layer flow,

and we find that, provided the mean velocity of flow remains unaltered, the velocity profile is then the same at all sections farther downstream—a most important fact from the practical point of view, to which we shall refer later.

It was Osborne Reynolds ¹ who first studied in detail the two types of flow in pipes, and showed the conditions in which each exists. Reynolds's experiments were carried out with water flowing along straight horizontal glass tubes, and he was able to distinguish between streamline and turbulent flow in a tube by watching the behaviour of a small quantity of colouring liquid introduced into the inlet by means of a small jet on the axis. At low speeds of flow, the colouring liquid was drawn out into a steady, well-defined filament along the axis of the tube. As the speed was increased, there came a time when the filament assumed a wavy appearance and showed clear evidence of the beginning of eddy formation in the water. With further increase of speed, these signs became more marked, until finally the filament completely lost its individuality and became diffused throughout the body of the water, in which turbulent motion had now become general. In a given size of tube and with water at a constant temperature, the speed at which eddy formation began was more or less constant ; but if the experiment was reversed, the speed being gradually reduced from an initial condition of turbulent flow, streamline motion began at a speed that was much more definite and rather lower than that at which eddy motion started in the previous experiment with ascending speeds. The transition from turbulent to steady flow in this reversed experiment was detected not by the colour method, which now could obviously not be applied, but by observation of the change in the relation between velocity and resistance * to flow that occurs when the type of flow changes : for streamline flow the resistance is proportional to the first power of the velocity ; for turbulent flow approximately to the square.

The speed at which the change of flow occurs is termed the critical speed ; and, from experiments with water at different temperatures and flowing at various speeds, Reynolds found that when the speed was gradually reduced, starting from turbulent flow, streamline motion always set in at a definite

* For an explanation of the term " resistance " see page 24.

value of the quantity $v d \rho / \mu$, where v * is the mean speed of flow, d is the pipe diameter, μ is the coefficient of viscosity, and ρ is the mass density (weight per unit volume/g) of the water. The quantity $v d \rho / \mu$ has come to be called the Reynolds number, and similar ratios are of fundamental importance in other branches of fluid motion. Its chief merit, which, since Reynolds's time, has been abundantly confirmed by experiment, is that it enables laws determined from experiments with a particular fluid to be generalised for all other fluids. Thus Reynolds found from his experiments with water that the critical velocity v_c is reached when

$$\frac{v d \rho}{\mu} = 2000,$$

so that

$$v_c = \frac{2000 \mu}{\rho d} = \frac{2000 \nu}{d},$$

if we adopt the usual convention of writing ν for μ/ρ , which is termed the coefficient of kinematic viscosity.†

For a 1-in. diameter pipe, this gives the critical velocity as about $3\frac{1}{2}$ in. per second for water at a temperature of 15°C . But the same expression holds for the flow of air, and indeed for any other fluid that can be assumed incompressible. Putting in the appropriate values of ρ and μ from Table 1, we find that for air at atmospheric pressure and 15°C . v_c is about 0.3 ft. per second in a 1-ft. diameter pipe. Generally, for air under these conditions $v_c = 0.32/d$ approximately (d in feet), so that the critical velocity is 1 foot per second for a pipe about $3\frac{1}{2}$ in. in diameter and is proportionately less for larger pipes. It follows that, from the engineering standpoint, the critical velocity for air is low; hence in the great majority of cases the engineer meets he will be dealing with turbulent flow. Fortunately, this causes no undue difficulties of measurement: with all ordinary degrees of turbulence, the airstream, if generated by a reasonably steadily running actuator, behaves, as far as the

* Throughout this chapter, unless otherwise stated, the velocity in a pipe is to be taken as the mean velocity, i.e., the volume of fluid passing in a given time divided by the cross-sectional area of the pipe.

† The reader with a knowledge of dimensional theory will see that the quantity $v_c d / \nu$ is non-dimensional, and hence that the value of the constant (2000) is the same for any set of self-consistent units.

instruments for measuring average speeds and pressures are concerned, virtually as though the motion were steady. The turbulent components superimposed on the average motion seem in most cases to be small enough to have little effect.

TABLE I

*Values of μ , ρ , and v for air at 760 mm. (29.92 in.) pressure
($g = 32.2 \text{ ft./sec.}^2$)*

Temp. ° C.	$10^7 \times \mu$ slugs per ft. per sec.	$10^3 \times \rho$ slugs per cubic foot.	$10^4 \times \mu/\rho$ ($= 10^4 \times v$) ft. ² per sec.
0	3.57	2.51	1.42
5	3.62	2.46	1.47
10	3.67	2.42	1.51
15	3.72	2.38	1.56
20	3.77	2.34	1.61
25	3.82	2.30	1.66
30	3.87	2.26	1.71
40	3.97	2.19	1.81
60	4.17	2.06	2.02
80	4.36	1.94	2.25
100	4.54	1.83	2.48
200	5.39	1.44 ₅	3.72
300	6.15	1.19 ₃	5.18
400	6.84	1.01 ₆	6.73
500	7.48	0.88 ₄	8.45

It should be remarked that the value $2000v/d$ for the critical velocity in a pipe is approximate only, and represents the average value under ordinary conditions. If special precautions are taken to avoid all disturbances, streamline flow may be made to persist up to Reynolds numbers of ten times this value, or even more. We must also bear in mind that all the discussion so far relates to smooth pipes, that is, pipes for which the average surface irregularities are small in relation to the pipe diameter. Further remarks on the subject of roughness will be found later (page 31), where an approximate quantitative definition of roughness is given.

The Resistance of a Pipe to the Motion of a Fluid along it.—There has been much discussion among engineers of the

nature of the resistance of a pipe to the flow of fluid along it and how this resistance should be measured. The answers to these questions can be logically developed from first principles. Let us consider firstly the simplest case, in which air or other fluid is flowing along a horizontal length of parallel pipe with a speed v which is equal at all points across the section of the pipe. Let p be the static pressure (above the atmospheric pressure) and ρ the mass density of unit volume of the air. A mass m of the air has kinetic energy $\frac{1}{2}mv^2$; and it has also potential energy by virtue of the work that has been done upon it (e.g., by the fan maintaining the motion) in raising its pressure to p . If V is the volume of the mass m of the air, this potential energy is pV , and the total energy referred to the mean level of the pipe is therefore $\frac{1}{2}mv^2 + pV$. Hence, since $m = \rho V$ by definition,

$$\text{total energy} = V(\frac{1}{2}\rho v^2 + p),$$

$$\text{and energy per unit volume} = \frac{1}{2}\rho v^2 + p.$$

It will be noted that each of the two terms in this expression has the dimensions of a pressure, and that their sum is, in fact, the total pressure of the fluid which would be measured by a facing tube (see page 7). If there were no resistance to the motion of the fluid along the pipe, and no loss of energy due to the formation and dissipation of eddies, this total pressure would remain constant. Actually, of course, such dissipation of energy does occur, and there is also a retarding force due to the viscous forces acting at the walls of the pipe, which causes a further loss of energy. If we consider two sections, one at each end of a straight length of pipe, we see that there will be a loss of energy as unit volume of the fluid passes from the upstream to the downstream section; a corresponding amount of work will consequently have to be done in forcing this unit volume along the pipe. The resistance of the length of pipe between the two sections may therefore be taken as equal to this quantity of work. Further, since, as we have seen, the energy per unit volume passing a section is equal to the total head at that section, the resistance of the length of pipe will manifest itself by causing the total head at the downstream section to be less than that at the upstream section.

It is thus apparent that the resistance offered by a length of

pipe to the passage of a volume of fluid along it may be measured in terms of the drop in total head experienced by the fluid in passing along this length; and this method of expressing the resistance is the only practicable one for general use (see page 26).

So far we have considered only flow in a parallel-walled straight pipe, in which both the velocity and the static pressure are constant across a section, so that the mean total head at that section can be obtained from a single measurement at any point in its plane. Such flow, however, rarely, if ever, occurs in practice. Experiments show that the velocities are not in general the same at all radii in a given section of a parallel pipe along which a fluid is moving steadily.* It is found that the velocity is usually a maximum at the axis of the pipe and falls away towards the walls (see Fig. 4). We must consider therefore how to measure resistance in an actual case, i.e., how to determine the mean total head across a section where the velocity is not uniform.

Before discussing the general case we may consider a straight parallel pipe, which presents no difficulties. In the first place, if the flow is parallel to the walls the static pressure across any cross-section must be constant. If this were not so, the air would tend to flow from the high-pressure regions to those of low pressure, and the resultant motion would not be parallel to the walls. Secondly, once the flow has settled down to the normal distribution in a straight pipe (see page 21), that is, at a sufficient distance from the inlet, we know that the distribution of velocity at all sections is the same. Now the mean total head at any section will be equal to the static pressure p at that section which, being uniform, may be obtained by a single measurement anywhere in the section, together with the velocity head corresponding to the mean kinetic energy of unit volume of the air passing the section. If we denote the velocity corresponding to this mean kinetic energy by $\bar{v}$, the velocity head is $\frac{1}{2}\rho\bar{v}^2$ (see page 13). We need not concern ourselves here with the question of how $\bar{v}$ is to be measured because;

* Steadily is here used in the sense that the motion is unaffected by external influences, such as the effect of the inlet to the pipe, and that there are no appreciable fluctuations in the speed of the fan or other actuator producing the motion. It is not to be taken as implying non-turbulent motion.

as we shall show immediately, it cancels out in the final result.

Consider now a length of pipe whose end sections are denoted by A and B. The mean total head at A will be greater than that at B by a pressure P, which is equal to the work done on unit volume of the fluid in moving from A to B, i.e., to the resistance of the length AB of the pipe. We therefore have the equation

$$\frac{1}{2}\rho_A \bar{v}_A^2 + p_A = \frac{1}{2}\rho_A \bar{v}_B^2 + p_B + P. \quad (1)$$

By our assumptions, the velocity distributions at A and B are identical, so that $\bar{v}_A = \bar{v}_B$. Further, if the static pressure difference $p_A - p_B$ is small compared with P, as it is found to be in most practical cases, we may assume $\rho_A = \rho_B$. Equation (1) therefore reduces to

$$P = p_A - p_B. \quad (2)$$

The physical significance of equation (2) is that, *in the special case of flow in a straight length of parallel pipe* in which the velocity distributions at the two ends are the same, the resistance is measured by the difference in static pressure at the two ends.

The fact that this is a special case cannot be too strongly emphasised. It is the failure to realise this that has led to much of the confusion that prevails in the measurement of pipe resistance. In the general case, one can only measure resistance by determining the loss of mean total head; and statements such as the one commonly made that resistance is measured by "a total-head gauge on the inlet side of a fan and by a static gauge on the outlet" are extremely misleading to anyone who accepts them without realising their limitations. Further reference to this particular statement will be made later (see page 199).

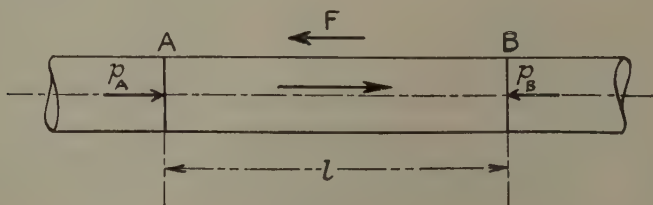


FIG. 5.

Returning to equation (2), we can derive an alternative expression for P by writing F for the frictional force opposing the flow *per unit area* of the internal surface of the pipe, whose diameter is d (Fig. 5). Then the total resistance of a length l is $F\pi dl$. For steady motion this must be balanced by an equal force acting on the fluid in the length l of the pipe along the direction of motion. This force is provided by the difference in pressure between A and B, and is easily seen to be $(p_A - p_B) \frac{\pi}{4} d^2$. Equating this force to the resistance $F\pi dl$, we get

$$(p_A - p_B) = \frac{4Fl}{d}. \quad . \quad . \quad . \quad . \quad (3)$$

The Resistance of Smooth Straight Pipes.—Many investigations have been made to determine the value of F in equation (3) for smooth straight pipes, and so to establish the relation between the pressure drop along a length of such a pipe and the rate of flow. In investigations of this kind it is important that there shall be sufficient length of pipe upstream of the experimental portion for the boundary layer to have spread to the axis. The flow is then in the condition that we may term normal pipe flow, and results obtained from one pipe will be general for normal flow along all other pipes of similar smoothness. The requisite upstream length seems to be about 100 pipe diameters for the highest accuracy, but considerably shorter lengths can be used without affecting the results seriously, particularly if a bell-mouthed entry is provided to reduce inlet disturbances. In general, the higher the Reynolds number of flow the greater the inlet length required, because the boundary layer becomes thinner as the Reynolds number increases.

Fig. 6 gives the results of the classical experiments of Stanton and Pannell² on the resistance of smooth straight pipes of drawn brass. This curve is in substantial agreement with the results of other important researches.* Although Stanton and Pannell used pipes of diameter ranging from 0.12 to 5 in., speeds of flow from 1 to 170 ft. per second, and three different

* In particular, the work of Nikuradse.³

fluids—air, water, and thick oil—they found that, when the results were plotted in the form of $F/\rho v^2$ against the Reynolds number, all the observations, irrespective of the fluid, the size of pipe, or the speed, could be represented by the right-hand of the two full curves shown in Fig. 6, provided that the Reynolds number was above about 3,000, i.e., in the condition of turbulent flow. In no case was the departure from the mean curve

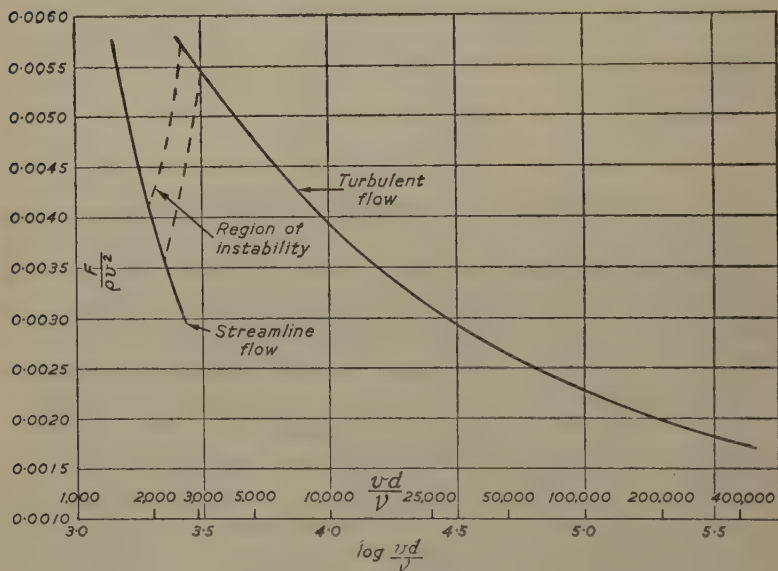


FIG. 6.—Resistance of smooth circular pipes.

greater than could be accounted for by the limits of the experimental accuracy. Similarly, for streamline flow at Reynolds numbers below the critical, the results could again be represented by a single curve—the left-hand curve in Fig. 6. The curves for the two different régimes were connected by a band of instability marking the transition from one type of flow to the other; within this band the observations were widely scattered between the two curves.

From an analysis of the data given by Stanton and Pannell, as well as those of other investigators, Lees⁴ proposed the

following empirical formula for the resistance per unit area in turbulent flow :

$$\frac{F}{\rho v^2} = \left\{ 0.0009 + 0.0763 \left(\frac{v}{vd} \right)^{0.35} \right\}^* \quad . \quad . \quad . \quad (4)$$

whence, by the use of equation (3), we get the following expression for the static pressure drop :

$$p_A - p_B = \frac{\rho v^2 l}{d} \left\{ 0.0036 + 0.305 \left(\frac{v}{vd} \right)^{0.35} \right\} \quad . \quad . \quad . \quad (5)$$

Using equation (5), we can calculate the pressure drop in smooth straight pipes for any speed of flow above the critical. It will be noted that the resistance, or the pressure drop per unit length, does not vary as the square of the speed. If it did, the quantity within the brackets $\{ \}$ would have a constant value, and the curve in Fig. 6 would be a straight line parallel to the axis of vd/v . In practice, it is sometimes convenient to assume that the resistance varies as some power n of the speed, i.e., that $F = kv^n$. For such purposes, sufficient accuracy will be obtained by using a formula due to Lees ⁴ for calculating n , namely

$$n = 2 - \frac{29.7}{85 + \left(\frac{vd}{v} \right)^{0.35}}.$$

Between $vd/v = 3,000$ (about the beginning of turbulent flow) and $vd/v = 400,000$, n varies from 1.71 to 1.83.

These results are strictly applicable only to smooth-walled pipes. It is not easy to define smoothness ; later work has

* According to modern boundary layer theory (see *Modern Developments in Fluid Dynamics* (1938), pp. 331 (Vol. II) *et seq.*), the resistance and the Reynolds number (R) should be related by an equation of the form

$$\left(\frac{F}{\frac{1}{2}\rho v^2} \right)^{-\frac{1}{2}} = A + B \log \left\{ R. \left(\frac{F}{\frac{1}{2}\rho v^2} \right)^{\frac{1}{2}} \right\} \quad . \quad . \quad . \quad (4a)$$

Analysis of the results of Stanton and Pannell and of Nikuradse has confirmed the theoretical form of the expression, and has shown that A and B are equal to -0.40 and 4.00 respectively. Putting these values in equation (4a), we obtain a formula connecting F and R , which is probably more accurate than that of Lees quoted in the text (equation (4)). In practice, it is best used to plot a curve of F against R by taking a number of arbitrary values of F within the range indicated by Fig. 6 and calculating R for each. Once plotted, this curve can be used to read off F for any given value of R in a smooth straight pipe.

shown that what can be considered smooth for a certain Reynolds number will be rough at higher Reynolds numbers. Stanton and Pannell used smooth drawn brass pipes in their work, but their results can probably be applied as a basis for resistance estimations in practice to all pipes that feel smooth to the touch, provided, of course, that there is an entry length of at least 40 diameters upstream of the length of pipe under consideration.

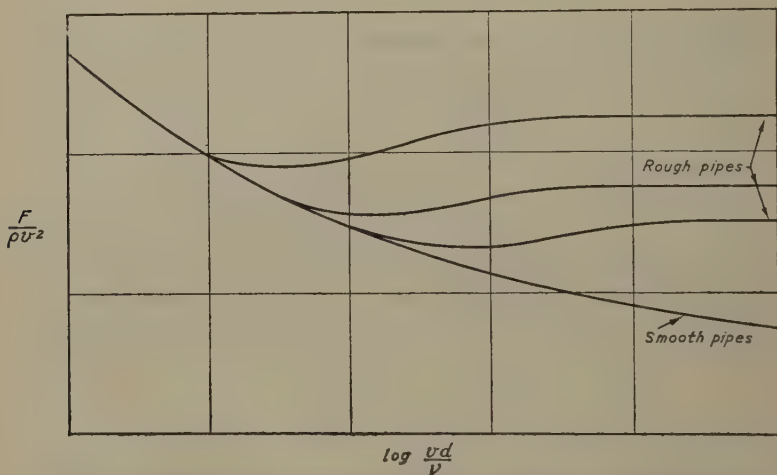


FIG. 7.—Resistance for turbulent flow in rough and smooth pipes.

Some experiments carried out by Nikuradse ⁵ in the turbulent range, with closely-spaced roughnesses formed by grains of sand stuck to the inside of the pipe, show the effect of different degrees of roughness, and give an indication of the limiting size of excrescence that can be permitted at different Reynolds numbers without causing a "smooth" pipe to become "rough." The effect of roughness is to cause the descending curve for turbulent flow in Fig. 6 to rise and approach a horizontal line, showing that the resistance tends to vary as the square of the speed. Nikuradse's results for different degrees of roughness formed a family of curves such as those sketched in Fig. 7. The values of the resistance all fell on the curve for smooth pipes at the lower Reynolds numbers, but departed from it as the rate of flow increased; the greater the roughness, the earlier the

departure from the smooth-pipe law and the higher the ultimate value of the resistance.

According to Nikuradse's results, and for the type of roughness he used, a pipe can be regarded as smooth provided that

the quantity $\frac{\epsilon\sqrt{F/\rho}}{v}$ (where ϵ denotes the average height of

the sand-grains) is less than 4. Therefore, if we equate this expression to 4, and for F we substitute its value from equation (4), we get a relation between the limiting size of excrescence ϵ beyond which the resistance will cease to follow the smooth-pipe law, and the rate of flow and pipe diameter. In this way we find that, for air at ordinary temperatures and pressures, the limiting size of excrescence for a 1-ft. diameter pipe is about 0.015 in. at a mean rate of flow of 10 ft. per second, and about 0.002 in. when the mean rate of flow is 100 ft. per second. Obviously, in smaller pipes the maximum allowable size of

excrescence is less, and vice versa. The value of $\frac{\epsilon\sqrt{F/\rho}}{v}$ of

about 4 marks the stage at which the curves of $F/\rho v^2$ shown in Fig. 7 begin to rise above the smooth-pipe curve. At a value

of $\frac{\epsilon\sqrt{F/\rho}}{v}$ of about 100 the curves become horizontal and

continue so for all higher values. The resistance then varies as the square of the speed, its actual value depending upon the degree of roughness, i.e., the size of the excrescences.

It should be noted that these remarks apply only to roughnesses consisting of closely spaced excrescences; the effects of a few isolated excrescences would, of course, be very much less.

Velocity Distribution Across a Section of a Smooth Circular Pipe.—It was stated on page 21 that, when normal pipe flow had been established at a distance sufficiently far from the mouth for inlet disturbances to be damped out, the velocity distribution across the section was the same at all sections farther downstream, and remained unaltered provided the rate of flow remained constant. This was a particular case

of the general statement which can be made now that the meaning of the Reynolds number has been explained: the velocity distribution in fully developed pipe flow depends only on the Reynolds number, i.e., if we restrict ourselves to a particular fluid at constant temperature and pressure, only on the product vd . For example, if the velocity at a certain radius r of a pipe along which air is flowing is, say, half the velocity at the axis, then it will be half this velocity at the same radius at all sections farther downstream. Moreover, for air at the same temperature and pressure, the velocity in a pipe double the diameter of the first will be half the velocity at the axis at the same fraction of the full pipe radius as in the first case, *provided that the value of vd is the same, i.e., that the mean velocity is half that of the first case.*

This statement, which we can predict from theoretical considerations, has been confirmed by Stanton by some experiments in smooth pipes.⁶ His curve of velocity distribution for a Reynolds number of 5.2×10^4 is shown in Fig. 8, which is based on observations obtained in two pipes, one of 4.93 cm. diameter and the other 7.4 cm., at the appropriate rates of flow. The velocity

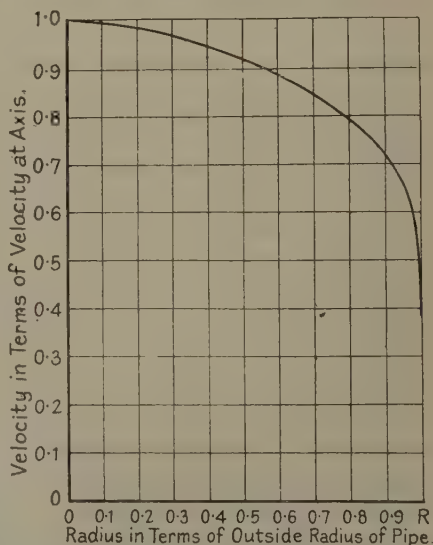


FIG. 8.—Distribution of velocity across a section of a smooth circular pipe.

distribution is approximately parabolic from the axis outwards to a radius of about $0.8R$, R being the full pipe radius. Thus, if v_r is the velocity at a radius r , where $r < 0.8R$, we can calculate v_r from the equation

$$v_r = v_a \left(1 - A \frac{r^2}{R^2} \right), \quad \dots \dots \dots (6)$$

where v_a is the velocity at the axis and A is some constant.

For the Reynolds number quoted, viz., 5.2×10^4 , Stanton's

results show that A is equal to about 0.35. Although, as we have just seen, A will vary with the Reynolds number, the variation is not great except at low Reynolds numbers; Stanton found no distinguishable difference in the value of A from another set of experiments made at a Reynolds number of 1.1×10^5 .

Ratio of the Mean Velocity to the Axial Velocity in Smooth Pipes.—It is often convenient in practice to determine the mean rate of flow in a smooth pipe from a single measurement of the velocity at the axis.* Since there is a characteristic velocity distribution for every Reynolds number, it follows that there must be a relation between v_a and the mean velocity v , which depends only on the Reynolds number, provided that we are working sufficiently far downstream for normal pipe flow, unaffected by inlet disturbances, to have become established. In 1914, Stanton and Pannell² determined the ratio v/v_a for smooth pipes for Reynolds numbers between about 2,400 and 100,000, and their results are given by the lower curve in Fig. 9. At the left, the Reynolds numbers are approaching that for the critical velocity at which the flow becomes streamline (see page 21); and we see that the value of v/v_a is tending towards $\frac{1}{2}$, which is its theoretical value for streamline flow (see page 37). Many years afterwards, Nikuradse³ published the results of similar measurements, also in smooth pipes, in which he extended the range of Reynolds numbers to considerably higher values. His results are shown in the upper curve of Fig. 9, and will be seen to be consistently higher than those of Stanton and Pannell. The reason for this discrepancy has not been established; Nikuradse made no attempt to explain it in his paper. All that can be said is that on several occasions when the present author has had cause to measure the ratio v/v_a , he has always obtained values that agree much better with those of Stanton and Pannell than with those of Nikuradse. Against this it must be remarked that the author's method of measuring velocities has been the same as that used by Stanton and Pannell, whereas Nikuradse used a method that had important points of difference. So far, however, neither method has been shown to be in error, and the

* See Chapter VIII.

difference in the results remains unexplained ; there appears to be a need for a third, independent, set of experiments.

Because of this uncertainty about the true value of v/v_a , the curve given in Fig. 9 cannot be used in practice, if accuracy within 2 or 3 per cent. is required, in order to estimate the mean

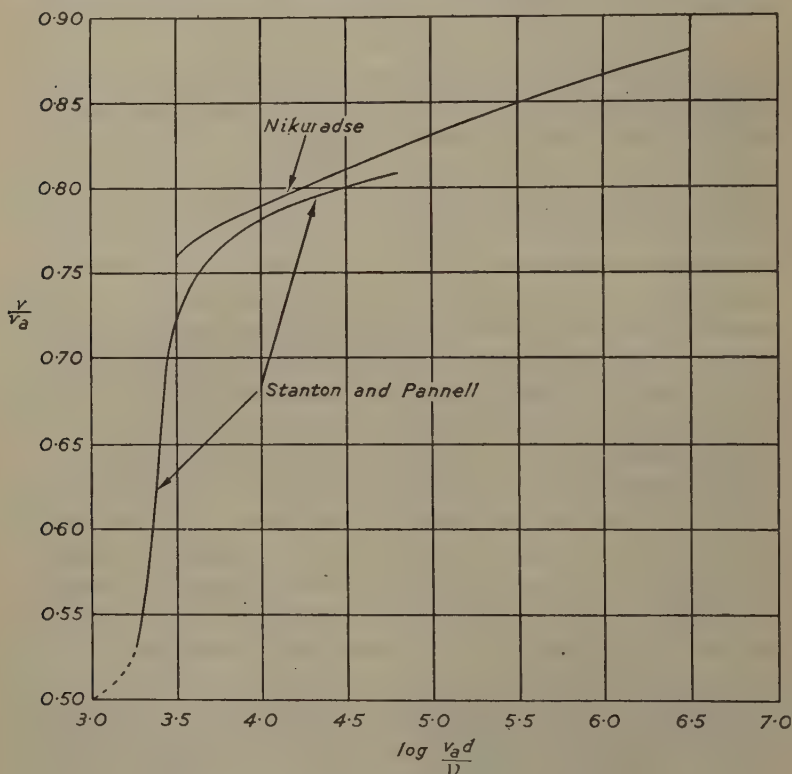


FIG. 9.—Relation between mean and axial velocity in smooth circular pipes.

rate of flow from a single reading at the axis. Anyone wishing to use this convenient method—practical details will be found in Chapter VIII—should make his own initial calibration, i.e., construct his own curve of v/v_a against rate of flow for subsequent use ; this will in any event be desirable in most practical cases, as there will generally be some uncertainty about possible disturbances at the section of measurement due to inlet conditions or other causes.

Velocity Distribution in Rough Pipes.—In rough pipes, for which the frictional resistance varied as the square of the velocity, Stanton⁶ found that the velocity distribution was parabolic up to a radius very nearly equal to the full radius of the pipe, and that this distribution did not, as it did in the case of the smooth pipes, vary with the value of vd/ν . The ratio v/v_a for these rough pipes had the value of about 0.76. It should be noted that the pipes in question were artificially roughened by cutting double screw-threads on the internal surfaces, in order that the frictional resistance should vary as the square of the speed—a point that was verified experimentally. It will be realised, therefore, that the pipes were appreciably rougher than those commonly used in practice.

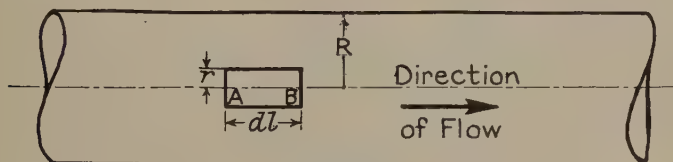


FIG. 10.

Stream-Line Flow in Pipes.—Let us imagine a fluid flowing along a straight parallel pipe under the influence of viscous forces and pressure differences only. If we consider any cylindrical element AB (Fig. 10) of the fluid, coaxial with the pipe, we see that there will be a viscous drag, which opposes the motion, acting on the cylindrical surface; and for the motion to proceed steadily this drag must be balanced by the difference in the pressures acting on the two ends A and B.

Let the length of the cylindrical element be dl and its radius r , and let R be the radius of the pipe.

Since the motion is assumed to be parallel to the walls of the pipe, there are no viscous forces acting radially, and therefore the pressure across any section perpendicular to the axis is uniform. We may therefore denote by p the pressure on the face A, and that on B by $p - dp$. The pressure difference maintaining the motion is therefore $\pi r^2 dp$, and this must be equal to the viscous retarding force acting on the curved surface of the cylinder, which, from the definition of the

coefficient of viscosity μ , is $2\pi r \cdot \mu dl \cdot dv/dr$, where v is the velocity at radius r .

Hence

$$\pi r^2 dp = - 2\pi r \mu dl \frac{dv}{dr},$$

or

$$\frac{dp}{dl} = - \frac{2\mu}{r} \frac{dv}{dr}.$$

Now the velocity will be some function of the radius only, since μ is constant and the flow is continuous and parallel to the walls of the pipe. In the above equation, therefore, the right-hand side is a function of r only; and since the pressure does not vary across the section it follows that dp/dl is a constant for

given conditions of flow. We may therefore write $\frac{p_1 - p_2}{l}$ for dp/dl , where p_1 and p_2 are now the pressures at two sections a finite distance l apart; i.e.,

$$\frac{p_1 - p_2}{l} r dr = - 2\mu dv,$$

and, on integration between $r = 0$ and $r = R$, we have

$$\frac{p_1 - p_2}{l} \cdot \frac{R^2}{2} = - 2\mu(v_r - v_a), \quad . \quad . \quad (7)$$

where v_a is the velocity at the axis.

If we assume that the velocity is zero at the walls,

$$v_r = 0,$$

and

$$\frac{p_1 - p_2}{l} = \frac{4\mu v_a}{R^2}. \quad . \quad . \quad . \quad (8)$$

The velocity at any radius r is obtained from (7) by substituting r for R and inserting the value of v_a from (8), and will be seen to be given by

$$v_r = \frac{p_1 - p_2}{4\mu l} (R^2 - r^2), \quad . \quad . \quad (9)$$

from which it appears that the velocity distribution is parabolic across the section.

The mean velocity is obtained as follows :

Imagine an annulus of the pipe at a radius r and of thickness dr ; the volume of fluid flowing through the annulus is given by $2\pi r.v_r.dr$, where v_r is obtained from (9). The total volume flowing is therefore

$$\begin{aligned} V &= \int_0^R 2\pi r v_r dr = \frac{\pi(p_1 - p_2)}{2\mu l} \int_0^R r(R^2 - r^2) dr \\ &= \frac{\pi(p_1 - p_2)}{2\mu l} \left[\frac{r^2 R^2}{2} - \frac{r^4}{4} \right]_0^R = \frac{\pi(p_1 - p_2) R^4}{8\mu l}. \end{aligned}$$

Now the mass of fluid passing in unit time is obviously constant; and if the pressure drop is small compared with either p_1 or p_2 (which are usually approximately atmospheric) we may, in the case of a gas, neglect changes of density and regard the volume flowing in unit time as constant. In the case of liquids, which are practically incompressible, this will be so even if the pressure drop is large. In either case, therefore, the volume flowing in unit time may be taken as constant and equal to $\pi R^2 v_m$, where v_m is the mean velocity of flow.

Hence

$$\pi R^2 v_m = \frac{\pi R^4 (p_1 - p_2)}{8\mu l},$$

or

$$v_m = \frac{p_1 - p_2}{8\mu l} R^2. \quad \dots \quad (10)$$

Now (8) may be written

$$v_a = \frac{p_1 - p_2}{4\mu l} R^2.$$

In streamline flow, therefore, the mean velocity of flow is half the velocity at the axis.

The Resistance of a Pipe of Varying Cross-Section.—It has been shown that the resistance of any given length of pipe is equal to the loss in energy experienced by unit volume of the fluid in passing along this length. When the pipe is parallel between the two sections considered and the velocity distribu-

tions at these sections are the same, we have seen that the resistance is equal to the difference between the static pressures at the sections ; but if the diameters of the two ends of the length of pipe are not the same, the resistance is equal to the difference between the mean energy of unit volume of the fluid at the two sections. The same is true for a parallel pipe in which the velocity distributions at the two ends are not identical. In the general case we have therefore to measure the mean energy per unit volume, i.e., the mean total head, at the two sections, and to subtract one from the other. This may be done as follows :

Consider any section of the pipe of radius R , and let v be the mean velocity at radius r through an annular element of width

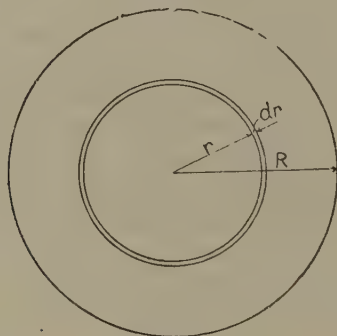


FIG. II.

dr . Also let p be the static pressure at this radius (p is taken above or below atmospheric pressure in the usual manner). The volume of fluid passing through the annulus in unit time is $2\pi r dr.v$; its mass is therefore $2\pi r dr.\rho v$, and its kinetic energy $2\pi r dr.\rho v.v^2/2$. The potential energy is $2\pi r dr.vp$. Hence the total energy is equal to $\pi r \rho v^3 dr + 2\pi r v p dr$. The total energy of the fluid flowing across the section per second is therefore

$$\pi \rho \int_0^R v^3 r dr + 2\pi \int_0^R p v r dr.$$

Also the total volume flowing per second is $2\pi \int_0^R v r dr$, so that the

energy per unit volume, or the mean total head, is equal to

$$\frac{1}{2}\rho \frac{\int_0^R v^3 r dr}{\int_0^R v r dr} + \frac{\int_0^R p v r dr}{\int_0^R v r dr}.$$

In order, therefore, to obtain the mean total head at a section, the values of the above integrals must be determined by taking sufficient measurements of v and p across the section. Thus, in

order to determine, for example, $\int_0^R p v r dr$, p and v must be

measured at a number of values of r ; a curve of the product $p v r$ is then plotted on a base of r , and its area gives the value of the integral. The units for the various quantities must be consistent: thus if v is in feet per second, p must be in pounds per square foot ($= 5.2 \times$ inches of water), r in feet, and ρ in pounds per cubic foot/32.2. The resulting total head will be in pounds per square foot. The total heads at each of the two sections forming the ends of the length of piping whose resistance is required must be obtained in this way, and their difference will be the resistance.

Usually it will be found that, although the pipe diameters at the two sections may be different, the sections themselves nevertheless form parts of parallel-walled pipes, so that the flow is axial and p may be taken as constant across the section.

The integral $\int_0^R p v r dr$ is then equivalent to $p \int_0^R v r dr$, so that the

expression for the mean total head at a section reduces to

$$\frac{1}{2}\rho \frac{\int_0^R v^3 r dr}{\int_0^R v r dr} + p. \quad \dots \quad (II)$$

It may be noted that if v were constant across the section it could be taken outside the integrals in which it occurs, and the expression would reduce to its familiar form of $\frac{1}{2}\rho v^2 + p$.

Now in fan testing and similar work, resistance has frequently to be determined, in addition to the volume of air flowing per second. The latter is deduced by measuring the mean velocity v_m across the section by one of the methods described in subsequent chapters. It is then often assumed that the mean total head at the section, for the purpose of computing resistance, is equal to $\frac{1}{2}\rho v_m^2 + p$. It will be apparent that this statement is incorrect, and in view of its wide acceptance it is important to examine the magnitude of the possible errors involved.

As a concrete example, we may take the case of the smooth parallel pipe for which Stanton measured the velocity distribution as given in Fig. 8. The following table shows the values from which this curve was obtained; here v is the velocity at radius r , v_a is the velocity at the axis, and R is the internal

TABLE 2

r/R	v/v_a	r/R	v/v_a
0	1.000	0.799	0.789
0.183	0.987	0.852	0.753
0.284	0.967	0.905	0.711
0.389	0.944	0.925	0.688
0.491	0.915	0.954	0.645
0.596	0.883	0.978	0.596
0.698	0.838	0.990	0.388

radius of the pipe. The integrals occurring in (11) can now be evaluated graphically thus :

For $\int_0^R v^3 r dr$ we may substitute its equivalent

$$v_a^3 R^2 \int_0^1 \left(\frac{v}{v_a}\right)^3 \frac{r}{R} \frac{dr}{R};$$

similarly $\int_0^R v r dr$ may be replaced by

$$v_a R^2 \int_0^1 \frac{v}{v_a} \frac{r}{R} \frac{dr}{R}.$$

If we now plot on a base of r/R the values of $(v/v_a)^3 r/R$ and $v/v_a \cdot r/R$ obtained from the above table, we shall derive two curves whose areas are respectively the two integrals required. Thus we shall find that

$$v_a^3 R^2 \int_0^1 \left(\frac{v}{v_a}\right)^3 \frac{r}{R} \frac{dr}{R} = 0.291 v_a^3 R^2, \quad (12a)$$

and

$$v_a R^2 \int_0^1 \frac{v}{v_a} \frac{r}{R} \frac{dr}{R} = 0.405 v_a R^2, \quad (12b)$$

so that the true mean total head, from (11), becomes

$$\frac{1}{2} \rho \times 0.718 v_a^2 + p.$$

Let us compare this with the mean total head incorrectly, but frequently, stated to be $\frac{1}{2} \rho v_m^2 + p$. In this case

$$\begin{aligned} v_m &= \frac{2\pi \int_0^R v r dr}{\pi R^2} = \frac{2v_a R^2 \int_0^1 \frac{v}{v_a} \cdot \frac{r}{R} \cdot \frac{dr}{R}}{R^2} \\ &= 0.81 v_a \text{ (from (12b))}. \end{aligned}$$

Hence

$$v_m^2 = 0.656 v_a^2,$$

and the mean total head calculated in this way is

$$\frac{1}{2} \rho \times 0.656 v_a^2 + p.$$

The true mean total head therefore exceeds that calculated by the usual method by $0.062 \times \frac{1}{2} \rho v_a^2$, i.e., by $0.095 \times \frac{1}{2} \rho v_m^2$. Now the head corresponding to the mean velocity v_m is, of course, $\frac{1}{2} \rho v_m^2$; so that we see that the error incurred in estimating the mean total head by this incorrect method is in this particular case $9\frac{1}{2}$ per cent. of the head corresponding to the mean velocity. The smaller the velocity at a section in relation to the static pressure, the less will be the error on total head. If we take the case, which is quite common in practice, in which the mean velocity head is about equal to the static pressure at a

section, we see that the mean total head deduced by the usual method may be about 5 per cent. too low—an error that is by no means negligible.

Methods of measuring the true mean total head at any section in practice are described in Chapter V.

The Resistance of Bends and Sharp Corners.—Any appreciable departure from straightness in a pipe line is accompanied by an increase in resistance, which becomes larger the sharper the bend. When air flows round a bend, centrifugal forces are set up, and these, combined with the effects of viscosity, cause the flow to leave the pipe walls at certain places. Thus, at and slightly upstream of the bend, there is a continuous increase of pressure across the section from the inside towards the outside of the bend ; this is due to the centrifugal action. At the outer wall (as at the inner) the air is already retarded by friction and its kinetic energy is reduced. If the curvature of the bend is sharp, the pressure at the outer wall may be raised considerably, and the energy of the moving air may not be sufficient to enable it to force its way into this region of high pressure. In that case, the air particles at the outer wall are brought to rest, or may even move in an upstream direction against the main flow, which then separates from the wall leaving a dead-air region in which there is more or less intense eddying motion. It is the energy lost in this eddying movement that accounts for the increase of resistance, which occurs whenever a stream separates from a solid surface along which it has previously been flowing. The purpose of “streamlining,” which plays so important a part in the design of aircraft, is to prevent such separation from taking place.

Separation may also occur at the inner wall slightly downstream of a sharp bend. This is due to the fact that the flow cannot change its direction so abruptly as to follow the bend, and it gives rise to a further increase of resistance.

A large amount of work has been done on the resistance of bends, but it is difficult to correlate the results because many of the investigators have not realised all the factors involved, so that their experimental conditions are not comparable. The biggest source of discrepancy, apart from differences of Reynolds number and of surface roughness, is the different lengths of

straight pipe upstream and downstream of the bend, particularly the latter. Experiments have shown that the disturbance produced by a bend may persist for more than 50 pipe diameters downstream, and that the loss of head due to the bend, as measured, say, 5 diameters downstream of the bend, may be less than half that measured 50 diameters downstream.

An idea of the magnitude of the resistance of 90° bends may be gathered from Curve 1,* Fig. 12, which shows the results obtained from a series of experiments⁷ with iron pipes, in

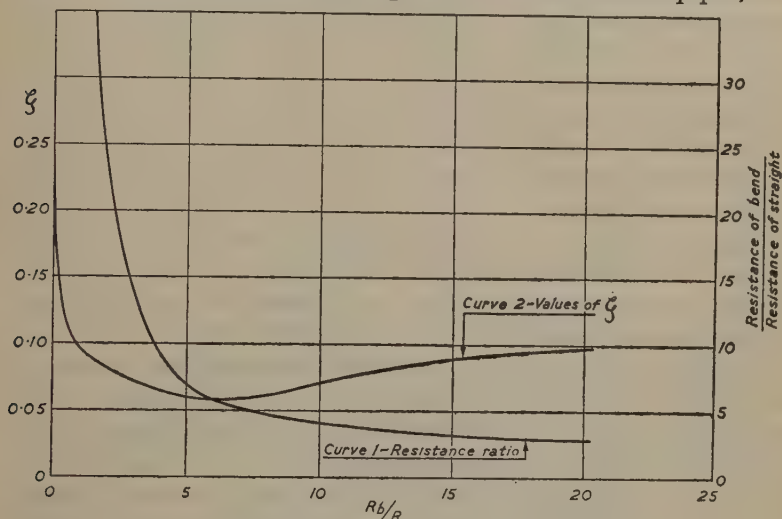


FIG. 12.—Resistance of bends.

which care was taken to have sufficiently long inlet and exit lengths to include the whole loss. The curve shows the ratio of the resistance of the length of pipe including the bend to the resistance of a length of straight pipe of the same axial length, this ratio being plotted against different values of the radius of the axis of the bend expressed in terms of the pipe radius. We see from this diagram how rapidly the resistance rises as the bend becomes sharper. Thus, when the mean radius of the bend R_b is twice the pipe radius R , the resistance of the pipe including the bend is almost 25 times that of the same length of straight pipe.

When the radius of the 90° bend becomes zero, we have a 90°

* The right-hand scale of ordinates should be used with Curve 1.

elbow, and the resistance is still higher. We may take as typical a result from some experiments with a 90° elbow, which showed that the resistance was about 45 times that of an equivalent length of straight duct. It should be noted, however, that the measurements were taken fairly close to the elbow, so that the total loss of head due to the elbow may not have been measured: a still higher value for the resistance might therefore have been found from measurements made farther up- and downstream.

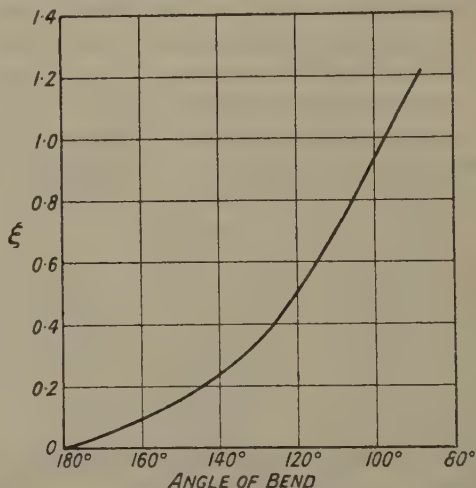


FIG. 13.—Resistance of elbows.

For elbows with larger angles than 90° between their branches, the resistance coefficient ξ is less, as will be seen from a typical curve given in Fig. 13, which is a mean of the results of a number of investigators for smooth pipes. In this diagram, ξ is the excess of the loss of head due to the bend over that of the same axial length of straight pipe. Again these figures must only be taken as an indication of the order of magnitude of the effects of bends.

Reduction of the Resistance of Bends and Elbows.—

Unfortunately, the sharp right-angled bend (90° elbow) is much used in pipe lines, including ventilation systems, because of the convenient way in which it connects lengths of pipes running along neighbouring walls of a building. It will be clear from the foregoing remarks that a system that includes a large number of such bends will be subject to very serious losses of energy; and that if the resistance of these bends could be appreciably reduced the running costs would be correspondingly lower, as would also the installation costs, because a less powerful fan (or fans) would be required.

As might be expected, rounding the corners of the elbow reduces the resistance when properly carried out ; this will already have been gathered from Fig. 12. It is, however, desirable that both inner and outer edges shall be rounded together. Experiments have shown, for instance, that rounding the outer corner while the inner is left sharp, so far from having a beneficial effect, actually increases the resistance if the radius of the corner is made greater than about one pipe diameter, while smaller radii lead to no appreciable change in the resistance. This is because the pipe area at the bend is reduced when only the outer corner is rounded. In fact, if the radius of the outer corner were made equal to $(2 + \sqrt{2})d$, where d is the pipe diameter, the inner corner remaining sharp, the outer wall would intersect the inner corner and the area at the bend would be zero.

Thus, if the outer corner only is rounded, there is a constriction at the bend followed by an expansion after the bend where the full pipe area is again reached. But, as we shall see later, when we discuss diffusers, a moving stream of air cannot expand at more than a certain rate : if the conditions demand too rapid an expansion, the flow simply leaves the walls, with the consequent increase of resistance we have already noted. This seems to happen when the expansion following the bend is greater than that obtained when the radius of the outer corner is more than about one pipe diameter while the inner corner remains sharp.

If the inner corner only of a 90° elbow is rounded off, there is a large decrease in the resistance, because the sudden change of direction at the inner walls is relieved. Again, however, this causes an expansion of area, this time at the bend itself ; but now the drop in resistance due to the elimination of the sharp corner more than compensates for the effect of the expansion, provided that the radius of the inner corner does not exceed about $1\frac{1}{2}$ pipe diameters. With this radius to its inner corner, and a sharp outer corner, the resistance of a 90° elbow is, according to one set of experiments, about one-fifth of the resistance when both corners are sharp. It should be remarked, however, that this reduction appears large when compared with the results of other experiments in which both corners were rounded, and is an instance of the difficulty already noted of comparing

the work of different investigators. For larger radii of the inner corner, the resistance of the elbow increases again.

There is no change of cross-sectional area of the pipe, and so no expansion at or after the bend, when both corners are rounded off together by radii struck from a common centre; and experiments show that this is the type of rounded bend that has about the minimum resistance. As the mean radius of the bend R_b is increased the resistance drops, but does not continue to do so indefinitely. This is shown by Curve 2⁷ in Fig. 12, in which the resistance coefficient ξ of the bend (see left-hand ordinate scale) is plotted against R_b/R , where R is the pipe radius.* The resistance is a minimum when the mean radius of the bend is 6 or 7 times the pipe radius. The reason for the existence of an optimum value of R_b is that, as R_b becomes larger, the total length, and so, also, the surface area, of the bend increases. Hence the wall friction increases, and there comes a stage at which the increase in friction balances the beneficial effect of increasing R_b ; any increase of R_b beyond that stage leads to a rise in the resistance. On the other hand, Curve 1 of Fig. 12, which has already been discussed, shows that the ratio of the resistance of the bend to the frictional resistance of the same length of straight pipe falls continually as R_b is increased.

The Effect of Guide Vanes.—The reduction of the resistance of elbows by rounding them off is an expedient which, although effective, is nevertheless often regarded with disfavour, as for example, when it is desired to have ventilation ducts following the walls of buildings without projecting at the corners. A very effective alternative method is available, which permits the retention of sharp corners but is somewhat more expensive to instal. It consists of the insertion at the elbow of a bank of guide vanes (see Fig. 14 (a)), which lead the air smoothly round the corner. A well-designed system of vanes reduces the resistance of an elbow to little more than the frictional resistance of the vanes themselves. In order to achieve the maximum reduction, the vanes should be of aero-

* The coefficient ξ is here the total pressure drop due to the bend divided by 8 times the velocity head. For a straight pipe, this coefficient would be equivalent to $\frac{Fl}{\rho v^2 d}$, where F is given by equation (3), page 27.

foil section, somewhat resembling aeroplane wings, but a considerable reduction of resistance is obtained by using simpler vanes made of thin sheet metal bent into the form of 90° circular arcs. Such vanes may therefore often be preferred to the more expansive aerofoil type. Obviously, guide vanes are most easily inserted in rectangular ducts, but they are effective also in circular pipes.

Some Russian experiments⁸ provide useful instructions on the number and spacing of the vanes. These experiments show that the lowest resistance is obtained when the inner and outer corners of the elbow are rounded equally, with a radius about one-quarter the pipe diameter. Six vanes are then sufficient, and they should be spaced so that the distances between the

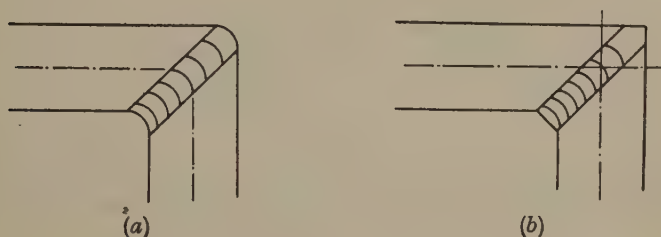


FIG. 14.

inner corner and the successive vanes are $0.135d$, $0.292d$, $0.471d$, $0.673d$, $0.898d$ and $1.145d$, where d is the diameter of the pipe, or the length of side in a square duct. With this arrangement, the vanes are spaced in an arithmetical progression, and the gap between the inner corner and the vane nearest to it is half that between the outer corner and the outer vane. If the inner and outer corners are not rounded, the arrangement shown in Fig. 14(b) is recommended. Here seven vanes are used, spaced at the following distances from the inner corner, which is chamfered off as shown: $0.118d$, $0.253d$, $0.404d$, $0.573d$, $0.759d$, $0.960d$, $1.180d$. The Russian figures show that the resistance of an elbow fitted with guide vanes in this way is from one-quarter to one-third of the resistance of the same bend without guide vanes.

Although this is still considerably above the purely frictional resistance, it does represent a valuable reduction. If the full

benefit of guide vanes is desired, aerofoil sections should be used, and the vanes should be equally spaced and about twice as numerous. Information on a suitable type will be found in the Russian journal already quoted or in a paper by Collar.⁹

Finally, it should be noted that the chords of the circular arc vanes should be inclined at an angle between 45° and 50° to the axis of the upstream arm of the bend, and the chord of the vanes should be about one-quarter to one-third of the pipe diameter.

Diffusers and Sudden Expansions.—The steady, linear flow of a perfect, frictionless fluid follows Bernoulli's law * that the total head is constant. Consider the flow of such a fluid through a diffuser (i.e., expanding cone) as shown in Fig. 15.

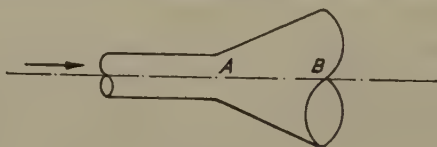


FIG. 15.

The total head at the inlet A is equal to that at the outlet B. Thus :

$$p_A + \frac{1}{2}\rho v_A^2 = p_B + \frac{1}{2}\rho v_B^2,$$

where p and v denote static pressure and velocity respectively, and since v_B is less than v_A because the quantity flowing is the same at both sections, it follows that the static pressure p_B at B is greater than that at A. An expansion of area is thus accompanied by an increase of static pressure. Air flowing into a region of higher pressure has to do work ; it does this work at the expense of its kinetic energy, which is represented by the velocity head $\frac{1}{2}\rho v^2$. In the absence of friction, the kinetic energy is just sufficient to overcome any increase in static pressure produced by the diffuser until the velocity is reduced to zero, i.e., until the outlet area of the diffuser becomes infinitely great.

But friction introduces an additional resistance against which the air has to do work, and so, in the case of a real fluid, a stage is reached, while the area is still finite, when the kinetic energy of the slowly moving particles in the boundary layer near the

* See Chapter II.

walls is insufficient to overcome both the friction and the rising pressure. The velocity at the walls is then reduced to zero, and the flow separates from the walls with the resulting increase of resistance already noted. Usually the flow is turbulent, and the retarded boundary layer receives appreciable energy from the faster moving particles nearer the axis of the pipe. If the rate at which work has to be done against pressure and friction is less than the rate at which this additional energy is received, no separation takes place. This explains the experimentally proven fact that there is a limiting angle for a diffuser below which no separation occurs. Numerous investigations have shown that this angle is about 6° or 7° for a conical diffuser, the angle being defined as the total angle, i.e., twice the inclination of the side of the cone to the axis of the pipe. If the angle is greater than this, the diffuser becomes inefficient.

A diffuser may be used either as a transition piece joining a small pipe to a larger one ; or else at the outlet from a pipe system where it discharges into atmosphere, in order to recover the velocity head, which would otherwise represent so much waste energy. It does this, as we have already shown, by converting the velocity head into pressure. In either case, it is generally desirable to have as short a length of diffuser as possible in order to save space, but the small limiting angle of the diffuser means that a long length will be necessary.

This conflict of requirements has led to numerous attempts to improve the efficiency of diffusers, and the mass of data that has been accumulated is so great that it is impossible to do more here than to summarise it very briefly. Readers desiring more detailed information are referred to a valuable paper by Patterson.¹⁰

The angle of a diffuser in a rectangular duct can be about doubled if one pair of walls diverges and the other remains parallel. The efficiency of a diffuser in which the rate of divergence does not exceed the limiting angle depends also to some extent on the length of duct that precedes and follows it. Obviously, since the breakdown in flow in a diffuser is a boundary layer phenomenon, the thinner the boundary layer at inlet the longer separation is delayed and the higher the efficiency. The inlet duct should therefore be as short as possible. On the other hand, it is found that a length of duct following the

diffuser improves the efficiency, because the conversion of velocity head to pressure is complete not at the outlet of the cone but at some distance downstream. Patterson states that if the inlet length is short the length of outlet duct should be about four times the maximum width or diameter of the diffuser, and with a long inlet the length of outlet should be about 50 per cent. greater.

Experiments show that, in straight-sided diffusers whose divergence exceeds the limiting angle, separation of the flow always occurs near the inlet. This is because the pressure gradient (i.e., the rate of rise of pressure) along the diffuser is steepest there. Hence Gibson and others have investigated the effect of curving the walls so as to reduce the rate of expansion near the inlet, and have found that this flared type of expansion appreciably increases the efficiency. Gibson¹¹ concluded that the best results were given by the form of diffuser shown in Fig. 16, starting with a gentle curve and terminating in a

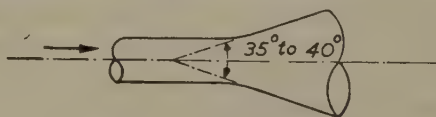


FIG. 16.—Flared diffuser.

straight-sided cone whose total angle should not exceed 35° to 40°. Patterson gives details of these and other experiments and quotes Gibson's equation for the curved portion of the wall.

Other methods of improving the efficiency are the insertion of guide vanes (which take the form of annular rings in conical diffusers) near the inlet, as in Fig. 17; suction of the boundary

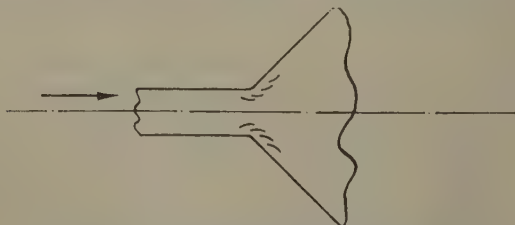


FIG. 17.—Diffuser with guide vanes.

layer in the diffuser; and imposing a certain type of rotation on the air before it reaches the diffuser. The method of suction is interesting, but probably of little practical use except in special

cases. There is, however, no doubt about its efficacy. One or two circumferential slits are made in the walls of the diffuser about one-third of the length from the inlet, and the retarded air in the thickened boundary layer is removed by applying suction to these slits from a separate suction pump or fan. The air thus removed is replaced by air from the central part of the diffuser having higher kinetic energy, and separation of the flow is avoided. Large increases in the efficiency of the diffuser are thereby possible for angles of expansion as high as 50° or more, even when account is taken of the power required for suction.

The method of imposing rotation is also interesting, and is quite simple to carry out. Fixed radial deflector vanes are inserted into the pipe ahead of the diffuser, so designed that the rotational velocity component they produce is proportional to the radius. That is, the air in the pipe, after it leaves the deflectors, rotates as if it were a solid body, the tangential velocity being zero at the axis and a maximum at the walls. Each deflector is twisted along its length somewhat like an aeroplane propeller, the angle of twist increasing outwards. Patterson gives full details for the design of a simple set of radial vanes, and the method is well worth applying where it is essential for diffuser losses to be reduced. It should be noted, however, that the rotation should be of the "rigid-body" type. A propeller-type fan in a duct, for example, produces a type of rotation such that the rotational component at a given radius is approximately inversely proportional to the radius. This type of rotation is found to improve the efficiency of diffusers only when the rotation is small.

REFERENCES

1. Reynolds, *Phil. Trans. Roy. Soc., A*, **174** (1883), p. 935.
2. Stanton and Pannell, *Phil. Trans. Roy. Soc., A*, **214** (1914), p. 199.
3. Nikuradse, *Verein deutscher Ingenieure, Forschungsheft No. 356* (1932).
4. Lees, *Proc. Roy. Soc., A*, **91** (1915), p. 46.
5. Nikuradse, *Verein deutscher Ingenieure, Forschungsheft No. 361* (1933).
6. Stanton, *Proc. Roy. Soc., A*, **85** (1911), p. 366.
7. Davis, *Trans. Amer. Soc. Civil Engineers*, **62** (1909), p. 97.
8. Abramovitch, *Transactions of the Central Aerodynamic Institute of Professor Joukowski*, No. 211 (1935).
9. Collar, *Reports and Memoranda of the Aeronautical Research Committee*, No. 1768 (1937).
10. Patterson, *Aircraft Engineering*, **10** (1938), p. 267.
11. Gibson, *Proc. Roy. Soc., A*, **83** (1910), p. 366; *Trans. Roy. Soc. Edinburgh*, **48** (1911-13), p. 97. See also *Hydraulics and its Applications* (1945), p. 92.

CHAPTER IV

THE DESIGN AND CHARACTERISTICS OF PITOT AND STATIC TUBES

It was shown in Chapter II that the velocity head can be obtained as a difference of two pressure observations—the total head and the static pressure—of which the former can be accurately determined by means of an open-ended tube facing the stream. The facing tube is, in fact, universally adopted for the measurement of total head, the tube being generally known as the pitot tube after the French scientist Pitot, who appears to have been the first to use it for this purpose. There is abundant experimental evidence to confirm that the pressure indicated by the facing tube is accurately equal to the total head, and that the form of the tube may be varied within wide limits without sensibly affecting the observed pressure. It is, in fact, true to say that on the surface of any solid body, no matter what its shape, immersed in a stream of fluid, there is some point (usually the most forward point) at which the pressure acting is the total head; hence the latter can be measured by arranging a pressure orifice at that point. No such latitude in shape is possible in the design of a tube to measure static pressure accurately: for this purpose it is essential to observe the condition we have already stated, namely that the flow past the static orifice must be as far as possible the same, both in speed and direction, as it was in the free stream before the measuring instrument was introduced.

On the basis of these requirements, the development of static-pressure heads has proceeded upon the general lines of the instrument shown in Fig. 18.

The head of the tube, which is introduced into the air current, is bent at right-angles to face the stream, its walls being parallel to the direction of flow; and it terminates in a conical plug of gradual taper merging smoothly into the tube, without a gap or change in external diameter. This form of head deflects the air only slightly, and at a short distance back from the tapered

portion, where the flow is once more sensibly parallel to the tube, a number of small holes is drilled. The other end of the tube is connected to a manometer ; and if the head and the

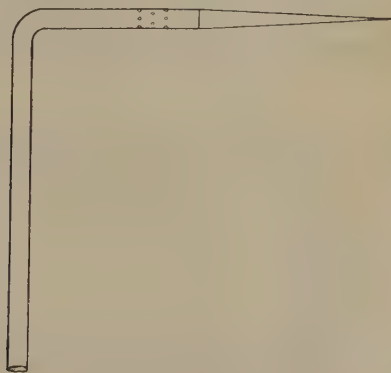


FIG. 18.—Static tube.

portion at right angles to it, generally called the stem, are suitably proportioned (see below), the true static pressure will be observed. In drilling the holes in the wall of the head it is essential that no burrs be left on the outside to disturb the flow locally.

General Principles of Design : Pressure Distribution along Surface.—We see, then, that the problem of designing a satisfactory pressure-tube anemometer can be reduced to the problem of the design of the static side, which, in turn, resolves itself into firstly, the design of a suitably shaped tube or other body to be immersed in the fluid ; and, secondly, the determination of the position of the static orifices in this body. We shall therefore find it helpful, before proceeding further, to consider from a general standpoint the salient features of the pressure distribution around solid bodies immersed in a current of fluid. Since, as we have already seen, one fundamental requirement is that the flow must be disturbed as little as possible, we shall confine ourselves to consideration of the type of body generally known as “streamline.”

Suppose that a solid of revolution of this type (Fig. 19) is immersed in a stream of incompressible fluid with its axis along

the direction of flow, and that it is provided with means for enabling the pressures to be measured at a number of points along a generating line. We shall then obtain a pressure distribution curve of the type sketched in the lower part of Fig. 19. At the foremost point, the pressure has a maximum positive value equal to the total head; as we proceed back

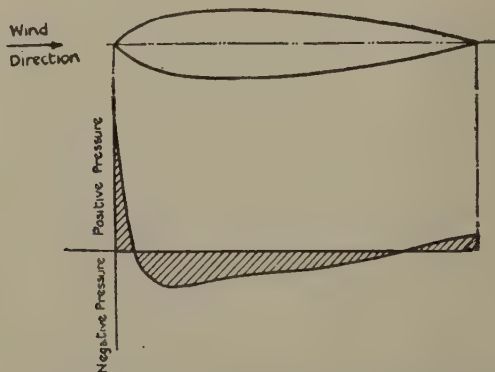


FIG. 19.—Distribution of pressure on streamline body.

along the surface, the pressure falls rapidly through zero * to a maximum negative value well forward on the body; and thereafter it rises slowly, generally passing again through zero to reach a small positive value at the tail.

As in the case of the facing tube (see page 14), the pressure at the forward point, where a filament of fluid is brought to rest, is equal to the velocity head, $\frac{1}{2}\rho v^2$, plus p_o , the static pressure in the fluid. The pressure acting at any other point on the surface of the body will be the local static pressure p_o , plus some fraction or multiple f (which may be either positive or negative) of the velocity head.† If, then, a tube from a small hole in the surface at the foremost point and another tube from a similar hole elsewhere on the surface are connected to opposite

* Remember that pressures are measured with reference to the prevailing atmospheric pressure.

† In Fig. 19, which relates to a typical streamline solid of revolution, the value of f is nowhere greater than 1, but in bodies of other shape, e.g., aeroplane-wing sections, f may reach a negative value considerably greater than 1.

sides of a differential manometer, the latter will indicate a pressure p given by

$$p = (1 - f) \cdot \frac{1}{2} \rho v^2 = K \cdot \frac{1}{2} \rho v^2. \quad (1)$$

If we determine the value of the factor K by an experimental calibration, we can therefore use such a streamline body, with the appropriate pressure connections, as a pressure-tube anemometer.

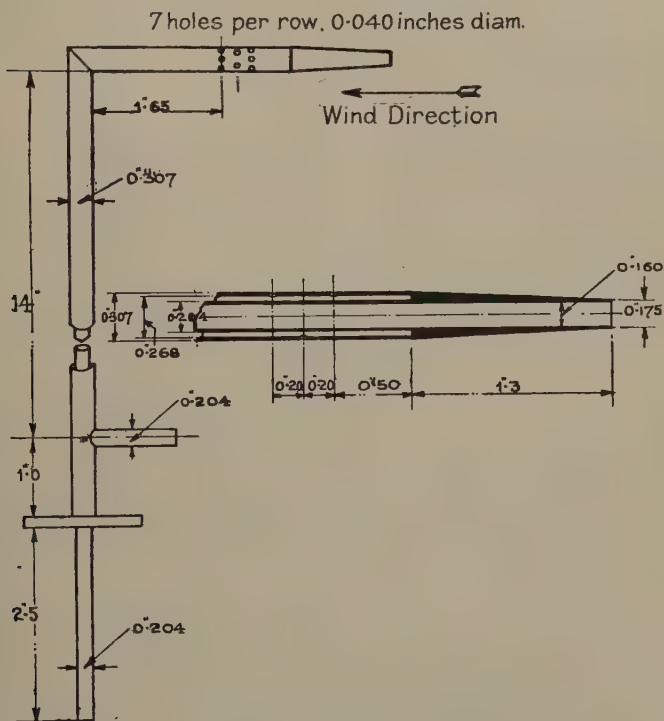


FIG. 20.—First N.P.L. standard pitot-static tube.

The Pitot-Static Tube.—The combined pitot-static tube, an early example of which is shown in Fig. 20,* is a special case of such a device, in which the value of f in equation (1) is

* The circular disc shown near the end of the stem in Fig. 20 was provided to simplify mounting the whole instrument in a wind tunnel; it is generally omitted, without effect on the factor of the instrument, in modern designs for use in pipes or ducts.

made to be zero. Hence, $K = 1$ and equation (1) reduces to $p = \frac{1}{2}\rho v^2$; i.e., the instrument reads the true velocity head.

It will be seen from Fig. 20 that the head of the combined pitot-static tube consists of two co-axial tubes: the inner tube faces the current and measures total head; the other tube is open to the stream only through the small static orifices a short distance back from the mouth. The head of the instrument is generally carried by a "stem" at right angles, which projects through the side of the pipe or duct in which measurements are being made and carries the pressure connections leading to the manometer.

The Characteristics of Pitot-Static Tubes.—In the absence of the stem, the pressure distribution along the surface of the head is similar in type to that shown in Fig. 19 for the streamline solid of revolution, but with the forward region of positive pressure relatively rather shorter. If the value of K in equation (1) is to be 1, in order that the true velocity head may be given by the differential pressure, the static orifices must be placed in a region where the pressure acting on the surface of the head is the true static pressure. The actual pressure distribution, and hence the correct position of the static orifices, must be expected to depend to some extent on the shape of the front of the head (the "nose"); and the matter is further complicated by the presence of the stem, which produces a pressure upstream of itself and so also affects the pressure at the static orifices.

The effect of these variables—the shape of the nose, the effect of the stem, and the position of the static orifices—was established by a fundamental investigation of the characteristics of static tubes carried out in 1926.¹ Observations were made with four differently shaped noses, three of them tapered as in Fig. 20, but of different degrees of taper, and one hemispherical; the pressure distribution along the parallel portion of the head was measured in each case, both without the stem and with the stem at different positions behind the static orifices. The general type of the observed pressure distribution over the parallel portion of the head in the absence of the stem, which was found to be almost independent of the shape of the nose, is shown by the lower curve of Fig. 21; the upper curve shows

the pressure produced by the stem at points on the tube upstream.* The zero of pressure for both curves is the true static pressure in the undisturbed stream. It will be seen that the pressure due to the flow around the nose is negative and of

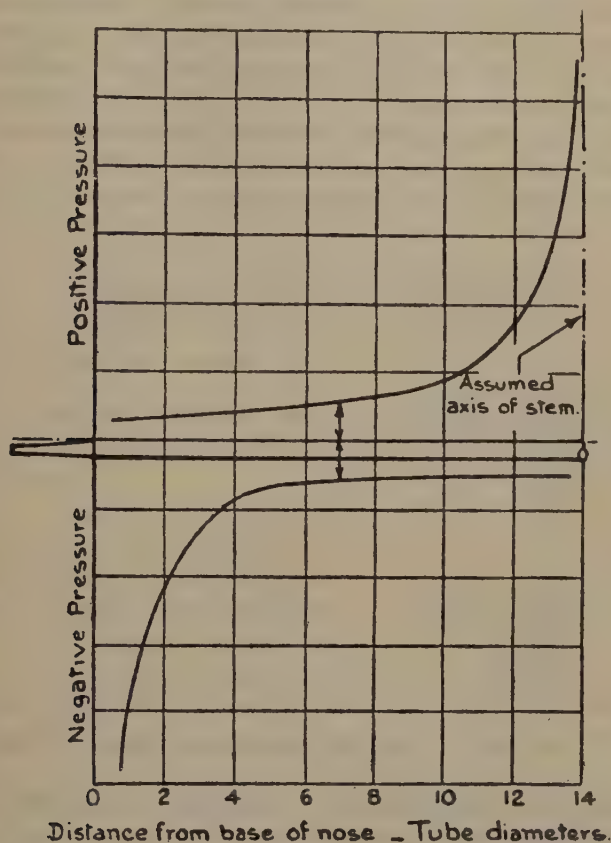


FIG. 21.

opposite sign to that due to the stem; the correct position for the static holes is that at which the ordinates of the two curves are equal in magnitude.

Obviously, this position of balance can be controlled by altering the distance between the stem and the base of the nose

* The ordinates of the two curves are not relatively to scale, being only approximately correct in this respect; the diagram is intended merely to illustrate what occurs.

where the parallel portion of the stem begins ; for each position of the stem there will be an intermediate position of the static holes at which the two opposing pressures will balance. As regards the suction due to the flow round the nose, it will be seen from Fig. 21 that the pressure gradient is steep at first, but becomes more gradual some 5 or 6 tube diameters back from the base of the nose. If the static orifices are in a region of steep pressure gradient, slight errors in manufacture may lead to appreciable differences in the value of K from one instrument to another ; on the other hand, if the static orifices are too far back, the head of the instrument will be undesirably long.

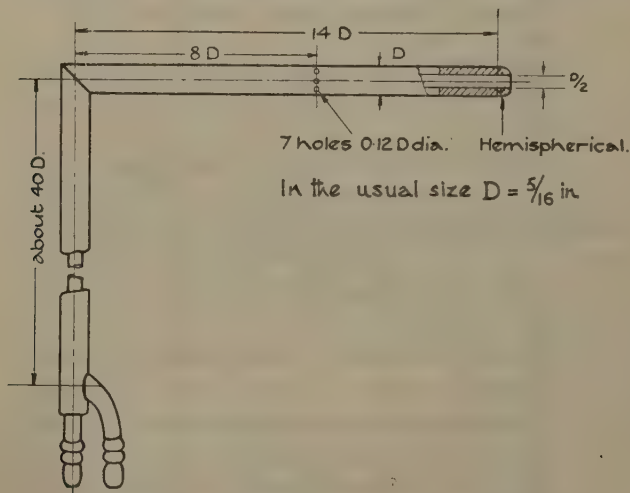


FIG. 22.—N.P.L. standard round-nosed pitot-static tube.

Hence a suitable position for the static orifices appears to be 6 diameters back from the base of the nose, where the pressure gradient, as indicated by the lower curve of Fig. 21, has become small. If the stem is then 8 diameters behind the static holes, the pressure at the holes will be the true static pressure.

The type of instrument shown in Fig. 20 is the first N.P.L. standard pitot-static tube, which was designed and calibrated at the National Physical Laboratory in 1912.² It is remarkable that, although little was then known about the properties of such instruments, so that the selected proportions were to a large extent arbitrary, nevertheless the factor of the instrument

was found to be 1 to an accuracy of 0.1 per cent. over a wide range of speed. Subsequent check tests have confirmed this calibration. The only real disadvantage of this instrument is the sharp edge of the total-head tube, which is liable to accidental damage. For this reason, the alternative design shown in Fig. 22 has been adopted at the National Physical Laboratory. This instrument has the static holes and the stem spaced respectively 6 and 14 diameters behind the base of the hemispherical nose, in accordance with what has already been said. Its factor of 1 is therefore more easily reproducible from one instrument to another than it is with the original standard, in which the static holes, being farther forward, are in a steeper part of the pressure gradient. It is, indeed, true to say that all instruments of this type made with ordinary workshop precision to the proportions shown in Fig. 22 may be confidently accepted, without calibration, as having a factor of 1 correct to 0.1 per cent. An additional slight advantage of the more recent instrument is the rather shorter overall length of the head.

A somewhat similar type of instrument has been designed by Prandtl in Germany (Fig. 23); here the static holes have been replaced by an annular slit, which has advantages as regards both symmetry and freedom from blocking in dust-laden streams. The factor for this instrument also is 1.

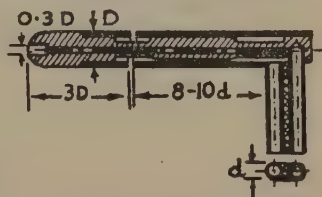


FIG. 23.—The Prandtl pitot-static tube.

The Effect of Alignment of the Pitot-Static Tube.—In use, the pitot-static tube must have its head aligned with the direction of the air current; for appreciable errors arise if the axis of the head is inclined at more than five or six degrees to the direction of flow. Fig. 24 shows the effect of inclination of the head on the velocity-head reading for two types of pitot-static tube—the original N.P.L. type shown in Fig. 20, and an instrument with a hemispherical nose similar to that shown in Fig. 22 but with the stem 15 diameters behind the static holes instead of 8; this instrument was constructed for a particular purpose and had a factor of 1.006.¹ The ordinates in Fig. 24

are the ratio of the pressure at inclination θ° to the pressure when $\theta = 0$.

It will be clear from this diagram that the position of alignment with the direction of flow corresponds to a minimum pressure reading ; hence, if, as sometimes happens in practice, the direction of flow is not known accurately, the correct alignment can be found by yawing the pitot from side to side of the

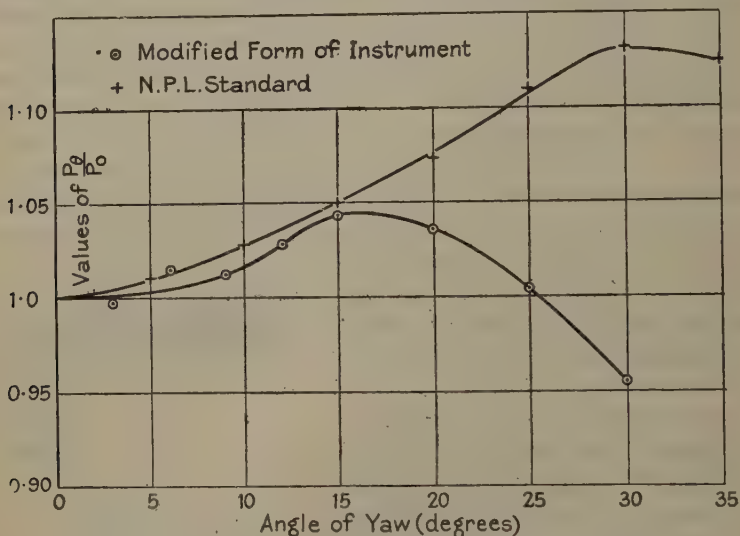


FIG. 24.—Effect of yaw on velocity-head reading.

approximate direction of flow and finding the position in which the differential pressure is least. Fig. 24 shows the variation in the velocity-head reading as the instrument is rotated about the axis of the stem ; there is a similar variation, although not quite symmetrical about zero, as the head is rotated about an axis perpendicular to this, but, here again, alignment corresponds to a pressure minimum.

Effect on Calibration of Size of Pitot-Static Tube: Geometrical and Dynamical Similarity.—Simple theoretical considerations provide us with a valuable insight into the effect of change of size on the pitot-static factor K (see equation (1)). If we restrict ourselves to incompressible flow, there are only four properties of the air and of the pitot-static tube itself that

can conceivably affect the pressure acting on it, namely the size of the instrument, and the velocity, density, and viscosity of the air. Thus, the pressure p may be considered as being given by an expression of the form

$$p \propto d^{\alpha} v^{\beta} \rho^{\gamma} \mu^{\delta}, \quad . \quad . \quad . \quad . \quad (2)$$

where d is some characteristic length of the pitot (e.g., the diameter of the tubing) which expresses its size,

v is the velocity of the stream,

ρ is the density of the air,

and μ is the coefficient of viscosity of the air. The indices α , β , γ , and δ are unknown, but we can deduce relations between them because the right-hand side of (2) must have the same dimensions as the left-hand side, i.e., the dimensions of a pressure. In terms of the fundamental units of mass (M), length (L), and time (T), the dimensions of pressure are + 1 in M, - 1 in L, and - 2 in T. Also, the dimensions of the four quantities on the right-hand side of (2) are as follows :

$$\begin{aligned} d &\equiv L, \\ v &\equiv LT^{-1}, \\ \rho &\equiv ML^{-3}, \\ \mu &\equiv ML^{-1}T^{-1}. \end{aligned}$$

Hence we have

$$\begin{aligned} 1 &= \gamma + \delta, \\ -1 &= \alpha + \beta - 3\gamma - \delta, \\ \text{and} \quad -2 &= -\beta - \delta. \end{aligned}$$

We can easily deduce from these three equations between the indices the following relations, which give α , β , and γ in terms of δ :

$$\begin{aligned} \alpha &= -\delta, \\ \beta &= 2 - \delta, \\ \gamma &= 1 - \delta. \end{aligned}$$

Hence, we can rearrange (2) thus :

$$p \propto \rho v^2 \left(\frac{\mu}{\rho v d} \right)^{\delta} . \quad . \quad . \quad . \quad . \quad (3)$$

It will be seen that the dimensions of the group of variables $\left(\frac{\mu}{\rho v d} \right)$ are zero in each of the fundamental units M, L, and T. This will be true whatever the value of the index δ . Thus,

the right hand side of (3) can contain any function or power, or groups of functions or powers, of the term $\frac{\mu}{\rho v d}$ without violating the dimensional conditions on which the above argument is based: the only restriction is that the quantities μ , ρ , v , and d shall occur only in the non-dimensional combination $\frac{\mu}{\rho v d}$, and not separately or in other combinations. Hence, writing ν for μ/ρ as in Chapter III, we can express (3) in its more general and better known form

$$p = \rho v^2 f(vd/\nu), \quad . \quad . \quad . \quad . \quad (4)$$

where $f(vd/\nu)$ represents some function of the quantity vd/ν .

The reader will recognise in vd/ν the quantity already defined in Chapter III as a Reynolds number, and will be interested to see that, whereas in that chapter we learned of the significance of this quantity on the basis solely of experimental results, we are now led to it from theoretical considerations.

We can use this relationship between the differential pressure developed by a pitot-static tube and the other quantities appearing in equation (4) to predict the general behaviour of the instrument. For we may rewrite (4) as

$$p = \frac{1}{2} \rho v^2 F(vd/\nu); \quad . \quad . \quad . \quad . \quad (5)$$

and, comparing equations (1) and (5), we see that K , the pitot-static factor, is a function of the Reynolds number.

If we restrict ourselves for the moment to one particular fluid, e.g. air, at constant temperature and pressure, ν is constant and the pitot-static factor is a function of the product vd only. It follows that under these conditions, if a given instrument has a particular value of the factor at a certain speed, a geometrically similar instrument n times as large will have the same value of the factor at $1/n$ th the speed. One of the major advantages of the pitot-static tube, which has been established by experimental calibration, is that $F(vd/\nu)$, i.e. the factor K , remains sensibly constant and equal to 1 over a very large range of the Reynolds number, i.e., in a given fluid at constant temperature and pressure, over a very large range of vd .

The Value of the Factor K for the N.P.L. Standard Pitot-Static Tube.—The N.P.L. Standard (Fig. 20) was first calibrated in air² over a speed range of about 20 to 70 feet per second; this range was subsequently extended by experiments in water to the equivalent of about 250 feet per second in air. In 1930, the calibration over the speed range 20 to 70 feet per second was repeated with extreme care and confirmed to a high degree of accuracy, the value of the factor being found to be 1 to 0.1 per cent. Thereafter, further experiments served to extend the range over which the factor was known both to higher and lower values of the Reynolds number. Using a model geometrically similar to the N.P.L. standard, but with a linear scale 25 times larger, Falkner* found that the factor was substantially unchanged up to an air speed of 90 feet per second. According to the law of similarity stated in the preceding section, this means that the factor of the standard instrument remains unchanged up to a speed in air at ordinary temperatures and pressures of 25×90 , i.e., 2,250 feet per second. In actual fact, this would not be the case, because long before this speed is reached the compressibility of the air has to be taken into account, a property that was neglected in the analysis from which equation (5) was deduced. If we were to extend the analysis to take account of compressibility, which we could do without difficulty, we should obtain, instead of equation (5), the following equation for p :

$$p = \frac{1}{2} \rho v^2 F(vd/v, v/a); \quad . \quad . \quad . \quad (6)$$

i.e. the pitot factor would now be a function not only of the Reynolds number, but also of the ratio of the velocity of flow v to a , the velocity of sound in air. The value of a for ordinary conditions is about 1,100 feet per second, and the effect of variation of v/a on the pitot-static factor is discussed below. All that need be said here is that the value of the factor will be affected long before a speed of 2,000 feet per second is reached.

In 1931, Ower and Johansen³ extended the low-speed end of the calibration of the N.P.L. standard by experiments covering the range 24 to less than 2 feet per second in air under room conditions. Fig. 25 shows the values of K obtained over this speed range. It will be seen that, as the speed falls below

* Results not published.

20 feet per second, K falls slowly but, down to a speed of 5 feet per second, its value never differs by more than 1 per cent from the value 1, which holds for speeds of 20 feet per second and upwards. At 4 feet per second K is equal to 0.989, and as the speed decreases further the value of the factor rises steeply, reaching 1.020 at 2 feet per second. It is clear from the equation between speed and differential pressure that, if K is nearly 1, any percentage error in the assumed value of K gives

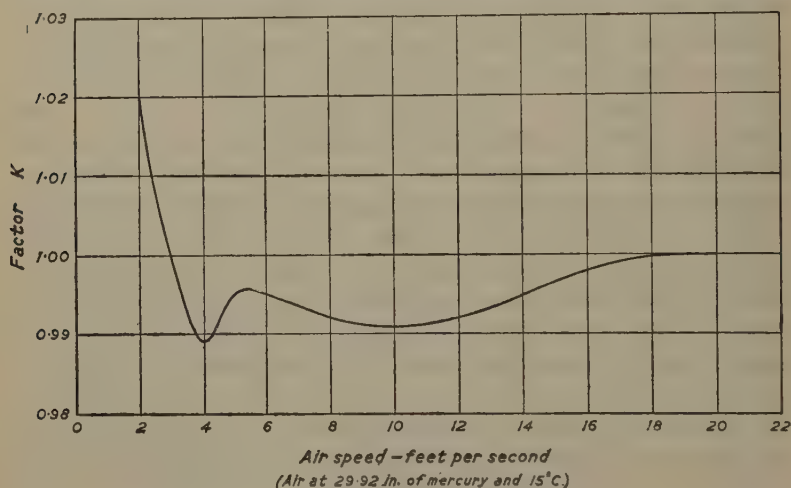


FIG. 25.—Factor of first N.P.L. standard pitot-static tube at low speeds.

rise to a percentage error of half this amount in v . Hence, down to a speed of 5 feet per second, the error due to the assumption that $K = 1$, which is true for the higher speeds, will not lead to an error exceeding 0.5 per cent. on air speed. Apart from the small speed range in the neighbourhood of 4 feet per second where the departure of K from unity is just over 1 per cent., we may say that K may be taken as unity down to about $2\frac{1}{2}$ feet per second without causing an error exceeding 0.5 per cent. on air speed. Between about $3\frac{1}{2}$ and $4\frac{1}{2}$ feet per second the error on speed arising from this assumption would be 0.6 per cent.

To sum up, we may assume that the factor of the N.P.L. standard pitot-static tube, or a geometrically similar copy of it, is equal to unity over a range of Reynolds number vd/ν extend-

ing from about 330 to at least 360,000. For the standard itself, having a tube diameter of about $5/16$ inch, used in air at ordinary temperatures and pressures, the corresponding range of air speed extends from about 2 feet per second to well over the speed at which compressibility begins to become important.

Factor of the Round-Nosed Instrument.—Tests in a wind tunnel showed that the factor of the pitot-static tube with the hemispherical nose shown in Fig. 22 agreed closely with that of the original N.P.L. standard at air speeds between 20 and 70 feet per second ; and there is no reason why any divergence should be expected at higher speeds below the region of compressibility. Data on the factor at low speeds were obtained during the course of the investigation just mentioned on the N.P.L. standard ; the figures are given in Table 3.

TABLE 3
Factors for Round-nosed Pitot-Static Tube

Air Speed, feet per second, 15° C. and 29.92 in. of mercury	Mean value of K, tube diameter $\frac{5}{16}$ in.
2	1.055
4	1.006
6	1.001
8	0.996
10	0.992
12	0.991
14	0.992
16	0.996
18	0.999
20	1.001

Limitations of the Pitot-Static Tube for the Measurement of Low Speeds under Industrial Conditions.—

As we have just seen, the factor of the pitot-static tube is constant, for all practical purposes, over a wide range of air speeds—a range that will cover a very great proportion of the cases with which the practical engineer has to deal. This, together with the ease of reproducibility of the factor from one

instrument to another of the same geometrical pattern, constitutes the main advantage that the pitot-static tube possesses as compared with all other types of pressure-tube anemometer. Against this, however, we have to admit that the small differential pressure set up at low air speeds is a serious objection to the use of the pitot-static tube; for it means that very sensitive manometers are needed if the necessary accuracy is to be achieved at such speeds, and these manometers are most inconvenient to use under industrial conditions. For example, to get an accuracy of 1 per cent. in the observation of the velocity head for an air speed of 2 feet per second, we need a manometer sensitive to less than one hundred thousandth of an inch of water-column; and an instrument of this sensitivity had to be specially designed and constructed* for the low-speed calibration of the N.P.L. standard pitot-static tube already mentioned. Such an instrument can only be used under laboratory conditions: numerous precautions to ensure steady temperature conditions and firm foundations have to be taken if full use is to be made of its sensitivity. If we take one thousandth of an inch of water as a reasonable limiting sensitivity for a manometer that can conveniently be used in industrial work, then the lowest air speed that can be observed to an accuracy of 1 per cent. with the pitot-static tube is about 15 feet per second.

Pressure-Tube Anemometers with Factors greater than Unity.—In order to overcome this disadvantage of the pitot-static tube, and to extend the practical range of pressure-tube anemometers in the low-speed region, a number of instruments have been designed having values of the factor K greater than 1. There is no practical difficulty about this; two of these instruments are illustrated in Fig. 26.

But such instruments are unfortunately not to be recommended, because they possess certain undesirable features. In the first place, experience shows that it may be taken as almost certain that if K appreciably exceeds 1 its value will vary appreciably with Reynolds number, and probably more noticeably in the lower ranges for which the instrument is primarily intended. In devising an instrument to have a higher value of

* See Chapter IX, page 232.

K than the pitot-static tube, we have, in fact, lost what we claimed on page 65 as one of the chief advantages of the latter, namely constancy of $F(vd/v)$ in equation (5) over a wide range of Reynolds number.

A typical calibration curve for an instrument of this type, similar to, but not identical with, that shown in Fig. 26(b), is shown in Fig. 27, from which it will be seen that the factor drops

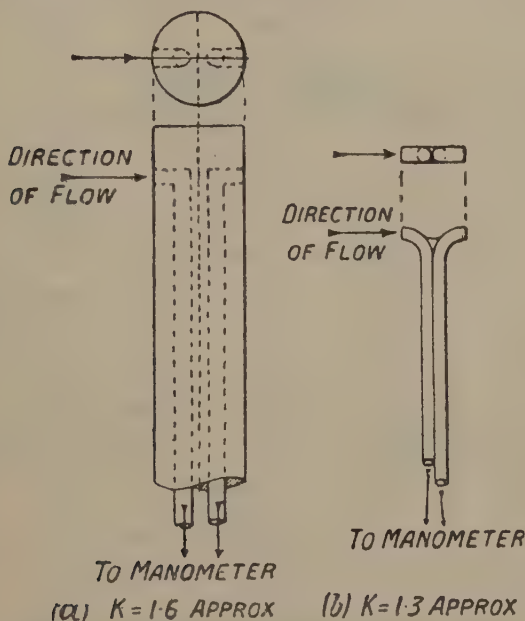


FIG. 26.—Pressure-tube anemometers with factors exceeding unity.

from nearly 1.59 to less than 1.53 within the narrow speed range 40 to 75 feet per second, and that at the higher of these speeds it is still falling steeply. There is no need to stress the inconvenience of a calibration that changes with speed, one result of which is that at least two calculations are needed to convert a pressure observation into velocity: the first has to be made with a guessed value of K, since the speed is initially unknown, and the approximate value of the speed thus obtained has to be used to obtain, from the calibration curve of the instrument, a nearer approach to the true value of K to be used in the second, more exact, calculation.

Another serious disadvantage of such instruments is that if K varies with Reynolds number its value at a given low Reynolds number is very likely to depend on the amount of turbulence present in the air stream; so that the calibration may be different, even at the same speed, in different pipe systems.

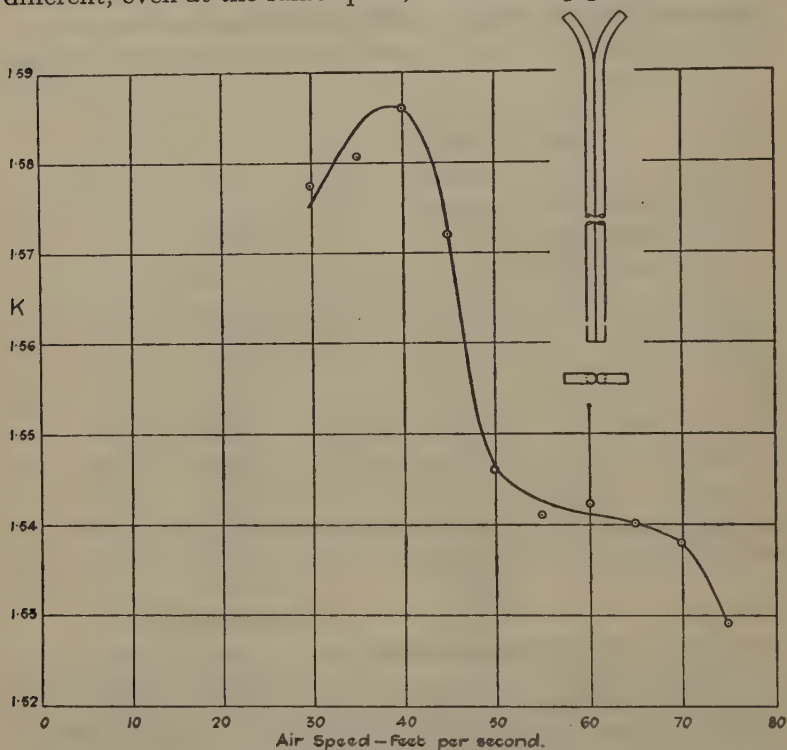


FIG. 27.

It must be remembered also that, since the velocity for a given pressure reading varies inversely as the square root of K , the extension of the lower range of speed that can be measured with a manometer of given sensitivity is not very large. Thus, taking as before 15 feet per second as the lowest speed that can conveniently be measured to an accuracy of 1 per cent. with the pitot-static tube under industrial conditions, we see that, by using an instrument with a factor of 1.6, we can only extend the lower limit of speed measurable to the same accuracy to about 12 feet per second—not a significant extension.

On the whole, therefore, there is nothing to be gained by using a pressure-tube anemometer having a higher value of K

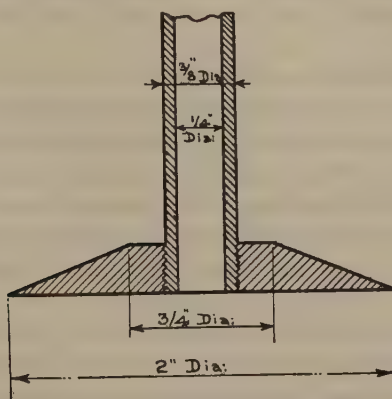


FIG. 28.—Disc type static head.

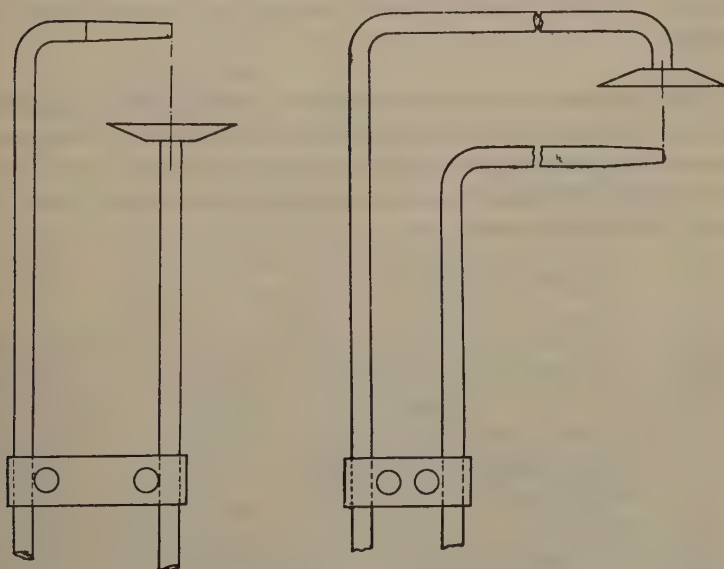


FIG. 29.—Combinations of disc static heads with total-head tubes.

than the pitot-static tube, unless it eventually proves possible to design an instrument free from the objectionable features from which all types so far proposed suffer. If the rate of flow is too low to enable the pitot-static tube to give the desired

accuracy with the manometer to be used, then one of the alternative methods described in later chapters of this book will have to be adopted.

Disc Static Heads.—In a dust-laden airstream, the small holes in a static head of the tubular type are liable to become blocked ; and for measurements under such conditions heads of the type shown in Fig. 28 are sometimes used. Experiments by Heenan and Gilbert⁴ have shown that these heads give all the accuracy ordinarily required in the measurement of static pressure. It is important that the face of the disc should be smooth and that the tube connecting the orifice to the manometer should end flush with the disc surface. Such heads may be used in conjunction with the usual form of total-head tube, as illustrated in Fig. 29, where two possible arrangements are shown ; or, alternatively, the measurements of static pressure and total head may be made separately, as explained in Chapter V.

The Readings of the Pitot-Static Tube at High Speeds.

—We have already shown (page 16) that the complete equation for the pressure p at the mouth of the total-head tube for a compressible fluid, such as air, at speeds below the velocity of sound in the medium is

$$p_1 = p_0 + \frac{1}{2}\rho v^2 + \left[\frac{1}{2}\rho v^2 \left\{ \frac{1}{4} \frac{v^2}{a^2} + \dots \right\} \right],$$

where a is the velocity of sound in the fluid. We have also seen that the terms within the brackets [] are significant for air-flow measurements only at speeds, under ordinary conditions of temperature and pressure, of about 200 feet per second and over. At lower speeds, the bracketed terms can be neglected and the relation between the total head and velocity reduces to its well-known form. At higher speeds, the velocity can be deduced from the differential pitot-static pressure by the use of the complete expression

$$p = \frac{1}{2}\rho v^2 \left[1 + \frac{1}{4} \frac{v^2}{a^2} + \dots \right],$$

on the assumption that the static reading is not affected by a compressibility effect similar to that which causes the departure

of the total-head relation from the form it has at the lower speeds. At very high speeds, however, this assumption is not valid; for Walchner⁵ has shown that, for the round-nosed Prandtl instrument illustrated in Fig. 23, there is a compressibility effect on the static reading that begins to become appreciable when the velocity is about 0.8 times that of sound, i.e. between 800 and 900 feet per second. For the round-nosed N.P.L. instrument (Fig. 22) the effect will presumably begin to be noticeable at about the same speed; for the N.P.L. standard, with its sharp nose, the critical speed may be somewhat higher.

The Effect of Turbulence on the Readings of Total-Head and Static Tubes.—It was explained in Chapter III that the flow in most cases that occur in engineering practice is turbulent; that is to say, although the bulk of the air as a whole is assumed to be flowing steadily along, small individual masses of it are moving about in all directions with definite velocities superimposed on the mean velocity of flow. The state of affairs is thought to be in some respects similar to what occurs in a mass of gas at rest, in which the molecules are in a continuous state of motion in all directions, the average length of their paths between collisions being called the "mean free path." Analogous to this mean free path in a gas at rest, a property known as the "mixture length" or the "mixing length" is attributed to a stream of fluid in motion; it represents the average length of path of the small masses of fluid during which they are distinguishable by their individual velocities from the general mass of the fluid. In experimental or theoretical investigations of turbulent flow, the turbulent velocities of the small masses of fluid are generally resolved into three components, one along the main direction of flow and two perpendicular to it and to each other. It is found that, in fully developed flow in smooth pipes, these three turbulent components have perfectly definite average values, which are functions of the Reynolds number only.

These turbulent velocity components affect the readings of both the total-head tube and the static tube. Goldstein⁶ has shown that the pressure measured by the former is not $p + \frac{1}{2}\rho v^2$ but $p + \frac{1}{2}\rho v^2 + \frac{1}{2}\rho (\bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2)$, where v is the mean

velocity of the flow as a whole and $\bar{v}_x$, $\bar{v}_y$, and $\bar{v}_z$ are the root mean squares of the three turbulent velocity components, $\bar{v}_x$ being in the same direction as v . The corresponding reading of the static tube has been determined by Fage⁷ as $p + \frac{1}{2}\rho(\bar{v}_y^2 + \bar{v}_z^2)$ instead of p , the reading in non-turbulent flow. Fage⁸ has also shown that, even in the case of steady mean flow along a straight pipe, turbulence causes a variation in static pressure along a diameter. This fact assumes significance in cases when one uses a hole in the side of the pipe (see Chapter VIII, page 186) for the measurement of static pressure, instead of a static tube; for, in consequence of this effect of turbulence, the static pressure at the side of the pipe is not equal to the static pressure at all points in the section.

The errors in pitot-static measurements due to these causes are, however, not large, and need only be taken into account when the highest accuracy is demanded. Values of the corrections to be applied can be calculated for the case of fully developed turbulence in straight smooth pipes, for which, as already remarked, the turbulent velocity components are definite and known. And since the corrections are small, the values thus calculated can be assumed to be sufficiently accurate for most of the cases in which fully developed pipe flow is not established.

Fage⁸ has calculated the corrections to be applied for fully developed flow when the total-head tube and side-hole are used as described in Chapter VIII, to measure the quantity of air flowing along a pipe. His method has been extended by Ower⁹ to find the errors in quantity when the combined pitot-static tube is used in the manner described in Chapter V; and also the errors in individual readings of velocity at different radii, both for the pitot-static tube and for the total-head tube and side-hole. The results of these various calculations are given in Table 4 (a) and (b), in the form of percentage corrections to be applied to the results of calculations of quantity and velocity carried out in the ordinary way from observations of total head and static pressure.

The figures in Table 4 (a) show that the errors on local velocity increase towards the wall of the pipe. They must, however, vanish actually at the wall; and it is probable that the maximum errors are not much different from the values

given at $r/R = 0.93$. At this radius the error is appreciable, for the reason that the resultant root mean square turbulent velocity is about 26 per cent. of the local axial velocity. The

TABLE 4
Percentage Corrections to be applied to allow for the effect of Turbulence in Pipes with Fully Developed Turbulent Flow.

(a) Corrections on Local Velocity.

$r/R = \frac{\text{radius of observation}}{\text{radius of pipe}}$	With pitot-static tube	With total-head tube and side-hole
0	- 0.2	+ 0.1
0.5	- 0.4	0
0.8	- 1.0	- 0.7
0.85	- 1.4	- 1.1
0.9	- 2.0	- 1.8
0.93	- 2.3	- 2.5

Note.—These corrections are calculated for a Reynolds number vd/ν of 18,340, for which $F/\rho v^2 = 0.00325$. For other Reynolds numbers the tabulated corrections must be multiplied by $(1/0.00325 \times F/\rho v^2)$. For the relation between vd/ν and $F/\rho v^2$, see page 28, *et seq.*; since $F/\rho v^2$ decreases as vd/ν increases, the necessary corrections at higher Reynolds numbers are smaller than those tabulated above.

(b) Corrections on Total Quantity obtained from $2\pi \int_0^R v r dr$ (see page 38).

vd/ν	$F/\rho v^2$	With pitot-static tube	With total-head tube and side-hole
3,000	0.0053	- 1.6	- 1.0
25,000	0.0031	- 0.9	- 0.6
100,000	0.0023	- 0.7	- 0.4
200,000	0.0020	- 0.6	- 0.4
400,000	0.0017	- 0.5	- 0.3

errors on the readings of the static tube and of the side-hole reduce the error on the differential pressure from which the velocity is calculated.

Over most of the pipe, the error with the total-head tube and side-hole is less than with the pitot-static tube; so that, as shown by Table 4 (b), the overestimation of quantity is appreciably less with the former combination.

Although it is not an effect of turbulence, we shall find it

convenient to consider at this stage another small correction that has to be applied in accurate work. This arises from the fact that the velocity across a section of a pipe is usually not uniform, but has a distribution such that the velocity at the axis is a maximum (see page 20). In consequence, there is generally a transverse total-head gradient across the mouth of the total-head tube ; and, under these conditions, Young and Maas ¹⁰ have shown that the effective centre of total pressure is displaced from the geometric axis of the tube towards the region

TABLE 5

Percentage Corrections to be applied to allow for Displacement of Effective Centre of Total-Head Tube in fully developed Pipe Flow.

(a) Corrections on Local Velocity.

$r/R = \frac{\text{radius of observation}}{\text{radius of pipe}}$	$\delta/d = 0.03$	$\delta/d = 0.02$	$\delta/d = 0.01$
0	0	0	0
0.5	- 0.2	- 0.1	0
0.8	- 0.8	- 0.3	- 0.1
0.85	- 1.1	- 0.5	- 0.1
0.9	- 1.1	- 0.5	- 0.1
1.0	0	0	0

(b) Corrections on Total Quantity obtained from $2\pi \int_0^R v r dr$.

$\delta/d = 0.03$	$\delta/d = 0.02$	$\delta/d = 0.01$
- 0.9	- 0.4	- 0.1

of higher pressure. The amount of the displacement depends, in addition to the gradient, on the ratio of the internal to the external diameter of the tube, and on the ratio of the tube diameter δ to the pipe diameter d . The result is that the local total head at the geometric centre of the tube is over-estimated. Fage ⁸ has calculated the effect of this over-estimation on measurements of the quantity of fluid flowing along a pipe at a section where the transverse velocity gradient is that corresponding to fully developed pipe flow ; and Ower ⁹ has extended the calculations to find the errors on local velocity due to this cause. The results, in the form of percentage corrections on local velocity and total quantity, are given in Table 5 ; they

relate to a total-head tube in which the inner diameter is 0.5 to 0.6 times the outer. In the standard instrument this ratio is 0.5.

It will generally be sufficiently accurate to assume that errors due to this cause are independent of the Reynolds number: the changes in velocity distribution across a section that occur with Reynolds number at values of the latter above about 5,000 do not affect the calculations to an extent that is significant for practical purposes. The figures in the table show that, by limiting the diameter of the total-head tube to one-fiftieth of the pipe diameter, we can keep these errors below one-half of 1 per cent. As in the case of the errors due to turbulence, the displacement errors are smaller nearer the entry to the pipe; for there the turbulence is less and the transverse velocity gradient is not so steep.

The Effect of Fluctuating or Pulsating Flow on the Readings of a Pitot-Static Tube.—We saw in the preceding section how to allow for the effect of the small-scale turbulent velocity components that are usually superimposed on a mass of air streaming steadily as a whole. Fluctuations or pulsations in this average velocity of the air stream introduce an entirely different kind of effect on the pitot-static readings, and one that it is very difficult to allow for; in fact, no simple and satisfactory method of measuring violently pulsating air flow has yet been discovered.

It is easy to see that the average pressure acting at the mouth of the total-head tube exceeds that due to the sum of the static pressure and the velocity head corresponding to the average velocity. Thus, let us take the simple case where both the velocity v and the static pressure p vary according to a sine law, both having the same period, i.e.,

$$v = v_0(1 + \alpha \sin qt)$$

$$\text{and } p = p_0(1 + \beta \sin qt),$$

where v_0 and p_0 are respectively the average velocity and static pressure about which the fluctuations take place. Then the pressure acting at the mouth of the total-head tube at any time t is

$$\frac{1}{2}\rho v_0^2(1 + \alpha \sin qt)^2 + p_0(1 + \beta \sin qt) = P \text{ (say).}$$

The periodic time is $2\pi/q$ and the average pressure at the

mouth of the tube will be the average pressure over a cycle, that is

$$\frac{q}{2\pi} \int_0^{2\pi/q} P dt;$$

i.e. average pressure =

$$\frac{q}{2\pi} \left[\frac{1}{2} \rho v_0^2 \int_0^{2\pi/q} (1 + \alpha \sin qt)^2 + p_0 \int_0^{2\pi/q} (1 + \beta \sin qt) \right].$$

Integrating the right-hand side and introducing the limits, we obtain, after reduction,

$$\text{average pressure} = p_0 + \frac{1}{2} \rho v_0^2 (1 + \alpha^2/2).$$

Hence, if a static tube is connected to the other side of the manometer to which the total-head tube is connected, we shall get a resultant reading of $\frac{1}{2} \rho v_0^2 (1 + \alpha^2/2)$. But, by hypothesis, v_0 is the average velocity; hence we see that there is an error on the velocity head equal to $\alpha^2/2$. Now α represents the amplitude of the velocity pulsations; if $\alpha = 0.5$, for example, the maximum departures of velocity during a cycle above and below the average value of v_0 are $\pm 0.5 v_0$. The percentage error on the average velocity is thus equal to $100 [\sqrt{1 + \alpha^2/2} - 1]$, or approximately, if α is not too large, to $100 \cdot \alpha^2/4$.

We see, therefore, that for small values of α , i.e. for small amplitudes of pulsation, the error is not large. Thus, even if α has the value 0.25, which indicates that the velocity fluctuates by 25 per cent. of its mean value on either side of the mean, the error is only about $1\frac{1}{2}$ per cent. But a fluctuation of this order—25 per cent. on each side of the mean—would be considered a very unsteady flow in practice; so that we deduce that the error due to pulsation is not large unless the flow is very unsteady. We must remember, however, that the error varies as the *square* of the velocity pulsation, so that, although it is only $1\frac{1}{2}$ per cent. when α is 0.25, it increases rapidly with α , being about 6 per cent. when $\alpha = 0.5$ and about 23 per cent. when $\alpha = 1$.

We ought therefore always to try, as far as we can, to avoid taking readings in pulsating flow or, if this is not possible, to reduce the pulsations. This can be done as shown in Chapter VI, page 142, by the introduction of a large air-box or receiver between the source of pulsation and the section of

measurement, but this method has obvious practical disadvantages. A simpler expedient is to take advantage of the fact that if the stream is contracted to $1/n$ th of its original area, the amplitude of the axial fluctuations after contraction is proportional to $1/n$ th of the amplitude before contraction. If, therefore, we introduce into the pipe a shaped nozzle (see Chapter VI, page 129) and take pitot-static readings in the outlet plane, we shall find the flow considerably steadier. There will also be two incidental advantages: firstly, that of the higher speed, as a result of which the pressures to be measured will be higher; and, secondly, the fact that the velocity in the outlet plane of a properly shaped nozzle is practically constant across the entire section, so that there should be no need to take a number of readings at various radii, as described in Chapter V, in order to measure the quantity of air flowing; a single reading at the axis will suffice.

The Calibration of Pitot-Static Tubes.—It may be of interest to describe briefly how a fundamental calibration of a pitot-static tube is carried out. The principle of the method is to move the instrument at a series of known speeds through the air and to observe the differential pressure set up at each speed. Simple though this sounds, its practical execution calls for experimental skill and technique of a high order.

It is obviously impracticable to move the instrument in a straight path; and some sort of "whirling arm" has always been used in the comparatively few fundamental calibrations that have been carried out in this country and abroad. A whirling arm consists of a braced horizontal arm that rotates about a fixed vertical axis at one end. To the free end, which thus travels in a circular path, the pitot-static tube is firmly attached; and rubber connections from the two pressure leads of the instrument run along the arm, and are connected to two specially designed liquid seals on the axis that enable the pressures from the moving instrument to be transmitted to the manometer which, with the observer, is generally situated at some convenient spot outside the circular orbit. The major difficulty in the investigation arises from the fact that the rotation of the arm and the pitot-static tube sets in motion the air in the room in which the whirling arm is housed, with the

result that a circular "swirl" is set up in the path pursued by the pitot-static tube. The speed of the latter relative to the air is thus not equal to its speed relative to the ground, which can be calculated simply from a knowledge of the radius of the arm and observation of its rotational speed, but to some lower speed, which is obtained by subtracting the swirl speed from the ground speed.

It is the measurement of the swirl speed that presents the greatest problem. For the calibration of the N.P.L. standard pitot-static tube, carried out in 1912,² a special sensitive vane anemometer* was devised and calibrated on the arm itself in the position subsequently occupied by the pitot-static tube. The arm was rotated at various speeds within the range to be covered, and at each speed an approximate relation was obtained between the readings of the anemometer, which were electrically recorded, and the air speed of the end of the arm, *on the assumption that the swirl speed was zero*. The anemometer was then removed from the arm and replaced by the pitot-static tube, the anemometer itself being successively mounted in each of eight fixed positions equally spaced round the room. In each position, the anemometer was just cleared by the pitot-static tube as it swept past on the end of the arm. The swirl set up by this motion caused the anemometer to rotate; and the approximate calibration of the anemometer which had been obtained when the latter was carried by the arm was used to obtain a first approximation to the swirl speed. The first approximation was then used to correct the anemometer calibration and so to obtain a second approximation to the swirl speed, and so on. Obviously, the process could be continued as far as desired, but the difference between the second and third approximations was small enough to make further application unnecessary.

This method of obtaining the swirl was repeated in the calibration of 1930 (page 63), for which the improved low-speed anemometer described in Chapter VII, page 149, was designed to measure the swirl. It may be mentioned that the swirl speed was about 7 or 8 per cent. of the ground speed, so that at the lowest air speed—about 20 feet per second—the anemometer had to indicate a speed of about $1\frac{1}{2}$ feet per second.

* The vane anemometer is described in Chapter VII.

For the low-speed calibration of 1931 (page 63), no vane anemometer sufficiently sensitive could be made; and hot-wire anemometers * had to be used to measure the swirl speed, which ranged from about 0.12 to 3 feet per second. Special precautions to exclude draughts had to be taken in these experiments, and it was found necessary to construct a small whirling arm of about 4-ft. radius (as compared with 30 feet for the earlier high-speed calibrations), and to enclose the moving end in an annular tunnel of 2 feet square cross-section, within which the pitot moved and was shielded from outside disturbances. The hot wires used to measure the swirl were, as the anemometers in the earlier experiments, calibrated on the arm itself.

With the swirl speed established, the determination of the pitot-static factor over a range of speeds presents relatively little difficulty. We may note that the method may be used either to check the accuracy of each tube separately, or to observe the differential pressure directly. The pressure at the mouth of the total-head tube is nearly zero. It would be accurately zero if there were no swirl, i.e., if the ground speed were equal to the air speed; for it happens that the suction at the mouth of the tube due to the centrifugal force is $\frac{1}{2}\rho v^2$, i.e., equal to the velocity head with no swirl. Similarly, if the static tube reads correctly, the pressure at the static orifices will be the centrifugal pressure ($-\frac{1}{2}\rho v^2$). For further details of these calibrations, the reader is referred to the original papers quoted in References 2 and 3, page 80.

This laborious method of calibration is obviously unsuitable for use in all cases when new pitot-static tubes have to be tested; it has, in fact, been used in this country only on the three occasions already mentioned, all on the N.P.L. standard instrument. After the first calibration, the standard instrument was used to calibrate all the wind tunnels † at the N.P.L., and these in turn are now used for tests of new pitot-static tubes or other anemometers.

* See Chapter X.

† A wind tunnel is an apparatus for providing an air stream of known and variable speed, and generally of considerable cross-section (up to 100 square feet or more), in which aerodynamic experiments, calibrations of anemometers, and other tests can be carried out.

In a wind-tunnel test of a pitot-static tube, the latter and the N.P.L. standard (or a copy of it which has itself been calibrated by the method now being described) are mounted in the tunnel side by side, some two feet or so apart, and the static sides of the two instruments are connected to opposite sides of a sensitive manometer, which thus measures the difference between the static readings of the two instruments. This difference is observed for each of a number of air speeds; the two instruments are then interchanged, so that each now occupies the position previously occupied by the other, and the differences in the static readings are again observed through the same speed range. As a result of this change in position, the differences between the static readings of the two instruments at the various speeds are obtained free from the effect of any errors due to the possibility that the actual static pressures in the two positions in the tunnel occupied by the instruments might have differed slightly. (It is usually not possible to avoid very small differences in static pressures from spot to spot in a cross-section of a wind tunnel.) In this way, the error on the static side of the instrument under test is obtained, since the fundamental calibration on the whirling arm has shown that the static side of the standard instrument reads correctly. A similar comparison could, if desired, be made of the total-head sides of the two instruments, but this is never done now because, as already indicated, experience has shown that, at any rate for instruments basically of conventional design but differing widely in detail, the total-head side always reads correctly.

REFERENCES

1. Ower and Johansen, *Reports and Memoranda of the Aeronautical Research Committee*, No. 981 (1926).
2. Bramwell, Relf and Fage, *Reports and Memoranda of the Advisory Committee for Aeronautics*, No. 71 (1912).
3. Ower and Johansen, *Proc. Roy. Soc., A*, **136** (1932), p. 153; *Reports and Memoranda of the Aeronautical Research Committee*, No. 1437 (1932).
4. Heenan and Gilbert, *Proc. Inst. Civil Engineers*, **123** (1895-6), p. 272.
5. Walchner, *Jahrbuch der deutschen Luftfahrtforschung*, **1** (1938), p. 578.
6. Goldstein, *Proc. Roy. Soc., A*, **155** (1936), p. 570.
7. Fage, *Proc. Roy. Soc., A*, **155** (1936), p. 576.
8. Fage, *Engineering*, **145** (1938), p. 616.
9. Ower, *Trans. Inst. Chem. Engineers*, **18** (1940), p. 87.
10. Young and Maas, *Reports and Memoranda of the Aeronautical Research Committee*, No. 1770 (1937).

CHAPTER V

MEASUREMENTS OF FLOW AND RESISTANCE WITH PITOT-STATIC TUBES

It was shown in Chapter III that the speed of flow at all points in a cross-section of a pipe is not in general the same, the velocity being a maximum at the centre and falling away towards the walls. Now instruments such as the combined pitot-static tube or the vane anemometer will, of course, only indicate the local velocity in the regions which they occupy ; so that, in order to determine the volume of air flowing in unit time by the use of such instruments, it is necessary to measure the velocity at a number of points across a given section.

Determination of the Mean Velocity.—We shall, as before, define the mean velocity of flow v_m in the pipe as that velocity which, when multiplied by the area A of the pipe section, gives the volume V of air passing the section per second.

If we consider a small element of area a_1 of the pipe through which the mean velocity of flow is v_1 , the volume flowing through this element is a_1v_1 , and

$$V = \Sigma a_1v_1 = Av_m, \quad . \quad . \quad . \quad (1)$$

so that

$$v_m = \frac{1}{A} \Sigma a_1v_1.$$

Suppose now that we divide the section of the pipe into n equal parts of area a , through which the mean velocities are v_1, v_2, v_3 , etc., then we shall have

$$\begin{aligned} v_m &= \frac{1}{A} \Sigma av_1 = \frac{1}{A} (av_1 + av_2 + av_3 + \dots + av_n) \\ &= \frac{a}{A} (v_1 + v_2 + \dots + v_n), \end{aligned}$$

so that, since $A = na$,

$$v_m = \frac{1}{n} (v_1 + v_2 + \dots + v_n). \quad (2)$$

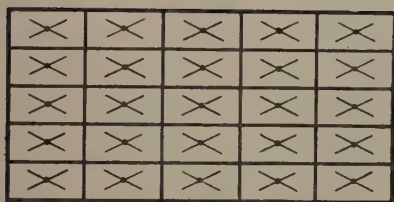
We see therefore that the mean velocity of flow through the pipe is obtained by dividing the section into a number of parts of equal area, and measuring the mean velocity through each. If the number of parts is sufficiently large, i.e. if each part is sufficiently small, the mean velocity through any part may be taken as the velocity of flow at its centre. The number of velocity readings necessary will then be equal to the number of parts into which the section is divided. It will be clear that the more irregular the velocity distribution across the section, the greater will be the number of parts into which the section must be divided, if the velocity measured at the centre of area of each part is to approximate to the mean velocity across that part. Further, if we consider two pipes, similar in section but different in size, and assume that the velocity distributions across any pair of corresponding sections are similar, then for the same percentage accuracy in the determination of v_m , the two sections must evidently be divided into the same number of parts. This point is often not appreciated; it must be remembered that if, for practical reasons, fewer measurements are taken in a small pipe, some accuracy will be sacrificed. The number of readings taken should depend primarily on the nature of the velocity distribution across the pipe, and not on the size of the pipe.

In considering how best to divide up a pipe section in practice, we need discuss only pipes of circular and rectangular cross-section, since it is exceptional to use sections of any other form.

Pipes of Rectangular Cross-section.—The section should be divided, by lines parallel to the sides, into a number of equal areas, each geometrically similar to the whole section (Fig. 30); and the velocity should be measured at the centre of each. The mean velocity and the volume flowing per minute can then be obtained directly by the use of equations (1) and (2). As regards the number of parts into which the section should be divided, a good working rule, subject to the remarks made above, is to make the minimum number 16 for pipes up to 576 square inches in cross-sectional area, and for larger pipes to

take the number such that each part is not greater than 36 square inches in area.* (For pipes of less than 5-inch side see remarks for small circular pipes.)

Pipes of Circular Cross-section.—The pipe should be divided into n equal concentric zones with centres at O , as in Fig. 31, by circles of radii r_2, r_4, r_6 , etc. If now we take any radius OP of the pipe, and consider a particular zone— AB , say—a velocity measurement must be made along this radius at a



Positions of Axis of Measuring Instrument shown thus:— 
FIG. 30.

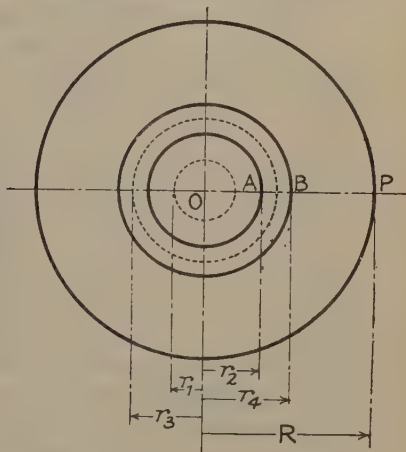


FIG. 31.

point such that the circle through it with O as centre divides AB into two equal areas. The positions of the points of measurement in the other zones are determined in the same manner; and it will be evident that the whole section is in effect divided into $2n$ parts by circles of radii r_1, r_2, r_3 , etc., and

* The British Standards Institution recommends¹ that extra observations should be taken in the sections adjoining the walls of the duct, particularly at the corners. It appears from Figs. 28 and 29 of B.S. 1042, however, that, under the B.S.I. Specification, the smallest areas into which the duct is divided may be much larger than the 36 square inches recommended by the author: Fig. 29 of the Specification, for example, shows an area 24 in. $\times$ 12 in. With such large sections, extra readings, as recommended by the B.S.I., are certainly needed in the corner and wall areas; but there does not seem to be any reason why the average value of the velocity head in the corner sections, as obtained by the somewhat complicated method advocated by the B.S.I., is likely to be nearer the true average than the ordinary arithmetic mean.

that n velocity measurements are made at the radii r_1, r_3, r_5 , and so on, the radius of the outermost point of observation being r_{2n-1} .

If R is the length of the radius of the pipe, the following equations express the conditions that all the zones are of equal area :

$$\pi(R^2 - r_{2n-1}^2) = \pi(r_{2n-1}^2 - r_{2n-2}^2) = \text{etc.} \\ = \pi(r_2^2 - r_1^2) = \pi r_1^2,$$

so that

$$r_2^2 = 2r_1^2, \\ r_3^2 = 3r_1^2, \text{ etc.,} \\ r_{2n-1}^2 = (2n-1)r_1^2,$$

and

$$R^2 = 2nr_1^2.$$

Hence

$$r_1 = R\sqrt{\frac{1}{2n}}, \\ r_2 = R\sqrt{\frac{2}{2n}}, \text{ etc.,}$$

and

$$r_{2n-1} = R\sqrt{\frac{2n-1}{2n}}.$$

The n readings of velocity are therefore to be taken at radii

$$R\sqrt{\frac{1}{2n}}, R\sqrt{\frac{3}{2n}}, R\sqrt{\frac{5}{2n}}, \text{ etc., the final radius being } R\sqrt{\frac{2n-1}{2n}}.$$

In practice, it is not sufficient to take readings along one radius only. The mean velocity for each zone should be obtained from at least four readings, one along each of four radii of which two form one pipe diameter and the other two the perpendicular diameter. It follows that $2n$ readings of velocity will be taken along each of the two chosen diameters, making a total number of $4n$ observations ; and it will usually be found convenient to locate the positions of observation by measuring from one end of the diameter. Expressed in this way, the distances at which readings of velocity are to be taken will be

$$\frac{D}{2}\left(1 - \sqrt{\frac{2n-1}{2n}}\right), \frac{D}{2}\left(1 - \sqrt{\frac{2n-3}{2n}}\right), \text{ etc.,}$$

$$\text{up to } \frac{D}{2}\left(1 - \sqrt{\frac{1}{2n}}\right)$$

on one side of the centre of the pipe ; and

$$\frac{D}{2} \left(1 + \sqrt{\frac{2n-1}{2n}} \right), \frac{D}{2} \left(1 + \sqrt{\frac{2n-3}{2n}} \right), \text{ etc.,}$$

$$\text{up to } \frac{D}{2} \left(1 + \sqrt{\frac{1}{2n}} \right)$$

on the other, where D is the internal pipe diameter.

The number of observations to be taken will be governed largely by the considerations set out above. As a practical guide, however, we may take it that n should be not less than 5, so that at least ten observations will be made along each of two mutually perpendicular diameters at distances from the wall of 0.026 D , 0.082 D , 0.146 D , 0.226 D , 0.342 D , 0.658 D , 0.774 D , 0.854 D , 0.919 D , and 0.975 D .

Special consideration is needed for the accurate measurement of the flow in small pipes of less than about 8 inches diameter ; this subject is discussed more fully in Chapter VIII.

The above rules for determining the number of areas into which pipes should be divided must not be regarded as rigidly applicable in all cases. They will be found to give reasonably accurate results, provided that the velocity distribution across a diameter of the pipe follows a more or less smooth curve ; but if any doubt is felt about the nature of the distribution in a particular case, a preliminary exploration, comprising velocity measurements at a number of points in the section, should always be made. If the velocity distribution is found to be unduly irregular, and it is not possible to make measurements at some other section of the pipe where conditions are more favourable, there will be no alternative but to take a larger number of readings than the above rules indicate.

Alternative Method of Dividing Circular Pipes.—The system just described of dividing a circular pipe into zones of equal area has the disadvantage that the outer zones come close to the pipe walls ; and so if a vane anemometer is being used to measure the velocity it may be impossible to place the instrument (which is generally at least $2\frac{1}{2}$ to 3 inches in diameter) with its centre at the outermost of the prescribed radii. Mr. H. C. Stevenson, of the Factory Department, Home Office, has suggested the following alternative in such cases : The pipe is

divided into annular zones and one circular central zone, as before ; but now the zones are not of equal area, each being of width equal to the radius of the central circular zone. Thus, if r_1 is the radius of this circular zone, r_2 the outer radius of the next zone, r_3 the outer radius of the next, and so on, then

$$r_1 = (r_2 - r_1) = (r_3 - r_2) = (r_4 - r_3), \text{ etc.}$$

The mean velocity in each zone is then determined. Let these velocities be v_1, v_2, v_3 , etc., and let n be the number of zones. Then the mean velocity v_m is obtained from the equation

$$v_m = \frac{1}{n^2} \left[v_1 + 3v_2 + 5v_3 + 7v_4 + \dots + (2n - 1)v_n \right]. \quad (3)$$

Alternatively, one velocity reading is taken in the central zone, three in the next, five in the next, and so on ; v_m is then the sum of these velocities divided by n^2 , the velocity readings in each zone being well distributed to get values in all parts of the pipe section.

Equation (3) can easily be established as follows : The areas of successive zones, starting from the centre, are

$$\pi r_1^2, \pi(r_2^2 - r_1^2), \pi(r_3^2 - r_2^2), \text{ etc. ;}$$

and, since the zones are of equal width,

$$r_2 = 2r_1, r_3 = 3r_1, r_4 = 4r_1, \text{ etc ;}$$

so that the areas become

$$\pi r_1^2, 3\pi r_1^2, 5\pi r_1^2, 7\pi r_1^2, \text{ etc., and finally } (2n - 1)\pi r_1^2.$$

Hence, the total quantity of air flowing through the pipe is given by

$$\pi r_1^2 [v_1 + 3v_2 + 5v_3 + \dots + (2n - 1)v_n]$$

and, by definition, this is equal to $\pi R^2 v_m$.

Equating these two expressions for the total quantity flowing, and putting in the condition that $R = nr_1$, we get equation (3). It will be seen that the factor by which the velocity in each zone is multiplied is equal to the ratio of the area of that zone to the area of the central zone. In the method first described, when the areas of all the zones are equal, equal weight is attached to the velocity in each zone.

Measurements with Pitot-Static Tubes.—Measurements of velocity head are made either by means of the combined pitot-static tube or, as described later (see page 98), by means of separate total-head and static tubes inserted successively at the appropriate points. In either case, the resultant reading at each point of measurement will be a differential pressure p which, as we have already learned, is related to the local speed v by the equation

$$p = \frac{1}{2} \rho v^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where, if v is to be in feet per second, ρ must be expressed in pounds weight per cubic foot/ g , and p in pounds per square foot.

Now ρ will depend upon the temperature and pressure of the air ; it is therefore necessary to measure the temperature in the pipe and the barometric pressure. Strictly, the air in the pipe will be at a pressure $b + p_s$, where p_s is the static pressure in the pipe measured above or below the barometric pressure b , but as a rule p_s can be neglected in relation to b . For the present we shall assume that this is permissible ; the correction to be applied when p_s is too large to be neglected is discussed later (page 89).

When the temperature is 15.6°C. (60°F.) and the pressure 760 mm. of mercury (29.92 inches), $\rho = 0.00237$, so that, for any other temperature $t^{\circ}\text{C.}$ and pressure b mm. of mercury,

$$\begin{aligned} \rho &= 0.00237 \times \frac{288.6}{273 + t} \times \frac{b}{760} \\ &= 0.0009 \frac{b}{273 + t} \quad \dots \dots \dots (5) \end{aligned}$$

The velocity pressure p in equation (4) will as a rule be measured on some form of water manometer (see Chapter IX), which will give it in terms of inches of water-column. Now 1 inch of water-column at ordinary temperatures is equivalent to a pressure of $62 \cdot 36 / 12$, i.e. 5.2 lb. per square foot ; so that, if h is the velocity head expressed in inches of water,

$$h = p/5 \cdot 2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and, on substitution of (5) and (6), equation (4) becomes

$$h = 0.0000865v^2 \times \frac{b}{273 + t}$$

which may be written

$$v = k\sqrt{h}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where

$$k = 107.5 \sqrt{\frac{273 + t}{b}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This equation is used when t is measured in degrees Centigrade and b in millimetres of mercury. If it is preferred to measure the temperature on the Fahrenheit scale and the barometric height in inches of mercury, the following expression for k should be used:

$$k = 15.89 \sqrt{\frac{460 + t}{b}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

We can now apply these results to the calculation of the mean rate of flow through a pipe. The observations will consist of a number of measurements of velocity head h_1, h_2 , etc., taken at points determined by one of the methods previously described. Thus, when the pipe has been divided into a number of zones of equal area we shall use equation (2), and for v_1, v_2 , etc., we may substitute their equivalents $k\sqrt{h_1}, k\sqrt{h_2}$, etc. We thus obtain for the mean velocity v_m ,

$$v_m = \frac{k}{N} (\sqrt{h_1} + \sqrt{h_2} + \sqrt{h_3} + \dots), \quad . \quad . \quad . \quad (10)$$

where N is the total number of observations ($= 4n$ for a circular pipe divided into n zones of equal area, since readings are then taken along four radii as previously explained).

If h_m is the velocity head corresponding to the mean rate of flow v_m , then

$$\sqrt{h_m} = \frac{v_m}{k} = \frac{1}{N} (\sqrt{h_1} + \sqrt{h_2} + \sqrt{h_3} + \dots). \quad . \quad . \quad . \quad (11)$$

The volume of air flowing in cubic feet per second will be, if A is the area of the pipe in square feet,

$$\frac{Ak}{N} (\sqrt{h_1} + \sqrt{h_2} + \dots), \text{ or } Ak\sqrt{h_m}.$$

Similarly, the weight of air passing in pounds per second will be

$$Ag\rho k\sqrt{h_m}.$$

Generally, engineers prefer to measure the flow in cubic feet V , or pounds Q , per minute. For these units we may write

$$\left. \begin{aligned} V \text{ (cubic feet per minute)} &= C_1 A \sqrt{h_m} \\ Q \text{ (pounds per minute)} &= C_2 A \sqrt{h_m} \end{aligned} \right\}, \quad (12)$$

where C_1 and C_2 have the following values :

(a) For the Centigrade and millimetres-of-mercury scales

$$\left. \begin{aligned} C_1 &= 6450 \sqrt{\frac{273+t}{b}} \\ C_2 &= 186.6 \sqrt{\frac{b}{273+t}} \end{aligned} \right\} \dots \dots \dots (13)$$

(b) For the Fahrenheit and inches-of-mercury scales

$$\left. \begin{aligned} C_1 &= 953 \sqrt{\frac{460+t}{b}} \\ C_2 &= 1264 \sqrt{\frac{b}{460+t}} \end{aligned} \right\} \dots \dots \dots (14)$$

We have already remarked that a correction may be necessary to allow for the fact that the pressure of the air in the pipe is actually not b , the barometric pressure, but $b + p_s$, where p_s is the static pressure in the pipe above or below atmospheric pressure. Now, since the value of b is nearly always about 30 inches of mercury, i.e., about 400 inches of water, and since only the square root of the pressure is involved in the equations (see (13) and (14)), it is clear that p_s can as a rule be as much as 8 inches of water (i.e., 2 per cent. of b) without causing the values of V and Q as computed from equations (12), (13), and (14) to be more than 1 per cent. in error. Hence, as remarked earlier, it is very often sufficiently accurate to assume that the air in the pipe is at atmospheric pressure. If the static pressure is more than ± 8 inches of water, the errors will exceed 1 per cent. ; and then, if higher accuracy is required, in place of b in the above formulæ we must use $b + p_s$, where p_s , which is usually measured in inches of water, must be reduced to the same units as those in which b is measured. If p_s is measured in inches of water, its value in inches of mercury is $0.074 p_s$, and in millimetres of mercury $1.8 p_s$; and the appropriate one of these two values must be added to or subtracted from b in the

formulae, according to whether the static pressure is above or below the atmospheric pressure.

If, instead of taking observations at the mean radii of zones of equal area, we use the alternative method in which the zones are of equal width, we use exactly the same equations (12), (13), and (14), remembering only that $\sqrt{h_m}$ in (12) is now given by

$$\frac{1}{N^2} (\sqrt{h_1} + \sqrt{h_2} + \sqrt{h_3} + \dots),^*$$

where h_1, h_2 , etc., are the individual readings of velocity head and N is the total number of readings as before. This can easily be verified from equation (3).

Summary of Procedure for Measuring Quantity with Pitot-Static Tubes.—We can now summarise as follows the procedure to be followed in obtaining, by means of pitot-static tubes, the quantity of air flowing in a pipe of cross-sectional area A square feet :

- (1) Measure the velocity head in inches of water at a number of points across the section determined by one of the two alternative methods indicated on pages 83–86.
- (2) Determine the square root of each velocity head.
- (3) Determine $\sqrt{h_m}$, the square root of the head equivalent to the mean velocity, as follows :
 - (a) if the equal-area method of division has been used—by dividing the sum of the square roots of the velocity heads by the number of observations ;
 - (b) if the equal-width method of division has been used with one reading in the central zone, three in the next, five in the next, and so on—by dividing the sum of the square roots of the individual velocity heads by the square of the number of observations.

* This assumes that, as suggested in the paragraph immediately following the statement of equation (3), one reading is taken in the central zone, three in the next, five in the next, and so on. If four readings in each zone are taken along two mutually perpendicular diameters, as advocated in connection with the equal-area method of division, the expression for $\sqrt{h_m}$ in the equal-width method becomes more complicated, viz. : $\frac{1}{4}n^2 (\sqrt{h_1} + 3\sqrt{h_2} + 5\sqrt{h_3} + 7\sqrt{h_4} + \dots)$, where h_1, h_2 , etc., are the means of the four readings taken in their respective zones and n is the number of zones.

- (4) Calculate the quantity of air flowing by the use of equations (12) and (13) or (12) and (14) (according to the system of units adopted), using the value of $\sqrt{h_m}$ as just determined from steps (1) to (3).
- (5) The quantity b in (13) and (14) may be taken as the observed barometric pressure so long as the static pressure in the pipe does not numerically exceed 8 inches of water.* Otherwise b must be increased or decreased by the static pressure, according to the units in which the barometric height is observed. The conversion factors are given above (page 89).
- (6) The above assumes that the pitot-static tube in use has a calibration factor (see Chapter IV) of 1. If the factor differs from 1, being equal to χ , say, this is simply allowed for by using χC_1 or χC_2 instead of C_1 or C_2 , as the case may be, in equations (12), (13), and (14).
- (7) In special cases, when the highest possible accuracy is required, it may be desirable to apply corrections for errors on the pitot-static readings due to turbulence and variation of velocity across the section. The necessary data will be found on pages 73 and 74.

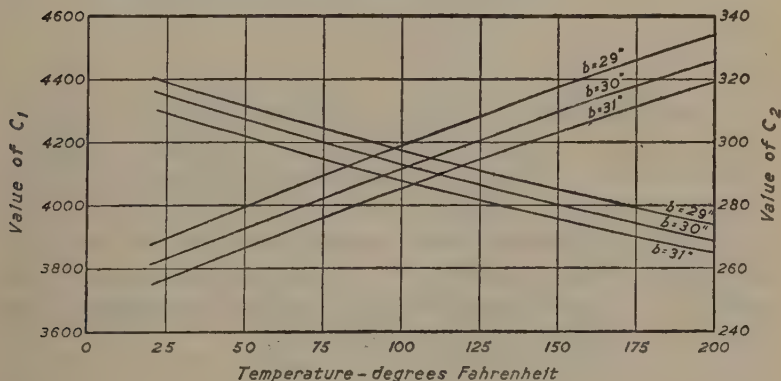


FIG. 32.

Temperature and Pressure Charts.—If long series of quantity measurements have to be undertaken, and the variation of air temperature throughout the series is likely to be

* If an error not exceeding 1 per cent. on the rate of flow is permissible (see page 89).

appreciable, it will save time to construct a chart like that in Fig. 32, from which the coefficients C_1 and C_2 can be read off directly. The chart is prepared by calculating C_1 and C_2 for each of a number of temperatures within the range to be covered, and plotting the results on a temperature base. The calculations are made by the use of equations (13) or (14), and it will usually suffice to carry them through for three values of b , say 29, 30, and 31 inches of mercury if British units are being used. Thus we shall get three curves for C_1 and three for C_2 , each appropriate to one of the selected barometric pressures. Values of the coefficients for intermediate barometric pressures can be obtained by interpolation.

Measurement of the Mean Total Head at a Section of a Pipe.—It was shown in Chapter III that the resistance of a length of pipe between any two sections A and B to the flow of air along it may be expressed as the difference between the true mean total heads T_1 and T_2 at A and B. It was also shown that, if v is the velocity at a section of a circular pipe at radius r , and R is the internal pipe radius,

$$T = \frac{1}{2}\rho \frac{\int_0^R v^3 r dr}{\int_0^R v r dr} + \frac{\int_0^R p v r dr}{\int_0^R v r dr} \quad (15)$$

In order, therefore, to determine T , the value of each of the integrals in (15) must be calculated. For the highest accuracy, p and v should be measured at a sufficient number of radii to enable curves to be defined showing the variation of v and p with r . From these data it is possible to obtain, at each value of r , values of the quantities $v^3 r$, $v r$, and $p v r$, and to plot them against r . The areas enclosed by the curves thus obtained and the axis of r will be equal respectively to

$$\int_0^R v^3 r dr, \int_0^R v r dr, \text{ and } \int_0^R p v r dr.*$$

The value of T is then calculable from (15).

* A numerical example of one such integration, $\int_0^R v r dr$, is given in Chapter XI.

This method is laborious, and often unnecessarily so. In many cases, it will be good enough to observe the mean velocity and the mean static pressure in each of a number of zones in which we imagine the pipe to be divided, exactly as in the procedure described on pages 81-82 for the measurement of mean velocity. Following the analysis there given, we have flowing through the small element of area a_1 a volume per second of air $a_1 v_1$ whose mass is $\rho a_1 v_1$ and kinetic energy $\rho a_1 v_1 \cdot v_1^2/2$. If p_1 is the static pressure at the element, the potential energy of the air is $p_1 a_1 v_1$. Hence, the total energy passing the whole of the section of the pipe per second is equal to $\frac{1}{2}\rho \Sigma a_1 v_1^3 + \Sigma p_1 a_1 v_1$.* Also, the total volume of air flowing per second (see equation (1)) is $\Sigma a_1 v_1$, so that the energy per unit volume, which is equal to the mean total head T at the section, is given by

$$T = \frac{1}{2}\rho \frac{\Sigma a_1 v_1^3}{\Sigma a_1 v_1} + \frac{\Sigma p_1 a_1 v_1}{\Sigma a_1 v_1} \quad (16)$$

We can choose one of the methods already discussed of subdividing the pipe into zones. Since measurements of mean total head will always be undertaken by means of pitot and static tubes, we need consider only the method of zones of equal area; for the method of zones of equal width is primarily intended for measurement by means of the vane anemometer (see above, page 85). The analysis that follows can easily be extended to the latter method, however.

If, then, the pipe is divided into zones of equal area, $a_1 = a_2 = a_3$, etc., so that (16) becomes

$$T = \frac{1}{2}\rho \frac{(v_1^3 + v_2^3 + \dots + v_n^3)}{(v_1 + v_2 + \dots + v_n)} + \frac{(p_1 v_1 + p_2 v_2 + \dots + p_n v_n)}{(v_1 + v_2 + \dots + v_n)} \quad (17)$$

We note the similarity of form of this equation to equation (2) for the mean velocity v_m . In fact, observations taken for the determination of v_m will suffice for the calculation of the first term of (17), while for the second term we need, in addition, only measurements of static pressure at the points of velocity determination.

The most convenient practical method of using (17) is as

* It is assumed that pressure changes are small enough to enable ρ to be regarded as the same for each element.

follows: Our measurements will consist of a number of readings, in inches of water gauge, of velocity head (h_1, h_2, h_3 , etc.), and of static-pressure head (h_{s1}, h_{s2}, h_{s3} , etc.). Using the relation between velocity v and velocity head h , viz.

$$v = \sqrt{10.4h/\rho}$$

(which follows immediately from equation (6) and the fundamental equation $p = \frac{1}{2}\rho v^2$), we obtain from (17), in which T is given in pounds per square foot,

T (inches of water) =

$$\frac{\rho \sqrt{\frac{10.4^3}{\rho^3}} (\sqrt{h_1^3} + \sqrt{h_2^3} + \dots) \sqrt{\frac{10.4}{\rho}} (h_{s1}\sqrt{h_1} + h_{s2}\sqrt{h_2} + \dots)}{10.4 \sqrt{\frac{10.4}{\rho}} (\sqrt{h_1} + \sqrt{h_2} + \dots) \sqrt{\frac{10.4}{\rho}} (\sqrt{h_1} + \sqrt{h_2} + \dots)} +$$

$$\text{or } T \text{ (inches of water)} = \frac{(\sqrt{h_1^3} + \sqrt{h_2^3} + \dots)}{(\sqrt{h_1} + \sqrt{h_2} + \dots)} + \frac{(h_{s1}\sqrt{h_1} + h_{s2}\sqrt{h_2} + \dots)}{(\sqrt{h_1} + \sqrt{h_2} + \dots)}. \quad (18)$$

As a rule, the static pressure will be fairly uniform across the section. Any departure from uniformity means that the flow is not parallel to the walls of the pipe, since the air will tend to flow towards the regions of low pressure. If the departure from axial flow is very pronounced, it is doubtful whether any method of measurement will give good accuracy, partly because both velocity-head and static-pressure readings at the various points of measurement may be in error on account of the inclination of the local direction of flow to the pitot-static head. Measurements at sections where such conditions prevail should therefore always be avoided when possible. If, then, we can assume that the static pressure is uniform across the section, $h_{s1} = h_{s2} = \text{etc.} = h_s$, and (18) simplifies to

$$T = \frac{(\sqrt{h_1^3} + \sqrt{h_2^3} + \dots)}{(\sqrt{h_1} + \sqrt{h_2} + \dots)} + h_s. \quad \dots \quad (19)$$

Practical Notes on the Use of Pitot-Static Tubes.—

As we have seen, in order to determine the mean rate of flow, or the mean total head at a section, the pitot-static tube must be inserted into the pipe through a hole cut in the side, and

traversed across the section, readings being taken at a number of pre-determined points. It is convenient to provide a number of wire rings fitting the stem of the instrument fairly tightly, whose positions can be adjusted before the insertion of the tube into the pipe, so that, when each ring in turn is up against the outside of the pipe, the axis of the head is at one of the desired positions inside. A linear scale marked on the stem further facilitates the correct location of the points of measurement.

The head of the instrument should, of course, point upstream, with its axis parallel to the direction of motion of the air, which is usually assumed to be parallel to the walls of the pipe. A useful method of ensuring that the head is in the desired attitude is to have, permanently fixed to the extreme end of the stem outside the pipe, a straight-edge, five or six inches in length, parallel to the head, which can be sighted along the outside wall of the pipe. The hole cut in the pipe wall should not be larger than is necessary for the introduction of the tubes, and should preferably be stopped up, as far as possible, when the instrument is in place.

The usual dimensions of the combined pitot-static tubes on the market, either of the N.P.L. pattern or of the alternative type shown in Fig. 22, are such that they can be used in pipes up to about 15 inches diameter *; but there is no reason why the stems should be limited in length to this amount. In practice, however, it frequently happens either that the lengths of the tubes available are too short to permit them to be traversed completely across a section of a pipe, or that the instruments are unsuitable for some other reason. In such cases, when modified forms of pitot tubes have to be made up to suit particular conditions, it obviously simplifies construction to have the total-head and static tubes separate, and to use them separately or clamped together. Satisfactory apparatus can be made from material generally to be found in any factory: steel or brass tubing of about $\frac{5}{16}$ or $\frac{3}{8}$ -inch external diameter is suitable for most purposes, provided the pipe diameter exceeds about 8 inches. Some care is necessary in shaping the heads of both tubes if it is

* The smallest diameter of pipe in which instruments of the normal tube diameter, viz., $\frac{5}{16}$ inch, should be used is 6 inches or preferably 8 inches, for the reasons stated in Chapter VIII, where methods for use in smaller pipes are described.

intended to use them clamped together. Although any open-ended tube facing the stream will indicate total head correctly,

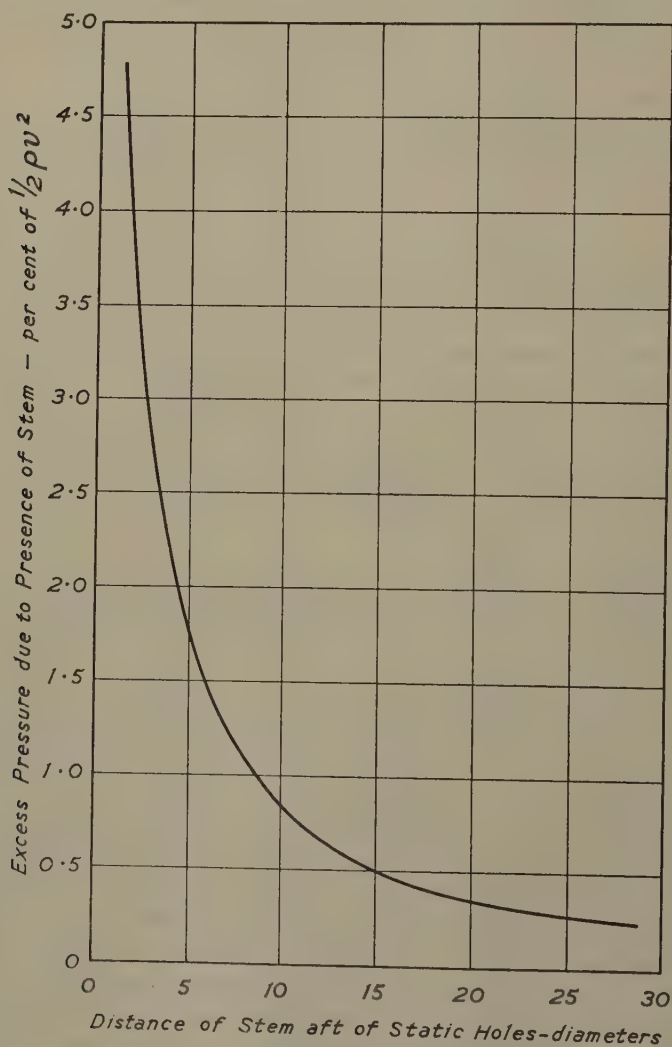


FIG. 33.

if the tubing is at all thick-walled the flow of air around the outside will be somewhat disturbed, and may therefore lead to error in the measurement of the static pressure by means of

another tube placed near the first. For this reason, it is advisable to file the outside of the total-head tube near the end to form a taper, so as to present only a thin annular edge to the wind. For the static tube, it will probably be found simplest to adopt the type having a hemispherical nose. This tube can easily be constructed by bending a straight piece of tubing to form the head and the stem, and closing the open end of the head by means of a hemispherical plug having a short shank that forms a push fit or screws into the tube, the joint being made airtight by finishing it off with solder. Care should be taken to leave no gap between the plug and the end of the tube: the outline of the head should be smooth and continuous. The static holes should be 6 tube diameters behind the junction of the plug and the head, and the stem 8 diameters behind the holes, if convenient. The tube will then indicate the static pressure correctly; but if, for some reason, it is more convenient to have the stem at some other distance behind the static holes, the correction to be applied to the observed pressure in order to obtain the true static pressure can easily be obtained by reference to Fig. 33, which shows the pressure at the orifices of a static tube due to the presence of the stem at different positions downstream, the pressure being expressed as a percentage of the velocity head.² The size of the static holes is not of great importance, provided that they are not too large; a diameter of about 0.05 inch in $\frac{3}{8}$ -inch diameter tubing is suitable. It is important that in drilling the holes the outside of the tube should be left entirely free from burrs.

When separate total-head and static tubes are used clamped together, the air flow round each will be affected by the presence of the other; so that, if the tubes are too near each other, the accuracy of the static-head reading will be impaired by the interference effect of the total-head tube, although the reading of the latter will not be correspondingly affected. Experiments made by the author have shown that the tubes may safely be clamped with their axes not less than 5 tube diameters apart. Similarly, all brackets and clamps should preferably be confined to the stems of the tubes outside the pipe, in order that they should not produce pressure effects at the static holes similar to those due to the stem.

It may be mentioned here that when separate tubes are used

for the measurement of total head and static pressure, these observations need not necessarily be made simultaneously, provided that the conditions of flow are reasonably steady and constant. It is often more convenient to read first the static pressure and then the total head at the point (or *vice versa*), the appropriate tubes being successively introduced into the pipe. This necessitates the cutting of smaller holes in the piping than when the tubes are inserted clamped together. The velocity head is then, of course, obtained by subtracting the observed reading of the static tube from that of the total-head tube, due attention being paid to the signs of the observed pressures (i.e., whether they are above or below the atmospheric pressure).

The Reduction of Unsteadiness due to Pressure Fluctuations.—Unsteady readings in pitot-static measurements may be caused by the random pressure fluctuations



FIG. 34.—Method of equalising the damping on both sides of a pitot-static tube.

that sometimes occur, particularly when one end of the pipe system is open to atmosphere and therefore influenced by draughts; it is remarkable what large disturbances can then occur on a windy day. A most useful method of reducing the resulting unsteadiness of the manometer readings in such conditions has been suggested by Mr. L. F. G. Simmons, of the National Physical Laboratory. It depends in principle on the fact that a pressure pulse acting on one end of a long tube is not transmitted instantaneously to the other end. Moreover, the amplitude of the transmitted pulse is not the same as that of the applied pulse. A tube of given dimensions exerts, in effect, a definite amount of damping on the motion of a pressure wave along its length. Now the amount of damping contributed by the total-head tube will obviously be less than that of the static tube on account of the larger resistance of the small static orifices. Hence, even if the same pulse acts simultaneously

both at the mouth of the total-head tube and at the static holes, the fluctuations will not be damped out to the same extent by the time the pressures are transmitted to the manometer, unless extra resistance is introduced in the total-head connecting tube. Such extra resistance may be provided by using a longer length of rubber tubing on the total-head side, but a more convenient method is to introduce a piece of glass capillary tube into the total-head connecting tube. It will generally be found that an inch or two of capillary is enough.

The necessary lengths of tubing can readily be determined by the method illustrated in Fig. 34. The head of the pitot-static tube is introduced through a cork into a length of brass tubing about $\frac{3}{4}$ inch in diameter, the other end of which is connected, as shown, to a convenient length of large-bore rubber tubing closed by a second cork. By squeezing this rubber tubing in the hand, fluctuating pressures can be applied simultaneously to the mouth of the total-head tube and the orifices in the static tube. If the other ends of these tubes are connected to the manometer in the usual manner, the lengths of the connecting tubes, or the amount of capillary inserted, can be altered until the oscillations of the manometric liquid become small even for large applied pressure fluctuations. When the connections have been thus adjusted by trial and error, it will be found that, even under unsteady conditions of measurement, sufficiently steady readings will generally be obtained on the manometer.

A system balanced in this manner under pressure variations only will not give entirely steady readings when used to measure flow in a pipe where the velocity and static pressure are both fluctuating; but it will, nevertheless, usually lead to greatly improved steadiness.

REFERENCES

1. British Standards Institution, *Flow Measurement*, British Standard Code, B.S. 1042 : 1943.
2. Ower and Johansen, *Reports and Memoranda of the Aeronautical Research Committee*, No. 981 (1926).

CHAPTER VI

THE PLATE ORIFICE, VENTURI TUBE, AND SHAPED NOZZLE

THE pitot tube, in conjunction with a properly designed static tube, has the important advantage that its calibration has been definitely established by careful experiment, so that it can be, and, in fact, is, regarded as a fundamental standard for the measurement of air speed. In the absence of any other standard of equal precision, it must play an important part in the calibration of all other instruments for air-speed measurements. It is, however, subject to certain shortcomings for practical work. In the first place, as we have seen, it is not well suited to a rapid determination of quantity, since the pipe section has to be traversed and a certain amount of arithmetical work has to be performed. Secondly, the pressures to be observed are very often small, so that sensitive manometers are required to give the necessary accuracy ; and it is very difficult to use these sensitive instruments under ordinary industrial conditions, except in the laboratory.

For these reasons, methods have been sought in which the pressures to be measured are automatically augmented, and in which single readings enable the mean velocity in a pipe to be determined. Although such methods lack the certainty of the pitot tube when used with a manometer of the requisite precision (as a general rule, in fact, it is safest to calibrate the apparatus employed by means of the pitot tube if the highest accuracy is required), they are undeniably superior to that instrument as regards the ease and rapidity with which the necessary observations can be performed. Briefly, these methods involve the insertion into the stream of some device, such as a nozzle or a plate with a circular hole cut in it, which constricts the free area through which the air can pass and so effects a local increase in velocity. It is then found that a comparatively large drop of static pressure is experienced by the air in passing from the full diameter of the pipe to the

section at which the constriction, and hence also the velocity, is a maximum, and that this pressure drop can be used as a measure of the air speed.

Equations for the Theoretical Flow through a Constriction.—Suppose that in a given pipe the walls are shaped as shown in Fig. 35, so that the area at section BB is less than at AA. Let p_1 , v_1 , ρ_1 , a_1 be respectively the absolute static pressure (measured from zero), the mean velocity, the density of the air, and the area of the section at AA; and p_2 , v_2 , ρ_2 , a_2 be the corresponding quantities at BB. Also imagine that the pipe walls at AA and BB and the directions of flow at these sections are parallel to the axis of the pipe.

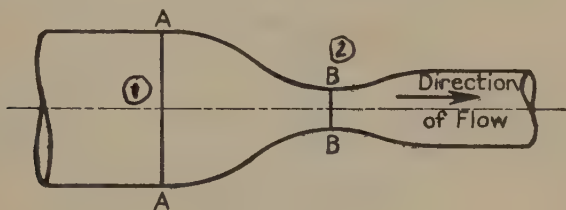


FIG. 35.

We shall assume the flow to be frictionless, and at a later stage introduce numerical coefficients to cover the error caused by this and other assumptions. Such coefficients will have to be determined experimentally.

The difference between the static pressures p_1 and p_2 will depend mainly on the ratio of a_1 to a_2 . In many cases, the ratio p_2/p_1 does not differ much from unity,* and in these circumstances ρ_1 and ρ_2 , the densities of the air at the two sections, may be considered as effectively equal.

We start with the general equation derived in Chapter II, page 13, viz.

$$\frac{v^2}{2} + \int \frac{dp}{\rho} = \text{constant},$$

* It must be remembered that p_1 and p_2 are measured from zero, so that their mean value in most cases will be approximately atmospheric pressure or, say, 400 inches of water. Hence in such cases they may differ by about 4 inches of water—an easily measurable amount—before the ratio of one to the other differs from unity by more than 1 per cent.

and apply it to the air between sections AA and BB. We then have

$$\frac{v_2^2 - v_1^2}{2} = \int_{p_2}^{p_1} \frac{dp}{\rho} \quad \dots \quad (1)$$

Case (a): p_2/p_1 Nearly Equal to Unity.—In this case, ρ_2 may be taken as equal to ρ_1 , which is equivalent to regarding the air as incompressible. Equation (1) becomes

$$\frac{v_2^2 - v_1^2}{2} = \frac{p_1 - p_2}{\rho} \quad \dots \quad (2)$$

Also, the weight of air flowing per unit time across AA is equal to that across BB, i.e.,

$$\rho a_1 v_1 = \rho a_2 v_2,$$

or

$$v_1 = \frac{a_2 v_2}{a_1} \quad \dots \quad (3)$$

Substituting (3) in (2), we have

$$v_2^2 = \frac{2(p_1 - p_2)}{\rho \left(1 - \frac{a_2^2}{a_1^2}\right)}, \quad \dots \quad (3a)$$

and the theoretical weight of air flowing per second * is given by

$$q_t = g \rho a_2 v_2 = a_2 g \sqrt{\frac{2\rho(p_1 - p_2)}{1 - \frac{a_2^2}{a_1^2}}}, \quad \dots \quad (4)$$

or, writing m for the ratio a_2/a_1 , we have finally

$$q_t = a_2 g \sqrt{\frac{2\rho(p_1 - p_2)}{1 - m^2}} \quad \dots \quad (5)$$

as the equation for the flow of an incompressible fluid through a constriction.

Case (b): p_2/p_1 Appreciably Less than Unity.—Compressibility can no longer be neglected, so that ρ_2 can not now be taken as equal to ρ_1 : the density will change with p according to the usual laws for gases. In practice the flow along the pipe is

* It should be remembered that ρ here has its customary significance of weight per unit volume/g, so that the weight of a volume V of air is $g\rho V$ units of weight.

almost always sufficiently rapid for the expansion (p_2 is less than p_1) in the constriction to be taken as adiabatic, so that we have

$$\frac{p_1}{\rho_1^\gamma} = C = \frac{p_2}{\rho_2^\gamma}, \quad \dots \quad (6)$$

where C is a constant and γ is the usual index for adiabatic expansion (1.408 for dry air).

Hence, in this case, (1) becomes, since, from (6), $\frac{1}{\rho} = \left(\frac{C}{p}\right)^{1/\gamma}$,

$$\frac{v_2^2 - v_1^2}{2} = C^{1/\gamma} \int_{p_2}^{p_1} \frac{dp}{p^{1/\gamma}} = C^{1/\gamma} \frac{\gamma}{\gamma - 1} \left\{ p_1^{\frac{\gamma-1}{\gamma}} - p_2^{\frac{\gamma-1}{\gamma}} \right\},$$

and, if we put $C^{1/\gamma} = p_1^{1/\gamma} / \rho_1$, this reduces to

$$\frac{v_2^2 - v_1^2}{2} = \frac{\gamma}{\gamma - 1} \cdot \frac{p_1}{\rho_1} \left\{ 1 - \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right\} \quad \dots \quad (7)$$

The equation for continuity of flow, i.e., equal weights of air passing sections AA and BB in unit time, is, as before,

$$\rho_1 a_1 v_1 = \rho_2 a_2 v_2,$$

or, if the value for ρ_2 / ρ_1 from (6) is inserted,

$$v_1 = \left(\frac{p_2}{p_1} \right)^{1/\gamma} \frac{a_2}{a_1} v_2.$$

Putting this value of v_1 in (7), and reducing, we have

$$v_2 = \sqrt{\frac{2 \cdot \frac{\gamma}{\gamma - 1} \cdot \frac{p_1}{\rho_1} \left\{ 1 - \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right\}}{1 - \frac{a_2^2}{a_1^2} \left(\frac{p_2}{p_1} \right)^{2/\gamma}}}; \quad \dots \quad (7a)$$

so that, if we write, as before, $m = a_2 / a_1$, the weight of air flowing per second becomes

$$\begin{aligned} q_t &= a_2 g \rho_2 v_2 = a_2 g \left(\frac{p_2}{p_1} \right)^{1/\gamma} \rho_1 v_2 \\ &= a_2 g \sqrt{\frac{2 \cdot \frac{\gamma}{\gamma - 1} \cdot \rho_1 p_2^{2/\gamma} p_1^{\frac{\gamma-2}{\gamma}} \left\{ 1 - \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right\}}{1 - m^2 \left(\frac{p_2}{p_1} \right)^{2/\gamma}}}; \end{aligned}$$

$$\text{i.e. } q_t = a_2 g \sqrt{\frac{2p_1 \rho_1 \cdot \frac{\gamma}{\gamma-1} \left\{ 1 - \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right\} \left(\frac{p_2}{p_1} \right)^{2/\gamma}}{1 - m^2 \left(\frac{p_2}{p_1} \right)^{2/\gamma}}} \quad (8)$$

This equation can be thrown into a form more convenient for practical purposes, involving the pressure drop $p_1 - p_2$ across the constriction, which is easily measurable. Moreover, the modified equation is more readily comparable with (5), its counterpart for the flow of an incompressible fluid.

Let us write π for the expansion ratio p_2/p_1 wherever it occurs in (8); then we may make the following substitution for p_1 when it occurs by itself (not as part of the ratio p_2/p_1):

$$p_1 = \frac{p_1}{(p_1 - p_2)} \times (p_1 - p_2) = (p_1 - p_2) \times \frac{1}{1 - \pi}.$$

Equation (8) may now be written thus:

$$q_t = a_2 g \sqrt{\frac{2\rho_1(p_1 - p_2)}{1 - m^2}} \sqrt{\frac{(1 - m^2)^{\frac{\gamma}{\gamma-1}} \cdot \frac{1}{1 - \pi} \left(1 - \pi^{\frac{\gamma-1}{\gamma}} \right) \pi^{2/\gamma}}{1 - m^2 \pi^{2/\gamma}}} \quad (9)$$

Comparing this equation with (5), we see that it is the equation for the discharge of an incompressible fluid, modified by the term under the second radical, which allows for the effect of compressibility. Equation (9) is the general equation for the flow of a fluid through a constriction.

The Practical Use of Constrictions.—In practice, the constriction in a pipe line consists of some device such as an orifice plate, a nozzle, or a venturi tube, each of which is considered in detail in the sequel. When these appliances are suitably proportioned to the flow, a pressure difference ($p_1 - p_2$) in equation (9) is set up of such magnitude that it can usually easily be measured to the necessary accuracy (within 1 per cent.) without a manometer of high sensitivity.

It will be seen that this pressure difference depends upon the ratio of the diameter of the pipe to that of the minimum section presented by the constriction, an increase in this ratio producing a corresponding increase in the pressure difference to be measured. Practical limitations may of course prevent the minimum section from being too small in relation to the pipe, but even so the pressure difference can be made much larger than the velocity head in the open pipe. As an example of the advantage to be derived from this feature of a constriction, we may take the case of a 6-inch diameter pipe in which the mean velocity is 20 feet per second. The corresponding velocity head is about 0.09 inch of water ; whereas if a 3-inch diameter plate orifice be inserted in the pipe, the pressure drop across it will be about 3.7 inches of water, i.e. about 40 times the velocity head.

A further merit of the method of measuring the flow in a pipe by means of a constriction is that, since no traverse of the pipe is necessary, as it is in the case of measurements by pitot tube, it lends itself readily to the taking of continuous records of the quantity of air flowing. For this purpose, one has merely to transmit the upstream and downstream pressures to a recording differential pressure gauge, the charts for which may be graduated to read in inches of water, or, if great accuracy is not of importance, directly in pounds of air per minute. In the latter case, of course, the corrections for compressibility and the variations of density of the air with temperature and pressure have to be neglected ; and, in order to reduce errors as far as possible, it is advisable to adjust the gauge to read correctly at the estimated mean conditions of temperature, pressure, and pressure drop across the constriction to which it is likely to be subjected when in use.

The chief difficulty in the practical application of this method of measurement arises from the fact that the actual rate of flow q is always less than the theoretical q_t . The ratio q/q_t is termed the discharge coefficient of the constriction, and its value naturally depends upon the type of the constriction. In practice, two main classes of constriction are employed, of which the venturi tube and the shaped nozzle on the one hand, and the thin plate orifice on the other are typical. The characteristic of the former type is that the stream of fluid is confined within solid boundaries at least as far as its minimum

area. In fact, the position of the minimum area, or "throat," of the constriction coincides with that of the stream, and the direction of flow at the throat is parallel to the axis of the pipe line. Thus, the conditions postulated in the derivation of the theoretical equations of flow are virtually reproduced, except that relating to the absence of friction and turbulence. And as losses of energy due to these causes are generally relatively small, we find that the coefficients of discharge of such constrictions are usually nearly unity.

Conditions are different for the plate orifice, which consists essentially of a thin plate inserted into the pipe on a transverse section, and having cut in it a hole, usually circular and coaxial with the pipe. In this case, the stream on issuing from the orifice does not do so as a parallel jet, but continues to contract for some distance downstream of the plate.* The section of minimum area of the stream itself is reached at a distance of 1 pipe diameter or so from the plane of the orifice, and is of the order of 0.6 or 0.7 times the area of the orifice. After passing this minimum section—sometimes called the "vena contracta"—the jet begins to expand, and the pipe again runs full some few diameters downstream.

Equations for the Flow through a Thin Plate Orifice.—

The conditions of flow are illustrated in Fig. 36. The suffix 2 now refers to the vena contracta, except that, for convenience, we still take a_2 as the smallest area of the constriction itself, in this case the area of the orifice, and call the area of the vena contracta χa_2 . Then, for the theoretical discharge when the flow can be treated as incompressible, we derive the following equation in precisely the same manner as we obtained equation (5):

$$q_t = \chi a_2 g \sqrt{\frac{2\rho(p_1 - p_2)}{1 - \chi^2 m^2}}, \quad \dots \quad (10)$$

and, if λ is the ratio of the true discharge q to the theoretical,

$$q = \lambda \chi a_2 g \sqrt{\frac{2\rho(p_1 - p_2)}{1 - \chi^2 m^2}}. \quad \dots \quad (11)$$

Now we have no means of calculating λ and χ from theoretical

* Except at very low speeds with which the engineer is not usually concerned (see Ref. 1).

considerations, and their separate determination by experiment is difficult. In fact, it is doubtful whether we could make satisfactory measurements of λ , which represents the frictional and turbulent losses of energy. The practical expedient that is therefore always adopted is to neglect the contraction of the stream after it passes the orifice, and to call the discharge calculated from (5) the theoretical discharge. An overall co-

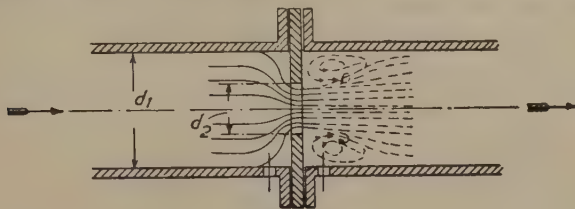


FIG. 36.

efficient of discharge α is then introduced, which is the ratio of the true discharge to the theoretical as calculated from (5). Thus,

$$q = \alpha a_2 g \sqrt{\frac{2\rho(p_1 - p_2)}{1 - m^2}}, \quad \dots \quad (12)$$

so that, from (11) and (12),

$$\alpha = \lambda \chi \sqrt{\frac{1 - m^2}{1 - \chi^2 m^2}}. \quad \dots \quad (13)$$

Although, as we have seen, we can conveniently determine neither λ nor χ , the overall coefficient α , which is a function of both χ and λ , in accordance with equation (13), can easily be measured experimentally; for if, for a given orifice installation, q is measured by one of the other methods available, α can be calculated from equation (12). (See numerical example in Chapter XI.)

It may be objected that the introduction of the quantity χ was an unnecessary complication, since no use has been made of it. But it is helpful in giving us a more complete conception of the nature of the flow through an orifice; and we shall see immediately that it has a very useful practical application, when we come to consider the flow of compressible fluids.

The flow must be treated as compressible when the ratio p_2/p_1 is no longer nearly unity. We then proceed as follows: The minimum area of the stream is now somewhat larger than its former value of χa_2 , for the gas expands in volume on account of the slight drop in pressure downstream of the orifice. We therefore take a new coefficient of contraction χ_c , and call the minimum area of the stream $\chi_c a_2$. It can then easily be verified that the equation corresponding to (10) is now

$$q_t = \chi_c a_2 g \sqrt{\frac{2\rho_1(p_1 - p_2)}{1 - \chi_c^2 m^2}} \sqrt{\frac{(1 - \chi_c^2 m^2) \cdot \frac{\gamma}{\gamma - 1} \cdot \frac{1}{1 - \pi} \left(1 - \pi^{\frac{\gamma - 1}{\gamma}}\right) \pi^{2/\gamma}}{1 - \chi_c^2 m^2 \pi^{2/\gamma}}}, \quad \dots \quad (14)$$

and, as before, $q = \lambda q_t$. (15)

Combining (14) and (15) and rearranging slightly, we have

$$q = \lambda \chi a_2 g \sqrt{\frac{2\rho_1(p_1 - p_2)}{1 - \chi^2 m^2}} \sqrt{\frac{\chi_c^2 (1 - \chi^2 m^2)}{\chi^2 (1 - \chi_c^2 m^2)}} \sqrt{\frac{(1 - \chi_c^2 m^2) \frac{\gamma}{\gamma - 1} \cdot \frac{1}{1 - \pi} \left(1 - \pi^{\frac{\gamma - 1}{\gamma}}\right) \pi^{2/\gamma}}{1 - \chi_c^2 m^2 \pi^{2/\gamma}}}. \quad \dots \quad (16)$$

We now introduce the *discharge coefficient* α for incompressible flow, by substituting in (16) the value of $\lambda \chi$ given by (13), and thus obtain

$$q = C \alpha a_2 g \sqrt{\frac{2\rho_1(p_1 - p_2)}{1 - m^2}}, \quad \dots \quad (17)$$

where

$$C = \frac{\chi_c}{\chi} \sqrt{\frac{(1 - \chi^2 m^2) \frac{\gamma}{\gamma - 1} \cdot \frac{1}{1 - \pi} \cdot \left(1 - \pi^{\frac{\gamma - 1}{\gamma}}\right) \pi^{2/\gamma}}{1 - \chi_c^2 m^2 \pi^{2/\gamma}}}. \quad \dots \quad (18)$$

As before, we have thus obtained the equation for compressible flow in a form in which it is exactly similar to that for incompressible flow, with the modification for the effect of compressibility confined to the single term C . It is important

to notice that the coefficient of discharge α entering into this equation is the *coefficient for incompressible flow*. In the sequel we shall have to consider various factors that can affect the value of α , but we need not include compressibility, since its effect is confined to the factor C .

It should be remarked that the orifice equations (12) and (17) have been derived on the assumption that the upstream and downstream pressures p_1 and p_2 are measured at sections of the pipe where the flow is parallel to the axis. For reasons explained on page 114, where this point is considered from the practical aspect, it is generally not convenient to measure the pressures in regions of parallel flow: as a rule, we measure two pressures, which we may denote by p_1' and p_2' , that differ somewhat from the values of p_1 and p_2 respectively appearing in the equations. This need not, however, cause us any concern: for example, taking the case of incompressible flow and introducing the *measured* pressure drop $p_1' - p_2'$ into (12), we have

$$q = \alpha a_2 g \sqrt{\frac{2\rho(p_1' - p_2')}{1 - m^2}}, \quad \dots \quad (12a)$$

where α is the overall coefficient of discharge which, as before, represents the ratio of the true to the theoretical value of q , the latter being again obtained from equation (5). The true discharge is still given by (11), so that, from (11) and (12a), we have

$$\alpha = \sqrt{\frac{1 - m^2}{1 - \chi^2 m^2}} \sqrt{\frac{p_1 - p_2}{p_1' - p_2'}} \quad \dots \quad (13a)$$

Comparing this with (13), we see that the value of α differs from that previously derived by the factor represented by the second radical in (13a). All that this implies in practice is that, if we measure the upstream and downstream pressures at sections that do not comply with the conditions of parallel flow postulated in deriving the equations, we shall have to use a different value of the discharge coefficient α , as we should have expected on general grounds. The system is, in fact, a different one and therefore has a different coefficient of discharge. But since α has in any case to be determined in the first instance by experiment, the only precaution we need take is to use the value of α determined for the appropriate disposition of the pressure taps at which the upstream and downstream pressures are observed.

Variables Affecting the Discharge Coefficient.—Before we consider the numerical values that have been established experimentally for the discharge coefficients of different types of constriction, it will be instructive to discuss from a general standpoint the variables that influence these numerical values. These variables are as follows :

- (a) the density ρ_2 of the fluid
- (b) the viscosity μ_2 of the fluid
- (c) a speed characteristic of the flow ;
- (d) the diameter d_2 of the constriction at the throat ;
- (e) the diameter d_1 of the pipe ;
- (f) the roughness of the pipe.

For the moment, we shall neglect item (f) in the above list, and apply the principle of dimensional theory to the other variables. In accordance with the principle, we have to arrange these variables into groups in the most general way, subject to the condition that each group must have the same dimensions in terms of the three fundamental units of mass, length, and time as the coefficient α , i.e. zero in each case since α , being a ratio, is a pure number. On this basis, we arrive at the following equation for α :

$$\alpha = f(v_2 d_2 / \nu_2, d_2 / d_1), \quad . \quad . \quad . \quad (19)$$

where f is some function of the two groups of variables concerned, and ν is the symbol generally adopted to represent the ratio μ/ρ , i.e. the viscosity of the fluid divided by its density. We notice immediately that one of the groups of variables in this equation corresponds to the Reynolds number for pipe flow, with d_2 , the constriction diameter, in place of the pipe diameter. We shall refer to this group as the Reynolds number for the constriction.

Equation (19) is of fundamental importance, and we may profitably devote some space to a discussion of its practical significance. If we consider two geometrically similar * constrictions of different sizes, the ratio d_2/d_1 for each will be the

* Two bodies, A and B, are geometrically similar when all linear dimensions of A bear a constant ratio to the corresponding dimensions of B. Thus, a 6-inch diameter orifice in a plate 1/16th-inch thick installed in a 12-inch diameter pipe is geometrically similar to a 3-inch diameter orifice in a 1/32-inch plate installed in a 6-inch pipe : any linear dimension in the former is twice the corresponding dimension in the latter. (Cf. page 60.)

same. If, further, we arrange the conditions of flow so that $v_2 d_2 / v_2$ is the same for the two constrictions, equation (19) indicates that the coefficients of discharge for the two constrictions must be equal, despite the fact that the latter are of different sizes and that the individual values of v , μ , ρ , etc., are not the same in the two cases. But if any of the quantities in equation (19) alter in such a manner as to change the value of the ratio in which they occur, the coefficient α is liable to change.

Pursuing this argument further, we see that for a series of geometrically similar constrictions, the value of α can only be changed by a change in the value of $v_2 d_2 / v_2$; hence, for such a series, equation (19) reduces to

$$\alpha = f_1(v_2 d_2 / v_2), \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where the form of the function f_1 is now the same for all members of the series, but may vary from one series to another.

From (20) it is apparent that if any one of the quantities in the ratio $v_2 d_2 / v_2$ is altered, the value of α will not change if one or both the other quantities are correspondingly altered by amounts such that the ratio remains constant. Thus, suppose for a given form of constriction a value of α is obtained for water flowing at a mean speed v_2 through the section of minimum area. The value of v for air is about 13 times that of water, so that the given constriction will have the same value of α with air, if the speed is increased thirteen-fold. Alternatively, a geometrically similar constriction, having a throat diameter thirteen times that of the original, will have a value of α for air equal to that of the original for water, when the throat speeds in the two cases are the same.

Although equations (19) and (20) do not provide a means of obtaining actual numerical values of α , they are of value in that they indicate the manner in which this coefficient will vary under different conditions. They therefore furnish a basis upon which the work of various experimenters can be correlated, so that their results can be given the widest possible application.

The Effect of Pipe Roughness.—A great deal of experimental work has been done to establish the numerical values of the discharge coefficients of different types of constrictions, particularly plate orifices. In the main, the results are con-

sistent, as indeed they must be if the values given are accurate, when they are compared on a similarity basis in accordance with equation (19). But there is evidence * of a slight variation in the value of α with size of pipe for geometrically similar installations, the variation decreasing as the size of the pipe increases. We are forced to the conclusion that geometrical similarity is not completely defined for similar constrictions by the ratio d_2/d_1 , i.e. the ratio of the diameter of the constriction to the diameter of the pipe. It seems that a second ratio t/d_1 is required, where t is some kind of average depth of the irregularities or roughness of the interior surface of the pipe: two pipes can not be regarded as entirely similar geometrically unless the ratio t/d_1 is the same for each. It is probable that in commercially used pipes of a given material—cast iron for example— t does not vary much with the pipe diameter, but is about the same in different-sized pipes; for it will always be the aim of the manufacturer to make the pipes reasonably smooth internally. Hence the larger the pipe the smaller the relative roughness, that is the more nearly will it approach the ideal case in which $t = 0$, and so the less the variation to be expected in the value of the discharge coefficient for a given orifice. When special precautions are taken to ensure a high degree of smoothness, the variation of α with the pipe diameter disappears. Further consideration is given below to the effect of pipe diameter on the numerical value of the discharge coefficient.

We now turn to a consideration of the various practical methods used to introduce a constriction into a pipe line in order to measure the flow from the pressure drop set up.

The Plate Orifice.—Mechanically, the plate orifice is the simplest of the devices under discussion in this chapter, and it is, moreover, the one most easily inserted into an existing pipe line with a minimum of alteration to the lay-out. Consequently, we find that it is the device to which most experimenters have devoted their attention in attempts to establish the values of the discharge coefficient. As a result of their work, and particularly that carried out in Germany, the values of the discharge coefficient can be used with confidence, provided that

* See page 119.

the conditions under which these values were measured are sufficiently closely reproduced.

As usually employed, the plate orifice consists simply of a thin plate or diaphragm clamped between two flanges of the pipe, and having a circular hole, coaxial with the pipe, cut in it, as in Fig. 36. Two pressure taps, one upstream and one downstream of the plane of the plate, serve as a means of measuring the static pressure drop ($p_1 - p_2$) from which the delivery is calculated according to equation (17).

Equation (17) can be thrown into a form which is more convenient for practical use. As it stands, it will give the quantity of air flowing in pounds per second, the area a_2 being measured in square feet, ρ_1 in slugs per cubic foot, and p_1 and p_2 in pounds per square foot. In practice, it is usually more convenient to derive the quantity Q in pounds per minute, and to measure the area a_2 of the orifice in square inches, and the pressure difference $p_1 - p_2$ in inches of water. It will also be convenient to insert the value for ρ under standard conditions of temperature and pressure (0.00237 at 29.92 inches of mercury and at 60° F.), and to apply a correction for the conditions prevailing at the time of the observations. The procedure in modifying equation (17) is similar to that performed in detail in equations (4), (5), and (7) of Chapter V, and need not be repeated here. The final form of the modified equation as recommended for practical use is

$$Q = 8.78 C \alpha a_2 \sqrt{\frac{1}{1 - m^2} \cdot (h_1 - h_2) \cdot \frac{b}{(460 + t)}}$$

pounds per minute, ... (21)

where a_2 , as stated above, is the area of the orifice in square inches, $(h_1 - h_2)$ is the pressure drop across the orifice in inches of water, t is the temperature of the air in the throat of the orifice in degrees Fahrenheit, and b is the barometric height in inches of mercury. If h_1 , the static pressure head in the pipe upstream of the orifice, is more than 8 inches of water above or below atmospheric pressure, b must be increased or diminished by the equivalent of h_1 in inches of mercury, according to the note on page 89.*

We have now to consider the value of the discharge co-

* If necessary, also, the density of the air must be corrected for humidity (see page 284).

efficient α and of the compressibility factor C . It will be remembered that the equations of flow were derived on the assumption that the pressure p_2 is measured at the section of minimum area of the stream, where the flow is parallel to the axis of the pipe. In practice, it would be necessary, in order to measure p_2 at this section, to find the position of the vena contracta of a given orifice before the downstream pressure tap could be located. Apart from the fact that this position is not fixed, but depends to some extent on the rate of flow, it is obviously undesirable to have to make the necessary preliminary measurements. It has therefore become common practice to measure p_2 either at a fixed distance downstream of the orifice plate—one pipe diameter or so—or actually at the downstream face. The former method has the disadvantage that, for every size of orifice in a given pipe, the pressure is in effect measured at a different place with respect to the constriction, so that it is difficult to compare the results of different series of experiments on a similarity basis. The majority of workers have therefore adopted the second course, which is not open to this objection. Thus, p_2 in the equations is taken to be the pressure at the downstream plane of the orifice plate, instead of at the vena contracta. Since an empirical value of α has in any case to be used, there is no valid objection to our measuring p_2 at any convenient place, provided that the place chosen introduces no difficulty in comparing the results on a basis of geometrical similarity. It only means that a somewhat different value of α is found than if p_2 were measured at the spot assumed for the theoretical equation. We may note here that the drop of pressure between the downstream plane of the plate and the vena contracta is normally only a few per cent. of the pressure drop across the orifice. For similar reasons, p_1 is measured at the upstream plane of the plate, where the pressure is slightly higher than it is a short distance upstream: there is a small rise of pressure as the stream approaches the orifice plate.

The shape of the edge of the orifice is a variable that has been shown to have an appreciable influence on the discharge coefficient, which increases as the edge is more rounded. It is therefore necessary to standardise some form of edge if we wish to make use of values of the discharge coefficient established once for all by careful calibration tests, and so avoid the

necessity of having to calibrate orifices whenever they are installed to measure flow. A slightly rounded edge has certain practical advantages, the chief of which is that it is less liable to damage or erosion than a sharp edge. On the other hand, a different radius of edge has to be used for each orifice size if geometrical similarity is to be preserved. This complication renders the round-edged type unsuitable for standardisation, and we therefore find that only sharp-edged orifices have been standardised.

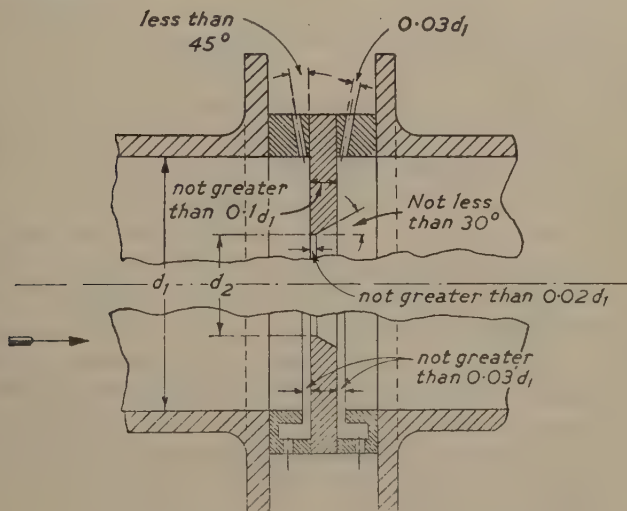


FIG. 37.—German Standard Orifice 1930.

The “German Standard Orifice 1930.”—Among the investigators who have measured the discharge coefficients of sharp-edged orifices with pressure taps at the upstream and downstream planes of the plate, we may mention Hodgson² in England, and Witte,³ Jakob,⁴ Kretschmer,⁵ Ruppel,⁶ Giese,⁷ Hansen,⁸ and Jaroschek⁹ in Germany. Exhaustive researches have also been made in America by Bean, Buckingham, Murphy, and Benesh,¹⁰ but not under conditions exactly comparable with those of the other work cited. The German results are by far the most complete, and have led to the standardisation by the Verein Deutscher Ingenieure¹¹ of a form of sharp-edged orifice for which values of the discharge coefficient α are given that can be applied when the device is

installed and used under prescribed conditions. In 1943, the same orifice, with one slight modification (see below), was adopted by the British Standards Institution ¹² in this country ; it has also been adopted by the International Federation of National Standardising Associations.

The essential features of this orifice, which is called the "German Standard Orifice 1930," are shown in Fig. 37. It will be seen that the passage through which the air flows has a short cylindrical entry, followed by a conical portion with a taper (each side) of not less than thirty degrees. The pressures are tapped off either through two single pressure holes, as shown in the upper portion of Fig. 37, or, as shown below, from two annular chambers which are in communication with the pipe pressure over the whole circumference. The latter arrangement is to be preferred when there is any possibility that the flow is not quite symmetrical, because of bends or obstructions in the pipe ahead of the orifice or downstream. It should be mentioned, however, that serious asymmetry would in any case render the standard coefficients invalid, so that the use of annular chambers must not be taken as a safeguard against anything more than a slight lack of symmetry of flow. Further details of the orifices will be found in the published Rules ¹¹ drawn up by the V.D.I. Anyone contemplating the use of the device is strongly recommended to study these Rules, which are admirably complete but far too long to be more than briefly quoted here.

The British Standard Code ¹² also gives details of the device ; but it must be noted that, whereas the V.D.I. stipulate that the length of the cylindrical entry preceding the tapered portion of the orifice shall not exceed $0.02 d_1$, the B.S.I. Code gives the maximum value of this dimension as $0.05 d_2$. Since the published values (see below) of the discharge coefficient α extend over a range of m up to 0.7 , for which $d_2/d_1 = 0.84$, it follows that, at this highest value of m , the maximum permissible length of the cylindrical entry under the B.S.I. regulations will be over twice the limit set by the V.D.I. The latter draw particular attention to this point in the 4th Edition of their Rules,¹¹ remarking that, in spite of the desire expressed by users for a longer cylindrical entry, the limiting value of this dimension was reduced from the value of $0.04 d_1$ allowed in the

3rd Edition, because some work by Ruppel (Ref. 6, second citation) had shown that the value of α was appreciably different with the longer length. No reference is made to this point in the B.S.I. Code ; it is therefore difficult to reconcile the increase in length which this code permits with the V.D.I. remarks just quoted. For this reason, and in the absence of a statement by the B.S.I., it appears desirable to retain the German limitation, if the V.D.I. values of α , which are reproduced identically in the B.S.I. Code, are to be used.

Discharge Coefficients of the German Standard Orifice 1930.—The published values of the discharge coefficient α for the Standard Orifice are based on the results of many series of careful experiments (Refs. 3 to 9), but mainly those of Witte with water, oil, and steam in *smooth* pipes of diameter ranging from 50 to 1,000 mm. (about 2 to 40 inches) ; they can be used in all pipes of not less than 2-inches diameter, provided that m lies between 0.05 and 0.7.* It is found that, when the *pipe* Reynolds number $v_1 d_1 / \nu_1$ exceeds a certain lower limit, which depends on the value of m , the value of α assumes the constant value for each m shown in the following table, in which the limiting values of the pipe Reynolds number that must be exceeded are also given.

TABLE 6
Discharge Coefficients for German Standard Orifice 1930.

m	α	Lowest value of $v_1 d_1 / \nu_1$ for which value of α is valid	m	α	Lowest value of $v_1 d_1 / \nu_1$ for which value of α is valid
0.05	0.597	20,000	0.40	0.606	125,000
0.10	0.599	30,000	0.45	0.605	145,000
0.15	0.601	40,000	0.50	0.603	170,000
0.20	0.603	50,000	0.55	0.599	195,000
0.25	0.604	65,000	0.60	0.594	225,000
0.30	0.605	80,000	0.65	0.585	260,000
0.35	0.606	100,000	0.70	0.574	300,000

It will be seen that, provided that m does not exceed 0.55, the value of α is approximately constant ; and the assumption of a mean value of 0.603 will lead to an error of 1 per cent. at

* The B.S.I. add the further restriction, which does not appear in the V.D.I. Rules, that d_2 must not be less than 0.25 inch.

the smallest values of m , while over most of the range of m up to 0.55 the error will be of the order of $\frac{1}{2}$ per cent. Similar conclusions regarding the small variation in α for m less than 0.5 were drawn by Hodgson,² who quoted a mean value of 0.608 for

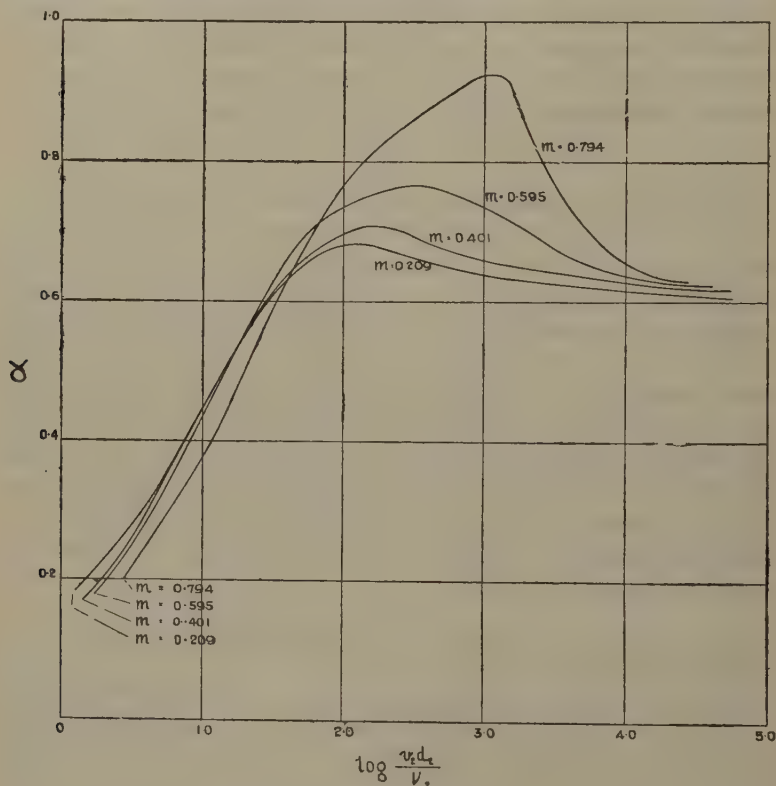


FIG. 38.

square-edged orifices very similar to the German Standard 1930: Hodgson stated that this value could be used provided that d_2/d_1 did not exceed 0.7 ($m = 0.49$).

At Reynolds numbers lower than those given in Table 6, the value of α rises and then falls rapidly as the Reynolds number is further decreased. This is shown in Fig. 38, which is due to Johansen.^{1*} A useful practical feature to be observed from

*Note that the abscissæ in Fig. 38 are $v_2 d_2 / \nu_2$ and not $v_1 d_1 / \nu_1$; also that, although these results relate to sharp-edged orifices, the type was not identical with the German standard.

these results is that the variation of α with Reynolds number is much greater for large contraction ratios than for small. Hence, when doubt is entertained whether the flow in a given case is likely to be fast enough for the critical Reynolds number to be exceeded, it is preferable to use an orifice with as small a contraction ratio m as possible. We must bear in mind, however, the fact that if m is too small, the fan may not be able to maintain the desired rate of flow against the increased loss of pressure (see page 124). Apart from this limitation, the use of small values of m is preferable also for other reasons, which are mentioned in the two following sections of this chapter.

Variation of Orifice Discharge Coefficients with Pipe Diameter.—We have already seen that the discharge coefficient of a constriction depends to some extent on the pipe diameter, unless special care is taken to ensure a smooth interior to the pipe. This effect, which is really a roughness effect, has been referred to by various investigators in relation to orifice discharge coefficients, and a useful analysis of existing data has been published by Johansen.¹³ He dealt with results obtained by Spitzglass, Witte, Jakob, and Kretschmer, and expressed the discharge coefficients obtained by these workers for pipes of different diameter in terms of the discharge coefficient obtained in a 10-cm. diameter pipe. The meaned results of the analysis are shown in Fig. 39. It will be seen that α decreases as the pipe diameter increases; the reduction is more rapid for small than for large diameters, a tendency being observable for α to approach a limiting value as the largest pipe diameters are attained. The effect is also more marked at large than at small values of m —a further instance of the advantage, already mentioned, of using a small rather than a large value of m .

The effect of roughness in increasing the discharge coefficient is in accordance with expectation; for Stanton¹⁴ has shown that the curve of velocity distribution in a rough pipe is more peaked in the centre than in a smooth pipe. That is, for the same mean rate of flow, the speed at the centre of the rough pipe is higher than in the smooth pipe; and hence the orifice discharge coefficient is higher—more fluid passes through the orifice for the same pressure drop. The effect would also be

expected to be more pronounced for large than for small values of m , as, in fact, it is.

The decrease in the diameter effect as the diameter increases has already been explained (page 112). For a series of commercial pipes of the same finish, the relative roughness will decrease with increase of diameter, until the larger pipes finally

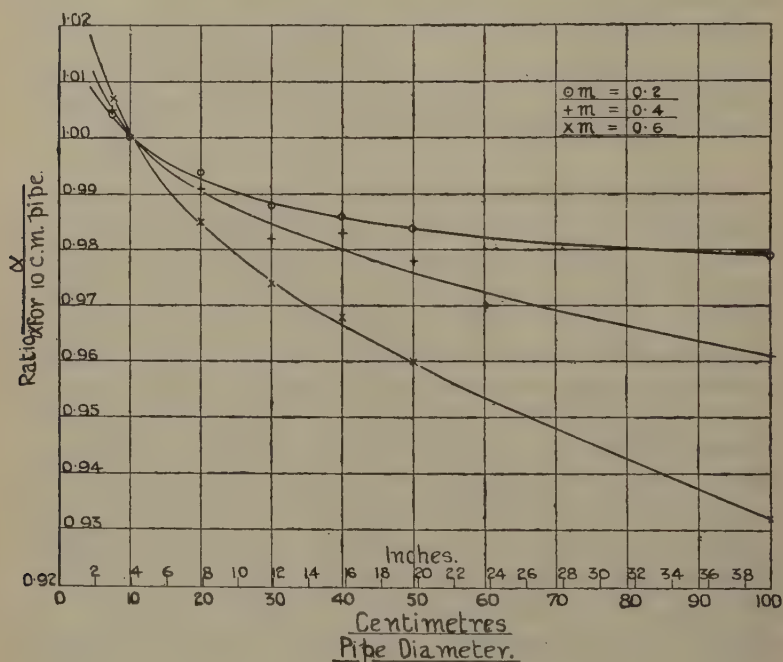


FIG. 39.

approximate to perfectly smooth pipes. Witte's work ³ has shown that, with pipes turned and polished internally, there is no measurable change in α for diameters of 5 to 100 cm., and also that the coefficients are in very close agreement with those obtained in 100 cm. diameter pipes of ordinary roughness.

The V.D.I. Rules give the corrections to be applied to α for the effect of pipe diameter. The corrections, which are given in Fig. 40, only extend up to diameters of 30 cm., but over this range they agree very well with the corresponding figures taken from the curves of Fig. 39.

The V.D.I. Rules state that these corrections apply to pipes

in a state of "service roughness," which is defined as the condition of cast-iron pipes that have rusted internally as a result of prolonged use, but whose exteriors show no signs of serious corrosion. It is further stated that a new cast-iron pipe, tarred inside and of 8-inch internal diameter, can be regarded as smooth provided m is less than 0.5.

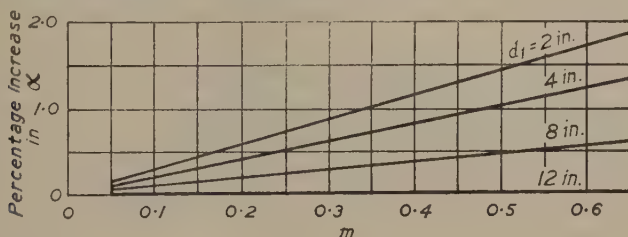


FIG. 40.

Other Factors affecting the Discharge Coefficient.—

According to the V.D.I. Rules, great care must be taken to make the upstream edge of the orifice truly sharp, if the standard values of α are to be used. Any rounding, however slight, increases α , and it is important that, besides being sharp, the edge should be true and free from dents. The extent to which the edge should approach perfection increases as the size of the orifice decreases. Thus, if the orifice diameter is 6 inches or more, ordinary visual examination of the edge is sufficient: any imperfections that the eye is unable to detect will not matter. For orifice diameters below 6 inches, the edge should be so sharp that it does not perceptibly reflect a beam of light directed upon it.

Another important consideration is the presence of bends or obstructions, or changes in the pipe section, upstream or downstream of the orifice: if there is an insufficient straight length of pipe between them and the orifice, the discharge coefficient will be affected. The requisite free lengths of pipe between various obstructions and the orifice, in order that α shall not be affected, are given in the V.D.I. Rules, from which Table 7 has been compiled.

The V.D.I. Rules state that, for values of m not exceeding 0.5, if the lengths of unobstructed pipe ahead and downstream of the orifice are only half those given in Table 7, the error on α will be only $\frac{1}{2}$ per cent. This may be the reason why the B.S.I.

Code allows unobstructed lengths appreciably less than those recommended by the V.D.I., but quotes tolerances (see below) that are from $\frac{1}{2}$ to 1 per cent. greater.

TABLE 7

*Lengths of Straight Pipe required between Obstructions and Orifice.
Lengths given in terms of Pipe Diameter (d_1).*

Type of Obstruction (Radii of all bends = d_1)	Position of Obstruction U = Upstream D = Downstream	Type of Orifice Pressure Tapping * S = Single Tap A = Annular Chamber	Value of m					
			0.1	0.2	0.3	0.4	0.5	0.6
Single Right-Angled Bend. (Radius equal to pipe diameter).	U	S	5	8	12	18	26	—
	U	A	6	9	12	17	21	26
	D	S	5	5	5	5	5	5
	D	A	5	5	5	5	5	5
Double Right-Angled Bend in One Plane (S-bend). (Radius as above).	U	S	5	5	5	8	15	24
	U	A	5	5	6	9	14	20
	D	S	5	5	7	9	13	17
	D	A	5	5	5	5	5	5
Double Right-Angled Bend in Two Planes. (Radius as above).	U	S	28	28	28	28	28	28
	U	A	4	5	7	9	12	16
	D	S	5	5	5	5	5	5
	D	A	5	5	5	5	5	5
Stop Valve.	U	S	16	27	40	54	68	83
	U	A	8	11	14	18	22	27
	D	S	5	5	5	5	5	5
	D	A	5	5	5	5	5	5

From Table 7 we note one more advantage of using as low a value of m as the fan will allow. In this case, the smaller the value of m , the shorter the length of unobstructed pipe necessary upstream of the orifice or, alternatively, the less susceptible the value of α to upstream obstructions.

Corrections and Tolerances on the Standard Discharge Coefficients.—Both the V.D.I.¹¹ and the B.S.I.¹² give correction

* See Fig. 37.

factors and "tolerances" on the values of α they publish for the Standard Orifice 1930. The values of α reproduced here in Table 6, page 117, apply only to orifices constructed and installed in full compliance with the specifications, and used in smooth pipes. Corrections have to be applied to these standard values of α to allow for (a) variation in Reynolds number, if the latter is below the limiting value for which α is constant, and (b) the roughness of the pipe which, as we have already seen, is equivalent to a correction based on the diameter. The B.S.I. and V.D.I. corrections for (b), pipe diameter, are identical; but for Reynolds number, the B.S.I. correction—when allowance is made for the fact that it is based on the *orifice* Reynolds number, whereas the corresponding V.D.I. correction is based on the *pipe* Reynolds number—is about 1 per cent. greater than the V.D.I. value. Against this, we note that the V.D.I. give a correction, which varies between 0 and 2 per cent., to be applied for "non-sharpness" of the edge, and which they recommend for use when any doubt is felt on this score. The B.S.I. make no mention of a correction of this kind, relying presumably on the requirement in their specification that the edge shall be "perfectly square and free from all burrs or rounding."

In addition to these corrections, which take account of *known* departures from conditions under which the standard values of α are valid, both authorities prescribe "tolerances" * to allow for uncertainties in the standard values themselves of α (basic tolerance) and of each of the correction factors. As a result of these tolerances, the final estimated values of the rates of flow as deduced with the aid of these orifices are subject to small uncertainties. In three examples of actual estimates of rates of flow given in the B.S.I. Code, the overall tolerances are given as ± 0.8 per cent., ± 2.4 per cent. and ± 0.85 per cent respectively.

If the temperature of the air stream is appreciably different from that at which the orifice diameter was measured, a correction has to be applied to allow for the change in diameter with temperature. This change is calculated in the normal way, using the coefficient of linear expansion of the material of which the orifice is made. A similar correction should be

* It appears from the original text that the "tolerance" is the quantity more commonly known as the probable error.

applied to the diameter of the pipe ; the value of m will be unchanged if the orifice plate and the pipe are made of the same material.

For full details of the various corrections and tolerances, the user should refer to the V.D.I. Rules¹¹ and the B.S.I. Code.¹²

Loss in Pressure due to the Passage of Air through an Orifice Plate.—As we have seen, the static pressure at the downstream face of an orifice plate is considerably less than that in the air upstream of the plate. There is a further slight decrease as the air leaves the orifice, a pressure minimum being reached at the minimum section of the jet about $\frac{1}{2}$ to 1 pipe diameter from the orifice. This pressure minimum is followed by a gradual recovery of pressure, which persists up to a distance of about 3 to 10 or more diameters downstream of the orifice ; the actual distance varies inversely as m , and probably depends also on the rate of flow. At some point, therefore, a pressure maximum occurs, which corresponds to the position where the jet, which at its throat had a section considerably less than that of the pipe, has expanded once more to the full pipe diameter. After this pressure maximum has been reached, the flow follows the normal laws of pipe flow ; and the pressure falls off gradually on account of the ordinary frictional loss at the walls. The point to be observed is that the static pressure at the downstream point of maximum recovery is considerably less than that upstream of the orifice ; it is also considerably less than the static pressure that would be experienced at that point if the orifice was removed, and the only source of loss of pressure was the normal frictional pressure drop in a parallel pipe. The orifice, in fact, introduces a resistance equal to that of a very long length of pipe, and causes a corresponding loss of head or energy in the flow. In other words, a fan delivering a given quantity of air per unit time along a pipe line would take more power if an orifice were introduced into the line and the quantity of air flowing were maintained at its previous value.

The last-mentioned point has an important practical significance ; for before a decision is made to instal an orifice in a given pipe system to measure the air flowing, it is necessary to make sure that the fan available will deliver the requisite

quantity against the increased head due to the loss of pressure through the orifice. Experiments show that a sufficiently close estimate can be made of the pressure loss from the rule that this loss, expressed as a fraction of the difference between the pressure at the upstream and downstream faces, is numerically equal to $(1 - m)$,

$$\text{i.e.,} \quad \frac{P_d}{p_1 - p_2} = 1 - m,$$

where P_d is the unrecoverable loss of head through the orifice.

The Contraction of the Jet from an Orifice.—Witte¹⁵ has used the published values of α for the German Standard Orifice to make approximate estimates of the contraction of the jet of an incompressible fluid after flowing through that orifice. That is to say, he has used equation (13) to calculate χ , the ratio of the area of the minimum section of the jet to the area of the orifice. For this purpose, he had to assume that λ , the coefficient representing the loss of energy due to friction and turbulence, was equal to unity, i.e., that such losses are small enough to be negligible. Although this assumption is near the truth for constrictions such as shaped nozzles and venturis, as is clear from the fact that the discharge coefficients for these constrictions are very near unity (except at low Reynolds numbers when frictional effects become relatively more important), it cannot be so nearly true in the case of an orifice, the sharp edge of which must cause eddying and consequent loss of energy. However, with this limitation in mind, we shall find it of some interest to see the results of Witte's calculations, as they will provide us with an indication of the order of size of the minimum section.

Taking λ as unity, we can rewrite (13) thus :

$$\chi = \frac{\alpha}{\sqrt{1 - m^2(1 - \alpha^2)}},$$

and, inserting the appropriate value of α from Table 6, Witte obtained the values of χ for different values of m shown in Table 8. According to these assumptions, therefore, the minimum section of the jet has an area between 60 and 70 per cent. of that of the orifice itself.

TABLE 8
Approximate values of χ for Incompressible Flow.

m	0.1	0.2	0.3	0.4	0.5	0.6	0.7
χ	0.60	0.61	0.62	0.64	0.66	0.68	0.70

The Compressibility Factor C for Orifices.—A number of investigators have discussed the compressibility factor C to be used for orifices, and formulæ for calculating it have been given by Witte,^{15, 16} Ruppel,¹⁷ Busemann,¹⁸ and Buckingham.¹⁹ These formulæ are, however, all somewhat complicated and based on various assumptions that need verification; it is better, therefore, to use experimentally determined values of C in practical work. Investigations by Witte^{15, 16} with air and superheated steam flowing through the Standard Orifice 1930 and the Standard Nozzle 1930 (see below) have provided the basis on which the V.D.I. have drawn up charts giving values of C for both these standard constrictions, for values of p_2/p_1 ranging from 0.5 to 1.0 in the case of the Standard Orifice. For air and other diatomic gases for which $\gamma = 1.41$, the values of C, according to these data, vary linearly with $(1 - (p_2/p_1))$, the formula being :

$$C = K.p_2/p_1 + (1 - K), \quad . \quad . \quad . \quad . \quad (22)$$

where K is a constant for any particular value of m . Values of K for various values of m , as calculated from the V.D.I. charts, are given in Table 9.

TABLE 9

m	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7
K	0.31 ₂	0.31 ₄	0.32	0.33	0.35	0.37	0.40	0.44 ₅

Thus, for example, when $m=0.5$, $K=0.37$; and if $p_2/p_1=0.85$, $C = 0.37 \times 0.85 + 1 - 0.37 = 0.95$.

These results, it should be repeated, apply only to air and other diatomic gases. For vapours, such as superheated steam

for which $\gamma = 1.31$, the variation of C is no longer linear with p_2/p_1 . It is found, however, that smooth curves, one for each value of m , can be plotted between C and $(p_2/p_1)^{1/\gamma}$ so that, for other values of γ , C can be obtained from Fig. 41, which is

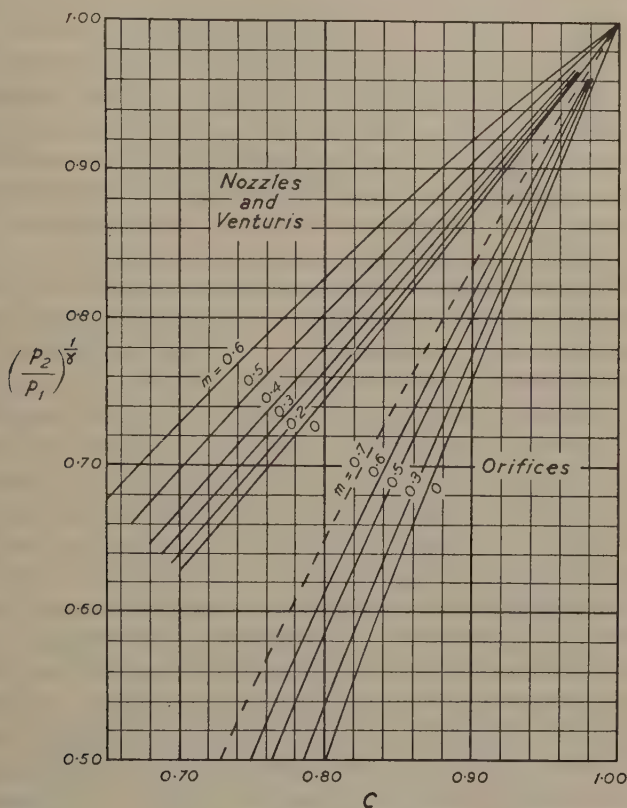


FIG. 41.

reproduced from the V.D.I. Rules. This diagram includes also values of C applicable to nozzles and venturi tubes (see below), in which the minimum area of the jet is equal to the minimum area of the constriction, i.e., there is no contraction of the jet after it passes the throat of the constricting device.

The "tolerances" or probable errors in the values of C obtained from the data reproduced above are given by the V.D.I. as ± 0.5 per cent, for values of p_2/p_1 between 0.99 and 0.80, and

± 2 per cent. for values of p_2/p_1 less than 0.80. These tolerances are for air and gas flow ; for vapours the figures are the same, except for the range of p_2/p_1 between 0.98 and 0.80, for which a figure of ± 1.5 per cent. is quoted.

Practical Notes on the Use of Orifices.—It will be gathered from the preceding pages that, provided the standard conditions laid down in the V.D.I. Rules are fulfilled, the discharge coefficient of the orifice shown in Fig. 37 can be safely taken from Table 6. The most important of these conditions are those relating to the sharpness of the edge, the smoothness of the pipes, if these are of small diameter, the location of the pressure taps close to the orifice plate, and the length of unobstructed straight pipe upstream and downstream of the orifice (see Table 7). If circumstances are such that for some reason it is impossible or inconvenient to reproduce the conditions sufficiently closely, a calibration of the orifice should be performed *in situ* over the range of flow at which it will be used in service. The calibration is not a matter of serious difficulty ; an example of the method of carrying it out for an orifice in a pipe is given in Chapter XI. If the pipe is sufficiently large for the quantity of air flowing to be measured by means of the standard pitot-static tube, as described in Chapter V, the work is somewhat simplified. It is only necessary to take measurements with this instrument at a convenient section of the pipe, and simultaneous readings of the pressure drop across the orifice, the air temperature in the pipe, and the barometric pressure. Equation (12) or (17) (according to the value of p_2/p_1) for the discharge through the orifice can then be solved for α by inserting the measured value of q . If an accuracy of 1 per cent. is good enough, the compressibility factor can be neglected, provided that p_2/p_1 is greater than 0.98 ; for the data given on page 126 show that C is greater than 0.99 for all values of p_2/p_1 between 0.98 and 1.00.

If it is decided that a calibration is necessary, there will probably not be much object in incurring the trouble and expense of making a standard orifice. A simpler type may be installed, with, perhaps, slightly rounded edges, which are easier to make and less likely to suffer damage than sharp edges. The pressure taps can be located at any convenient points near to the orifice

plate, but not necessarily in the upstream and downstream planes.

It is, perhaps, hardly necessary to point out that when an orifice is installed in an existing pipe system, the pressure drop between the inlet end and the upstream face of the orifice will be the same as the pressure drop between the same two points before the introduction of the orifice,* while the drop in the whole system is increased by the pressure loss in the orifice. Attention has already been drawn to the need for bearing in mind this increased pressure against which the fan will have to work to deliver the same quantity of air as before.

The Shaped Nozzle.—Another form of constriction, which has found favour in many quarters, is the shaped nozzle. In this device, the air is guided smoothly to the section of minimum area of the stream, which coincides with the minimum area of the constriction itself from which the stream issues as a parallel jet that does not contract further as it does in the case of the plate orifice. In consequence, the nozzle has a considerably higher discharge coefficient than the plate orifice; as we shall see in the next section, α for the nozzle is about 0.93 on the average as compared with 0.6 for the orifice. It follows from equation (21) that, for the same rate of flow and the same value of m , the differential head with the nozzle is about $(0.6/0.93)^2$, i.e., $1/2.4$ that of the orifice; and, since the unrecoverable pressure loss is roughly the same percentage of the differential head as it is for the orifice, the fan will have to overcome a considerably smaller resistance for the same rate of flow if a nozzle is fitted instead of an orifice. But the lower differential pressure ($1/2.4$ times that with the orifice) will generally be of no advantage; for the whole object of using a constriction at all is to provide a measurable pressure drop, and the size of the constriction (i.e., value of m) should be so chosen that the pressure drop is not unduly large: it should be just sufficient to be measurable to the requisite accuracy with the manometer available. Thus, if our manometer can be read to $1/20$ inch of water, the value of m should as a rule be such that the minimum pressure drop across the constriction does not exceed 5 inches of

* Apart from the negligibly small rise in pressure immediately in front of the orifice plate.

water ; the accuracy on $h_1 - h_2$ (see equation (21)) will then be 1 per cent. and the accuracy on Q will be $\frac{1}{2}$ per cent., which will usually be sufficient. Whether we use an orifice or a nozzle, therefore, we shall need a pressure drop of 5 inches of water, but to get it we can use a smaller value of m with the nozzle ; and this is a distinct advantage because, as we have already noted in connection with the orifice (and it is true also for the nozzle), the smaller the value of m the less the probable errors due to roughness and the presence of obstructions in the pipe ahead or downstream. Thus, the higher value of α for the nozzle gives the latter an indirect advantage over the orifice ; but the main superiority of the nozzle lies in the absence of any mechanical feature comparable with the sharp edge of the orifice, to the condition of which the discharge coefficient is so sensitive. This is of special advantage in the metering of dust-laden or tarry streams of gas or air, which might erode or tend to round off the sharp edge : in such conditions, the use of the nozzle is strongly to be recommended.

One further characteristic of the nozzle, which, as we have already seen (page 77), is of some practical importance, deserves mention. Careful explorations of the issuing jet in a number of instances have disclosed the fact that the velocity is practically constant over the entire section : it is only in the immediate vicinity of the walls that a slight falling-off is observed. For all practical purposes, the jet may be considered as having a uniform velocity equal to that measured by a single observation at the axis.

The “ German Standard Nozzle 1930.”—As in the case of the plate orifice, the V.D.I. have standardised a form of nozzle, known as the “ German Standard Nozzle 1930,” which has also been adopted by the B.S.I. and the International Federation of National Standardising Associations (I.S.A.). The last-mentioned body adopted it in 1932, and it is sometimes called the I.S.A. 1932 Nozzle. This device is illustrated in Fig. 42 (*a*) and (*b*), from which it will be seen that there are two types, one for use when $m < 0.45$ and one when $m > 0.45$. The inner contour has two radii, one equal to $d_2/3$ and tangential to the cylindrical part, and the other equal to $0.2 d_2$, tangential to the flat face. These two radii merge smoothly into each other, but when m

reaches 0.45, the curved part of the nozzle extends beyond the diameter d_1 of the pipe; a flat surface is then to be turned on the upstream side as shown in Fig. 42 (b).

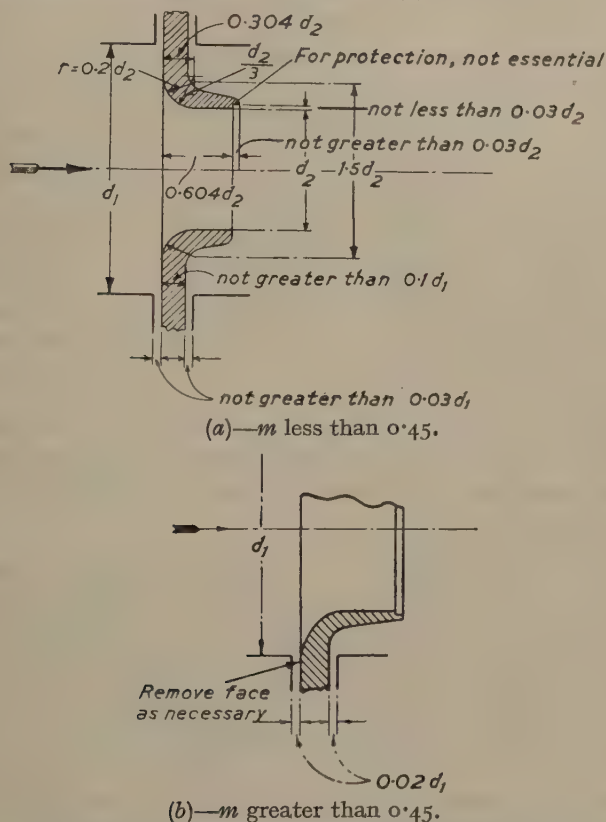


FIG. 42.—German Standard Nozzle 1930.

The arrangements for tapping off the pressures are the same as for the standard orifice (see Fig. 37); either single tappings or annular rings may be used. The mouth of the nozzle has a small lip as shown, to protect it from mechanical damage; its use is optional, so that, if there is no likelihood of damage, it may be omitted.

Discharge Coefficients of the German Standard Nozzle 1930.—As a result of a large amount of work by the same investigators quoted in connection with the standard orifice

(Refs. 3, 4, 6, 7, 8, 9, 20), values of α for the standard nozzle have been established and accepted by the V.D.I. and the B.S.I. They are as follows :

TABLE 10
Discharge Coefficients for German Standard Nozzle 1930.

m	α	Lowest value of $v_1 d_1 / v_1$ for which value of α is valid	m	α	Lowest value of $v_1 d_1 / v_1$ for which value of α is valid
0.05	0.986	70,000	0.40	0.954	145,000
0.10	0.984	73,000	0.45	0.946	170,000
0.15	0.982	80,000	0.50	0.936	185,000
0.20	0.979	90,000	0.55	0.925	195,000
0.25	0.975	100,000	0.60	0.914	200,000
0.30	0.969	115,000	0.65	0.899	200,000
0.35	0.963	130,000			

As in the case of the orifice, the values of α can be taken as constant, provided that certain minimum values of the pipe Reynolds number, shown in the 3rd and 6th columns of Table 10, are exceeded.

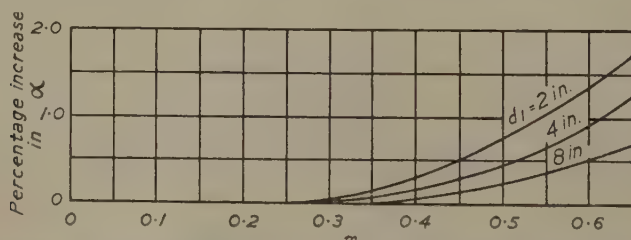


FIG. 43.—Corrections to nozzle coefficients for pipe diameter.

Again, as for the orifice, corrections have to be applied to α to allow for pipe size (i.e., roughness) and there must be certain minimum lengths of unobstructed pipe up and downstream. The corrections for pipe diameter are shown in Fig. 43; they are considerably smaller than the corresponding corrections for the orifice. Table 11 gives the requisite lengths of unobstructed pipe for the same cases as for the orifice.

The Compressibility Factor C for Nozzles.—It has been confirmed by experiment that the compressibility factor for the Standard German Nozzle 1930 agrees with the theoretical

values given by equation (18) when χ_c and χ are both equal to 1, as they are when there is no contraction of the jet beyond the throat of the constriction. The principal experiments on which this conclusion is based are those of Witte,* which were made

TABLE II

*Lengths of Straight Pipe required between Obstructions and Nozzle.
Lengths given in terms of Pipe Diameter (d_1).*

Type of Obstruction (Radii of all bends = d_1)	Position of Obstruction U = Upstream D = Downstream	Type of Pressure Tapping S = Single Tap A = Annular Chamber	Value of m				
			0.1	0.2	0.3	0.4	0.5
Single Right-Angled Bend.	U	S	7	9	11	16	20
	U	A	7	9	11	14	18
	D	S	5	5	5	10	28
	D	A	5	5	5	5	5
Double Right-Angled Bend in One Plane.	U	S	7	8	12	17	—
	U	A	6	6	7	14	—
	D	S	5	7	12	25	—
	D	A	5	5	5	5	5
Double Right-Angled Bend in Two Planes.	U	S	12	20	25	29	—
	U	A	5	6	8	12	16
	D	S	5	10	15	27	—
	D	A	5	5	5	5	5
Stop Valve.	U	S	15	28	43	63	—
	U	A	5	8	16	28	—
	D	S	5	5	5	5	5
	D	A	5	5	5	5	5

with superheated steam for values of m up to 0.4 and of p_2/p_1 down to the critical (see below, page 143). Confirmatory evidence for a somewhat different type of nozzle is quoted by the V.D.I.¹¹ in the form of some unpublished data obtained with air, and there are also some data (again unpublished) obtained by the author with air flowing through a venturi. It seems, therefore, justifiable to assume that C for nozzles can be calculated from the theoretical value over the whole of the range of flow, and this assumption has been made by the V.D.I. in putting forward the values of C for air which are

* Ref. 3 (1930).

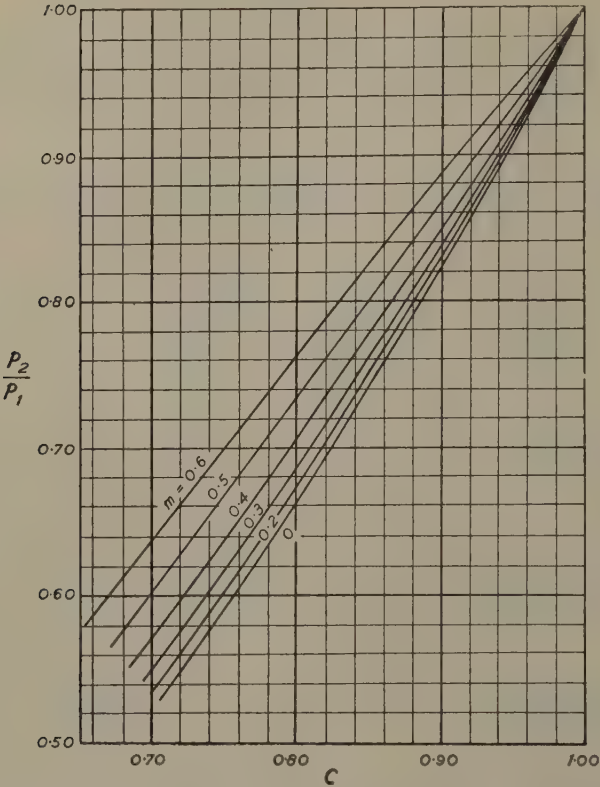


FIG. 44.—Values of C for nozzles ($\gamma = 1.41$ — air).

reproduced in Fig. 44¹¹. For media having different values of γ , the appropriate values of C can be obtained from Fig. 41. The following tolerances to be used with these values of C are given by the V.D.I. :

TABLE 12
Tolerances on C for Nozzles (for Air and Superheated Steam).

p_2/p_1	Tolerance per cent.		
	$m \leq 0.4$	$0.4 > m < 0.5$	$0.5 > m < 0.6$
1 to 0.9	0	0	0
0.9 to 0.8	0	± 1	± 1
less than 0.8	± 1	± 2	± 2

For other gases and vapours, the tolerances recommended are, for all values of m , ± 1 per cent. for p_2/p_1 down to 0.8, and ± 2 per cent. for smaller values of p_2/p_1 .

Venturi Tubes.—Most of the unrecoverable loss of head in an orifice or a nozzle is due to the sudden increase of area after the air has passed the section of minimum area: the rapid convergence of the stream on the upstream side contributes considerably less to the total loss of head. The cause of the loss of head on the downstream side seems to be that the space between the boundaries of the jet and the walls of the pipe, before

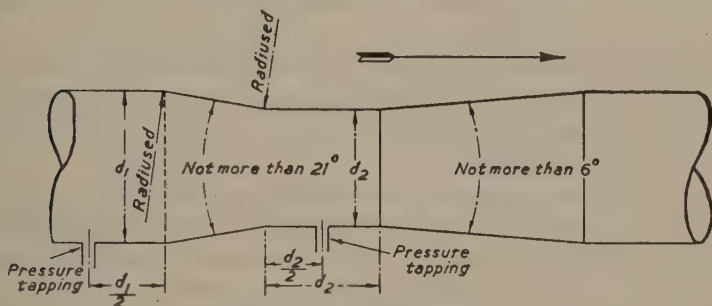


FIG. 45.—Venturi tube.

the stream again fills the whole of the pipe, becomes filled with "dead air" in a state of violent turbulence, in which much energy is dissipated. If the space that these eddies would otherwise occupy is filled by solid material, with suitably shaped boundaries, their formation can be largely prevented. In other words, we can recover most of the head by guiding the stream by means of a conical length of pipe, with its smaller end enclosing the jet and of the same cross section as the latter, and gradually expanding in size along the direction of flow until the full pipe diameter is reached. An arrangement of this kind, combined with a conical entry of sharper taper preceding the throat, is known as a venturi tube, a sketch of the inside of which, reproduced from the B.S.I. Code,¹² is shown in Fig. 45. Apart from the greatly reduced loss of head, the device has no outstanding advantages over the plate orifice or the nozzle; and it will probably only be used when this feature is of importance. It requires considerably more space in the pipe line than either

of the other two forms of constriction because, unless there is a long, gradually expanding portion after the throat, the whole purpose of the device is defeated.

The apex angle (total) of the diverging outlet cone should not exceed six degrees ; if it is appreciably greater the cone will not "run full," and a loss of head will ensue. Compared with the loss of head due to this cause, the loss due to friction in the long expanding outlet is small. The slope of the inlet cone can be considerably greater. Obviously, to reduce frictional losses, this cone should be as short as possible ; but there is a limit to the slope because, if the rate of contraction of the stream is too rapid, we tend to approach the conditions that exist in a plate orifice, i.e., the jet will contract after passing the smallest area of the constriction and the throat of the venturi will not run full. Experience has shown that the total apex angle of the inlet cone should not appreciably exceed the value of twenty-one degrees specified by the B.S.I.

An alternative type of venturi is also specified by the B.S.I.,¹² in which the internal contours of the inlet up to the throat are identical with those of the standard nozzle. Beyond being somewhat shorter than the former type it has no obvious advantage ; and it is more expensive to make because of the careful shaping that is necessary if the standard coefficients are to be used. Further, the pressure loss is probably rather greater than that of the type with the conical inlet.

Pressure tappings may be either single holes or a number of holes around the section communicating with an annular chamber from which the pressure is tapped off to the manometer ; the B.S.I. state that the diameters of any pressure holes must not exceed $0.1 d_1$ on the upstream side and $0.1 d_2$ at the throat.

Discharge Coefficients and Compressibility Factors for Venturis.—As for the nozzle, equation (21) is used for flow calculations based on venturi observations, with the value of C obtained from theory or Figs. 41 or 44. For α , we use the nozzle values in the case of the type having the shaped inlet ; and for the type with conical inlet, the values in Table 13 may be used, which have been taken from the B.S.I. charts.¹²

The values of α are subject to corrections and tolerances as for the other standard constrictions. For the type of venturi

having the shaped inlet, both corrections and tolerances are identical with those appropriate to the standard nozzle. For the type with conical inlet, the tolerances for pipe Reynolds number are included in Table 13; the correction factors for pipe diameter (roughness) are about 0.985, 0.99, 0.995, and 0.997 for $d_1 = 2, 4, 16,$ and 32 inches respectively, the

TABLE 13

Discharge Coefficients and Tolerances for Venturis with Conical Inlet.

$v_1 d_1 / v_1$	$m = 0.1$		$m = 0.3$		$m = 0.4$		$m = 0.5$	
	α	t per cent.	α	t per cent.	α	t per cent.	α	t per cent.
5,000	0.94	± 1.25	0.94	± 1.2	0.94 ₅	± 1.1	0.96	± 0.85
10,000	0.96	± 0.8	0.96	± 0.8	0.96	± 0.75	0.97	± 0.6
20,000	0.97	± 0.6	0.97	± 0.6	0.97	± 0.5	0.97 ₅	± 0.35
30,000	0.97	± 0.4	0.97	± 0.4	0.97	± 0.4	0.98	± 0.25
50,000	0.98	± 0.2	0.98	± 0.2	0.98	± 0.2	0.98 ₅	± 0.1
100,000	0.98 ₅	± 0.1	0.98 ₅	± 0.1	0.99	± 0.1	0.99	± 0

tolerances being the same as for the shaped nozzle. The basic tolerance for α for the conical inlet type is given as ± 0.8 per cent. at $m = 0.1, 0.2,$ and 0.3 ; ± 1 per cent. at $m = 0.4$; and ± 1.5 per cent. at $m = 0.5$. It appears from all these figures that the B.S.I. regard the published data on venturis with conical inlet as insufficient to enable the values of α to be specified as closely as they are for the other forms of standard constriction. It therefore appears advisable to carry out a calibration *in situ* whenever it is decided to use this device and the uncertainty that can be tolerated in the results is less than 1 or 2 per cent.

Loss of Head in Venturis.—With an outlet cone having an apex angle of five to seven degrees, the loss of head in a venturi, expressed as a percentage of the pressure difference between inlet and throat, ranges (approximately linearly with m) from about 13 per cent. at $m = 0.1$ to about $6\frac{1}{2}$ per cent. at $m = 0.5$, the corresponding values for the standard nozzle being about 83 and 33 per cent. These figures emphasise the great superiority, already noted, enjoyed by the venturi in this respect.

The Use of Orifices and Nozzles other than in Pipe Lines.—It is sometimes necessary to measure the quantity of air flowing between the atmosphere and large receivers or air drums ; and one of the most convenient methods of making such measurements is to instal an orifice in one wall of the drum and to measure the pressure drop across it. Any form of constriction can be used for this purpose but, because of its relative simplicity, the plate orifice is usually preferred. A familiar practical example of this use of the orifice arises in the measurement of the air supply to an internal-combustion engine on test. The air intake of the engine is connected to a large air box or drum, and all the air taken by the engine is sucked into this drum from the atmosphere through a small orifice in one side. Within the box, the pressure is less than the atmospheric pressure outside. All we need do, then, to measure the pressure drop across the orifice is to connect one side of a manometer, the other side of which is open to atmosphere, to a hole in the side of the box. If, as is usual in this method of measurement (see below), the orifice is small in relation to the dimensions of the box, the flow inside the latter is everywhere very slow and the pressure practically uniform throughout. The pressure tapping can therefore be situated at any convenient place in the box, but it should, for safety, be flush with the inner surface.

The flow is calculated by the use of the standard equations (12) or (17) already derived, according to the value of $p_1 - p_2$, i.e., the pressure difference between atmosphere and the interior of the box. The value of $p_1 - p_2$ will depend on the size of the orifice in relation to the rate of flow ; and it will usually be possible to select an orifice of such size that $p_1 - p_2$ is a small fraction of p_1 , about 1 per cent. or less. In that case the simpler equation (12) for incompressible flow may be used. A further simplification arises from the fact that, in this method of measurement, it is usual to make the orifice diameter small in relation to the cross-section of the box, so that we can put $m = 0$ without appreciable error. Hence, in this application of the orifice, the flow can generally be calculated from the equation

$$q = \alpha a_2 g \sqrt{[2\rho(p_1 - p_2)]}.$$

The value of α to be used in these conditions will naturally depend, as in the case of pipe flow, on the particular

type of orifice used. Various investigators have determined coefficients for other types of square-edged orifices installed in the sides of air-boxes. The work of Watson and Schofield²¹ is, perhaps, the most comprehensive, but other researches that must be noted are those of Durley²² and Müller.²³ Durley, working with square-edged orifices, found that α for a given orifice was independent of the cross-sectional area of the box, provided that the ratio of this area to that of the orifice was greater than 20 (i.e., $m < 0.05$). This result is, however, not altogether in agreement with that of Watson and Schofield, who found small variations, of the order of 1 per cent., in α for the same orifice for different values of m . The orifice diameters used by Watson and Schofield were $\frac{1}{2}$, 1, $1\frac{1}{2}$, and 2 inches, and three air-boxes were used, of dimensions $28 \times 13 \times 13$ inches, $38 \times 17 \times 17$ inches, and $57 \times 25 \times 25$ inches. It will be seen that the smallest of these had a cross-sectional area considerably greater than twenty times that of a 2-inch orifice.

However, if we neglect the results for the $\frac{1}{2}$ -inch diameter orifice, we find that none of the values of α given by Watson and Schofield differs by more than 1 per cent. from 0.596, irrespective of the size of box used or the pressure difference, which ranged from about $\frac{1}{2}$ inch to 2 inches of water. It is interesting to observe that Watson and Schofield used square-edged orifices cut in thin plates, and that the mean value of 0.596 agrees closely with that deduced—albeit for the case of pipe flow—for the case $m = 0$ for the square-edged German standard orifice (see Table 6). Similar agreement was found by Müller, who worked with orifices 0.92-inch, 1.73-inch, and 2.47-inch diameter, with pressure differences ranging from 0.2 inch to 2.3 inches of water. Müller's orifices, which were all tested in a 23.8-inch diameter drum, were square-edged on the upstream side and bevelled off at forty-five degrees on the downstream side, and thus resembled the German standard type more closely than those of Watson and Schofield. Müller's mean value of α was 0.597. We may note also that, whereas in Watson and Schofield's experiments the air passed into the box from the atmosphere, in Müller's the direction of flow was the reverse of this.

It is not apparent why the values of α obtained by Watson and Schofield for the $\frac{1}{2}$ -inch diameter orifice were usually rather

more than 1 per cent. above those for the others under similar conditions. One point that may have a bearing on this result, however, is that all the orifices were cut from plate of the same thickness, and so were not geometrically similar. This departure from similarity would, moreover, be more noticeable in the orifices of smaller diameter. Another feature the reason for which is not quite clear is that there was a definite, although small, progressive change in α with the pressure difference when the small and large boxes were used, whereas with the medium-sized box no such change was apparent. Further, it was in opposite directions with the large and small boxes, so that, even if the pressure differences had been large enough (which was not the case) to make the application of the compressibility correction necessary, this variation of α with pressure could not have been attributed to the fact that Watson and Schofield did not apply the correction. A similar tendency to a slight progressive change of α with $p_1 - p_2$ is discernible in Müller's published data, although he does not comment upon it. Durley also noticed a change in α with pressure difference.

Summing up the evidence, therefore, we may say that, although further research is required to define completely the characteristics of orifices used under these conditions, the obscure points are not of much practical importance. Provided that the orifices are square-edged, and not less than 1 inch in diameter, and are used in air-boxes of cross-section greater than twenty times that of the largest orifice, a value of 0.596 may be used for α if errors not exceeding about 1 per cent. are permissible. Further, it does not seem to be material whether the air is flowing into or out of the box.

Stach²⁴ has determined the discharge coefficients of the German standard orifice and the standard nozzle installed at the ends of pipes discharging into or sucking from the atmosphere; and his results are summarised in the V.D.I. Rules¹¹ as follows:—

When the air flows into the pipe, the upstream face of the orifice or the nozzle should be of diameter at least 1.5 times the diameter of the aperture, and there should be no obstructions within a distance of $20 d$ in front of the constriction or $10 d$ from the axis. Under these conditions, the value of α may be taken as 0.99 ± 1 per cent. for the nozzle and 0.60 ± 1.5 per cent.

for the orifice, provided that the pipe Reynolds number exceeds 55,000.

When the air flows from the pipe into the atmosphere, we may use the value of α appropriate to the value of m (see pages 117, 132), reducing the value for the nozzle by $\frac{1}{2}$ per cent. The probable errors are the same as those just given for air flowing into the pipe; and the limiting values of the pipe Reynolds number below which these values of α do not apply without correction are the same as for ordinary pipe-line installations. The dimensions of the zones on the outlet side that should be free from obstructions are about half those given above for flow in the reverse direction.

Very similar rules are given by the B.S.I.,¹² which include also directions for the use of the standard constrictions installed in the walls of large air-boxes or receivers, as described in the earlier parts of the present section; the value of α recommended by the B.S.I. for this case is that appropriate to $m = 0$, i.e., 0.595 for the orifice and 0.988 for the nozzle. The value for the orifice thus agrees well with that deduced from the results of the work of Watson and Schofield, Müller, and Durley quoted above.

Measurement of Unsteady Flow.—The fact that the method of measurement described in the preceding section is often used in order to measure the air supply in tests of internal-combustion engines raises the question of how to allow for, or damp out, pulsations in the flow. It is clear that there will be considerable pulsations in the intake, since the inlet valve is open only for a portion of the cycle. Similar problems, although generally in not so acute a form, occur in other cases of air flow measurement. The subject is a difficult one upon which there is not sufficient experimental evidence to enable the effect of pulsations to be estimated; but there is no doubt that serious errors can arise unless large pulsations are eliminated.* Watson and Schofield showed this clearly in their research just mentioned; others who have worked on the problem are Judd and

* Since the pressure that is used as a measure of the flow depends on the square of the air speed, the error is of the same nature as that discussed on page 75 in connection with the pitot-static tube. Its effect is to cause the constriction to produce an average pressure higher than that corresponding to the average rate of flow.

Pheley,²⁵ Hodgson,²⁶ Bailey,²⁷ and King.²⁸ Although these workers differ on certain points, they are in general agreement that, in order to enable pulsating air flow to be measured accurately by means of an orifice or other constriction, it is necessary to prevent appreciable fluctuations from reaching the orifice. This can usually be done by inserting capacity (for example, a large air-box or receiver) or a throttling device, or both if necessary, between the source of pulsation and the orifice. A suitable form of throttling device is another orifice in series with the measuring orifice. A large enough capacity would probably always suffice by itself; but if the pulsations are very violent the size of receiver required will probably be inconveniently large, and in such circumstances a combination of throttling and receiver capacity is best used.

Hodgson²⁶ has shown that it is possible to calculate approximately the combination of throttling and capacity that is necessary when the form of the flow-time curve is known; and Bailey²⁷ and King²⁸ have given approximate methods of calculation applicable to the case of the measurement of the air supply to an internal-combustion engine by the method described in the preceding section. Bailey's method involves both throttling and capacity, while King gives the following simple formula for the minimum size of receiver to ensure sufficiently steady conditions for measurements to be made:

$$\text{Volume of receiver in cubic feet} = 600 \frac{\text{H.P. of engine}}{\text{No. of cylinders} \times \text{R.P.M.}}$$

King states, however, that this formula should not be taken as applying to extreme cases, such, for example, as that of a single-cylinder engine having one suction stroke per two revolutions. Also, he recommends that an air-box of less than 5 cubic feet capacity should not be used, even if the formula indicates that a smaller size would suffice. King mentions a convenient and simple method of enabling one to tell whether the pulsations have been sufficiently reduced. The method is based on a device used by Watson and Schofield. A thin rubber sheet is secured to one end of a short piece of tube about 3-inch diameter, whose other end is in communication with the inside of the air-box. The rubber diaphragm will be bowed inwards by the suction in the air-box, and when in this position should show no

more than a trace of vibration. In extreme cases, if the largest available air-box is not enough, two boxes in series may be used, the one nearer the engine being solely for damping purposes.

A rather different kind of pulsation effect sometimes occurs, namely, the effect on the measuring manometer or pressure gauge when, although the rate of flow is satisfactorily steady, fluctuations in static pressure are present. These pressure changes will be communicated to the manometer through the connections that lead to it from the points at which the pressures are tapped off. If the connections on the two sides of the manometer have different capacities or damping, it is easy for quite large errors to be introduced. Such errors can generally be avoided by equalising the damping and capacity of the leads on the two sides of the manometer (see page 98).

The Critical Pressure Drop across a Constriction.—If we examine equation (8), the theoretical expression for the flow of a compressible fluid through a constriction in which there is no contraction of the jet after the minimum area of the constriction is passed, we see that there is a value of the pressure ratio p_2/p_1 for which the flow is a maximum. For simplicity, assume that the upstream diameter d_1 is so large compared with that of the constriction d_2 that we may take m as zero. Then, for given values of p_1 and ρ_1 , the quantity q is a maximum when

$$\frac{\gamma}{\gamma - 1} \left\{ 1 - \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right\} \left(\frac{p_2}{p_1} \right)^{2/\gamma}$$

is a maximum. That is to say, the flow is a maximum when the differential coefficient of this expression with respect to p_2/p_1 is zero. For maximum rate of flow we therefore have

$$\frac{2}{\gamma} \left(\frac{p_2}{p_1} \right)^{\frac{2-\gamma}{\gamma}} \left\{ 1 - \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right\} - \frac{\gamma-1}{\gamma} \left(\frac{p_2}{p_1} \right)^{2/\gamma} \left(\frac{p_2}{p_1} \right)^{-1/\gamma} = 0.$$

On reduction, this becomes

$$\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = \frac{2}{1 + \gamma}.$$

Inserting the value of γ for air (1.41), we find the solution of this equation to be $p_2/p_1 = 0.527$, which is the value of p_2/p_1 for which the flow is a maximum.

The physical interpretation of this is that, for a given value of p_1 , as the pressure ratio p_2/p_1 decreases the flow increases until p_2/p_1 reaches the value 0.527. The velocity at the minimum section of the jet is then equal to the velocity of sound in air, i.e., to the velocity of wave propagation in air. Hence, if the pressure into which the discharge is taking place is reduced below the value that makes $p_2/p_1 = 0.527$, the flow does not increase further because the lower pressure cannot transmit itself backwards along the issuing jet and affect the discharge. The rate of flow therefore remains constant at the value appropriate to $p_2/p_1 = 0.527$. This value of the pressure ratio is called the critical pressure ratio or critical pressure drop.

REFERENCES

1. Johansen, *Proc. Roy. Soc., A*, **126** (1930), p. 231; also *Aero. Res. Com. Rep. and Mem.*, No. 1252 (1930).
2. Hodgson, *Proc. Inst. C. E.*, **204** (1916-17), Pt. 2, p. 108, and **212** (1920-21), p. 272. Also *Trans. Inst. Naval Architects*, **64** (1922), p. 184; *Engineering*, **117** (1924), p. 314; *Inst. C. E. Selected Engineering Papers*, No. 31 (1925).
3. Witte, *Technische Mechanik und Thermodynamik (T.M.T.)*, **1** (1930), pp. 34, 72, and 113; *Forschung* (new title of T.M.T.), **2** (1931), pp. 245 and 291; *ibidem*, **5** (1934), p. 205.
4. Jakob and Kretschmer, *Verein deutscher Ingenieure, Forschungsheft*, No. 311 (1928).
5. Kretschmer, *Forschung*, **4** (1933), p. 93, and (with Wälzholz) **5** (1934), p. 25.
6. Ruppel, *T.M.T.*, **1** (1930), p. 151. Also *Zeitschrift des Vereines deutscher Ingenieure (Z.V.D.I.)*, **80** (1936), p. 1381.
7. Giese, *Forschung*, **4** (1933), p. 11.
8. Hansen, *Forschung*, **4** (1933), p. 64.
9. Jaroschek, *Z.V.D.I.*, **80** (1936), p. 643.
10. Bean, Buckingham, Murphy and Benesh, *Bureau of Standards Journal of Research*, **2** (1929), p. 561, and **7** (1931), p. 93.
11. *Regeln für die Durchflussmessung mit genormten Düsen und Blenden (V.D.I., Durchfluss-Messregeln. DIN 1952)*, 4th Edition (1937).
12. British Standards Institution, *Flow Measurement*, British Standard Code, B.S. 1042 : 1943.
13. Johansen, *Engineer*, **149** (1930), p. 679.
14. Stanton, *Proc. Roy. Soc., A*, **85** (1911), p. 366.
15. Witte, *Forschung*, **2** (1931), p. 291.
16. Witte, *T.M.T.*, **1** (1930), p. 113.
17. Ruppel, *T.M.T.*, **1** (1930), p. 151.
18. Busemann, *T.M.T.*, **1** (1930), p. 338.

19. Buckingham, *Bureau of Standards Journal of Research*, **6** (1931), p. 765 ; also *ibidem*, **9** (1932), p. 61.
20. Witte, *Z.V.D.I.*, **72** (1928), p. 1493.
21. Watson and Schofield, *Proc. Inst. Mech. Eng.*, Parts 1-2 (1912), p. 517.
22. Durley, *Trans. Amer. Soc. Mech. Eng.*, **44** (1923), p. 853.
23. Müller, *Z.V.D.I.*, **52** (1908), p. 285.
24. Stach, *Z.V.D.I.*, **78** (1934), p. 187.
25. Judd and Pheley, *Trans. Amer. Soc. Mech. Eng.*, **44** (1923), p. 853.
26. Hodgson, *Trans. Inst. Naval Arch.*, **64** (1922), p. 184 ; also *Trans. Inst. Mining Eng.*, **60** (1921), p. 275.
27. Bailey, *Trans. Amer. Soc. Mech. Eng.*, **61** (1939), p. 301.
28. King, *Engineering*, **115** (1923), pp. 456 and 481.

CHAPTER VII

THE VANE ANEMOMETER

ALTHOUGH it is subject to certain practical limitations, which will be indicated later, the vane anemometer has been extensively employed for many years. As a means of measuring air speed, particularly in the range of low speeds where, as we have already seen, such small pressures are set up by the motion that their accurate determination becomes a matter of considerable difficulty, its simplicity renders it an exceedingly useful instrument; and if properly used it is capable of giving all the precision ordinarily required in engineering practice.

The vane anemometer is, in effect, simply a windmill, consisting of a number of light, flat vanes mounted on radial arms attached to a common small steel spindle, which rotates in two jewelled bearings. Eight vanes, made of thin sheet aluminium or aluminium alloy, are almost invariably used; they are inclined at an angle to the axis of the spindle, which is set along the direction of flow when the instrument is in use; and the air forces acting on the vanes cause the spindle to rotate at a rate depending mainly on the air speed.* By means of suitable gearing, the motion of the spindle is communicated to a pointer or, more usually, to a number of pointers moving over graduated dials. Normally, the dials are marked with scales of feet; and experience has taught manufacturers of these instruments how to proportion the various dimensions, vane inclinations, and gearing ratios, so that the number of feet indicated on the dials in unit time is approximately equal to the distance traversed by the air in the same time, i.e., to the air speed. In order, therefore, to determine the air speed it is necessary to observe with the aid of a stop watch the time taken for a number of "feet of air," as shown by the indicating mechanism, to pass the instrument. This gives the "indicated" air speed, from which the true air speed is obtained by the aid of the calibration curve for the particular anemometer in use.

* See pages 162-168 for the effect of air density.

Instruments of this type are mainly of value for measuring air speeds in large ducts or ventilating shafts, or in smaller pipes (subject to the restriction on the limiting size mentioned below) where access can conveniently be made to an open end, preferably an outlet end. They are probably best used when mounted on light rods and inserted into the airstream from one side. It is important that the observer should not stand too close to the instrument whilst readings are being obtained, as his presence will disturb the conditions of flow even if he is



FIG. 46.—Vane anemometer.

situated downstream of the position occupied by the instrument. In order to determine the quantity of air flowing, readings must be taken with the axis of the vane circle at a number of points across a section (see page 81).

Two types of anemometer in common use are illustrated in Figs. 46 and 47. In the former, the plane of the dials is perpendicular to the plane of rotation of the vanes ; this design is not so compact as the other, but it is probably to be preferred on account of the greater facility which it offers for taking observations from the side.

In many instruments, provision is made for throwing the indicating mechanism into gear or disconnecting it from the vane spindle by the movement of a small lever. In some

instruments, also, the reading can be set at zero at any stage. These devices appear to be designed to obviate the use of a stop watch, the intention, presumably, being that readings should be taken by throwing the indicating mechanism into gear at a given instant and disconnecting it when a certain time

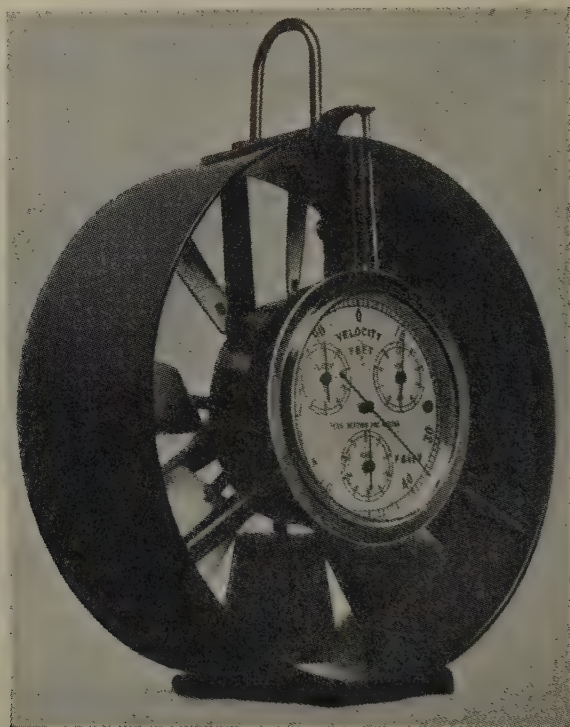


FIG. 47.—Vane anemometer with concentric dial.

has elapsed, which could be observed on any watch provided with a seconds hand. This method of using vane anemometers is not to be recommended ; more accurate results will certainly be obtained by observing with a stop watch the time taken for a given number of complete revolutions of one of the pointers. The other method should only be adopted if a stop watch is not available. In order to reduce starting and stopping errors to small percentages of the total readings, no observations should

occupy less than 60 seconds, or involve a reading of less than 100 feet on the instrument. It is advisable also to take two check readings of every observation, for it will often be found that the flow is subject to fluctuations, so that the average over a given period is not quite constant.

It is often stated that vane anemometers are unreliable and that their indications are subject to unaccountable inconsistencies. The author's experience with these instruments does not support this contention. Trouble frequently arises through the use of an anemometer to measure air speeds for which it is not designed. If the speed is too high, the vane setting may suffer a permanent distortion, which will cause a corresponding permanent alteration in the calibration curve. On the other hand, if the speed is too low, the friction of the bearings and gearing exercises an appreciable effect (see below). The normal types of instrument, such as those illustrated in Figs. 46 and 47, are suitable for the measurement of air speeds ranging from about 5 to 50 feet per second, and may confidently be relied upon in this range. For higher speeds, special instruments are made, which are either of heavier construction or, alternatively, incorporate a shutter device by means of which the quantity of air actually passing through the vanes can be reduced ; such instruments can be used for air speeds up to 100 feet per second or more.

Low-Speed Anemometers.—For low air speeds, it is essential to have an instrument in which the friction is very small. In order that the starting torque due to the wind shall be large compared with the resisting frictional torque, it is desirable that the vanes shall be large and set at a large radius from the axis. These dimensions are, however, limited by considerations of lightness ; for if the inertia of the moving parts is too great, the instrument will not follow rapid fluctuations in the speed of the air, and will give readings that differ perceptibly from the mean air speed. Moreover, it must be remembered that the resisting frictional torque is composed partly of a torque due to the weight of the moving parts, and partly of a torque due to the end thrust of the spindle on its downstream bearing, arising from the downstream component of the air force on the vanes ; the first of these factors will

increase with the weight of the vanes and the second with their size. As is usual in design generally, a compromise has therefore to be effected between conflicting requirements.

A form of low-speed anemometer that gives consistent readings down to air speeds of about 0.6 per second has been designed by the author,¹ and is illustrated in Fig. 48. The vanes, which are $1\frac{1}{8}$ inches deep, are of the usual pattern,

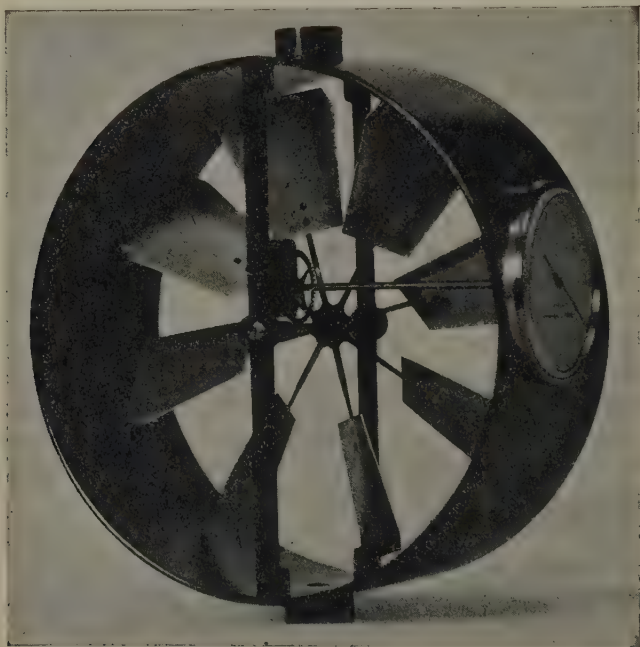


FIG. 48.—Author's low-speed vane anemometer.

inclined at an angle of forty-five degrees to the axis, and the outer radius of the vane circle is $1\frac{7}{8}$ inches. There is no gearing except a worm drive through which the rotation of the vanes is transmitted, with a reduction of 50 to 1, to a pointer moving over a dial attached to one side of the casing. The pointer is fixed directly to the worm-wheel spindle, and the only markings on the dial are four short radial lines, to indicate quarter-revolutions of the pointer. The calibration of the instrument is given in terms of the relation between the air speed and the revolutions per second of the pointer over the dial. Three

tapped bosses are fitted to the casing (the third, which cannot be seen in the photograph, is opposite to the dial) for convenience in mounting the instrument to suit various conditions of use.

A different type of low-speed anemometer is that designed by Rees,² which we have already briefly noted (see page 4). Although perhaps not as simple to use as the vane anemometer, it has the advantage over the latter that it can be used to measure appreciably lower speeds—down to about 0.15 feet per second. On the other hand, the instrument can only be used to measure horizontal air velocities, whereas the vane anemometer is subject to no such restriction, provided that, for accurate work, it is calibrated in the attitude in which it is to be used.

Calibration Curves.—It is important that every anemometer should be calibrated in an air stream whose speed can be controlled and measured by independent means, or by moving it at known speeds through still air; for it is not possible to predict the readings of an instrument accurately from its mechanical constants. Anemometers used to be sent out by the makers accompanied by a printed form stating that a certain number of feet per minute must be added to the readings to give the true air speed, the number being the same for all speeds. Such instructions, which are now fortunately rarely given, should be regarded with suspicion and not accepted without verification: during the course of a large number of calibrations of vane anemometers carried out under his supervision, the author does not recollect a single instrument whose error was constant over the whole range.

The method of carrying out an anemometer calibration is in itself a simple matter, but considerable care must be taken to obtain an air stream suitable for the purpose. When the amount of work necessitating the use of an anemometer does not warrant the provision by the user of his own special testing equipment, the instrument may be submitted to the National Physical Laboratory for calibration.

Readings in an Inclined Stream.—The readings of a vane anemometer are not seriously affected by fairly large variations of the direction of the air stream from its normal path perpendicular to the vane circle. Although it is not possible, on

account of the numerous differences in details of shape and construction that occur amongst existing types of vane anemometers, to draw generalised conclusions from the observed behaviour of a particular instrument, Fig. 49 will serve to show what order of error may be expected if the wind is inclined to the axis of the vane circle. The results relate to experiments * carried out by F. C. Johansen and W. J. Duncan with a calibrated anemometer of more or less normal proportions. The instrument was mounted in a wind tunnel side by side with a second calibrated instrument, which served as a standard. A sufficient distance was allowed between the two instruments,

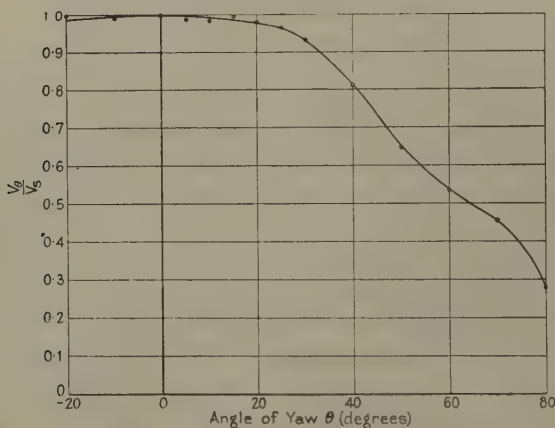


FIG. 49.—Effect of yaw on readings of vane anemometer.

which were situated in the same cross-section of the tunnel, to render negligible any interference one might have on the other. Simultaneous readings of the two instruments were taken, the standard being kept stationary in normal attitude and indicating the true mean speed over the intervals of observation; whilst the other was rotated successively to the wind direction, by stages of five or ten degrees at a time, through large angles on each side of its normal attitude. At each setting, the indicated speed of each instrument was observed; and the ratio V_θ/V_s was calculated, V_θ being the speed as given by the yawed instrument at an inclination θ° and V_s the true air speed as indicated by the standard.

* Results not published.

It will be seen from Fig. 49, where V_θ/V_s is plotted against θ , that the instrument could be yawed through about twenty degrees before an error of 1 per cent. on the indicated speed was incurred. It is probable, therefore, that little apprehension need be felt about the accuracy of the readings of a vane anemometer if the mean direction of the air current is known within reasonable limits.

Characteristics Indicated by Theory.—The theory of the vane anemometer will be considered in the second part of the present chapter; but, since the theoretical treatment is unavoidably somewhat involved, it will be useful at this stage to state the main deductions to which it leads. Under normal conditions, an anemometer is generally used to measure air speeds appreciably higher than the lowest speed that will just set the vanes in rotation. In these circumstances, it is found that the effect of friction at the bearings can be neglected, and it will be shown that the rotational speed of the vanes will then depend only on the velocity of the air and not on its density. The readings of a well-made anemometer, in which the frictional resistance is very small, do not therefore require correction for atmospheric pressure and temperature, provided that the vanes are not rotating too slowly. For special low-speed work, variations of the density of the air may, however, become important, and can be taken into account by the method shortly to be described.

As we have already seen in dealing with the flow of air in pipes, there is in general a variation in velocity from point to point across a section of a pipe; from which it follows that, when an anemometer is used to measure the speed of air flowing along a pipe, there will usually be a variation of air speed across the anemometer disc. Theory shows that, under these conditions, the anemometer does not indicate accurately the speed of the filament of air passing through the centre of the disc, and that quite important errors may arise from this cause. In order to avoid such errors, the diameter of the anemometer should be fairly small in comparison with that of the pipe, so that the variation of speed across the instrument itself is small. Unless precautions are taken to equalise the velocity at different points along the pipe diameter, if a marked variation originally

exists, a vane anemometer should not be used in a pipe whose diameter is less than about six times, preferably eight times, that of the instrument (see page 180).

Finally, when used in a pulsating air stream, a vane anemometer overestimates the average speed ; but provided that the extremes of speed are not more than about 15 per cent. on either side of the mean, the error will probably not appreciably exceed 1 per cent. On the other hand, since the error is proportional to the square of the amplitude of the pulsations, it increases rapidly if the amplitude exceeds 15 per cent. : for a 50 per cent. pulsation the error will be of the order of $12\frac{1}{2}$ per cent.

Measurement of the Rate of Flow in Pipes with Vane Anemometers.—Like the pitot tube, the vane anemometer measures a local velocity which, subject to the limitations mentioned above, may be taken as the velocity of the filament of air passing through the centre of the vane wheel. In order, therefore, to determine the quantity of air flowing along a pipe by the use of this instrument, readings of velocity must be taken at a number of points across the section, whose positions are determined in the manner indicated in Chapter V ; in other words, readings must be taken with the axis of the vane wheel at the points at which the velocity head would be observed if a pitot tube were being employed for the purpose. If v_1, v_2, v_3 , etc., are the air speeds in feet per second thus observed, the mean speed is given by equations (2) or (3) of Chapter V, according to whether the zones into which the pipe is divided are of equal area or of equal width.

The volume of air flowing in cubic feet per minute is

$$V = 60Av_m,$$

where A is the area of the pipe section in square feet ; and, since 1 cubic foot of dry air at 60° F. and 29.92 inches of mercury barometric pressure weighs 0.0763 pound, the weight of air flowing in pounds per minute is

$$\begin{aligned} Q &= 60Av_m \times 0.0763 \times \frac{b}{29.92} \times \frac{520}{460 + t} \\ &= 79.6Av_m \times \frac{b}{(460 + t)}, \end{aligned}$$

where b is the barometric pressure in inches of mercury and t is the temperature of the air in the pipe in degrees Fahrenheit.

It is clear from this equation that the remarks made in Chapter V regarding the allowance that must be made for the static pressure in the pipe, if this is appreciably above or below atmosphere, apply also to cases in which the vane anemometer is used. We observe that the permissible departure of the static pressure from atmospheric is, in this case, ± 4 inches of water if the error on v is to be limited to 1 per cent. ; whereas in the equations of Chapter V, in which b occurred under a radical sign, ± 8 inches could be allowed. It is, however, very unlikely that such static pressures will be experienced in any practical case in which the vane anemometer can conveniently be employed.

THE THEORY OF THE VANE ANEMOMETER *

Aerodynamic Characteristics of Flat Plates.—The vane anemometer consists of a number of small flat plates inclined at an angle to the direction of the wind ; let us therefore consider the system of forces that acts on a flat plate when exposed to an air current. In Fig. 50, AB represents an end view of the plate inclined at an angle ϕ to the wind direction, this angle being usually termed the angle of incidence of the plate.

Let V be the wind speed, ρ the density of the air, and S the area of the plate. Experiment shows that the aerodynamic reactions on the plate may be reduced to a single force F acting along OP, the point O, whose position along AB depends upon the incidence ϕ , being known as the centre of pressure. It is found also that OP usually makes a small angle γ , as shown, with

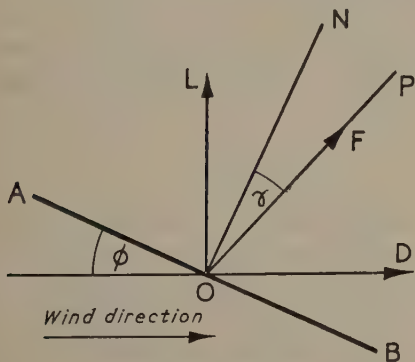


FIG. 50.

* Most of the material in this section has been taken from References 1, 3, 4, and 5, page 184.

the normal to the wind direction through O, and that the magnitude of the force F is given by the equation

$$F = K\rho V^2 S\phi, \quad \dots \dots \dots (1)$$

where K is a numerical coefficient whose value depends upon ϕ . Usually it will be found more convenient to resolve the force F

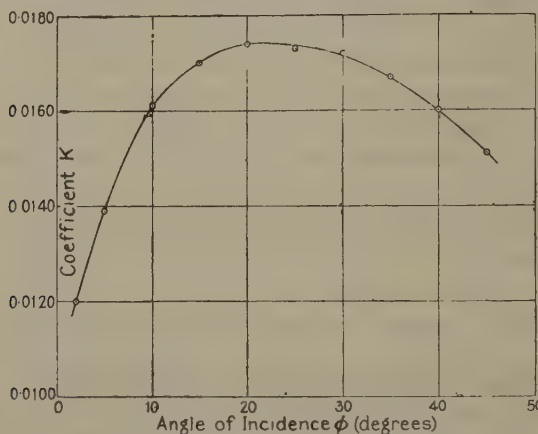


FIG. 51.

into two components L and D , respectively perpendicular to and along the wind direction. Thus,

$$\left. \begin{aligned} L &= K\rho V^2 S\phi \cos \gamma \\ D &= K\rho V^2 S\phi \sin \gamma \end{aligned} \right\} \quad \dots \dots \dots (2)$$

Values of K at different angles of incidence have been obtained experimentally for typical anemometer blades,⁴ and are given in Table 14 and in Fig. 51. Values of γ deduced from

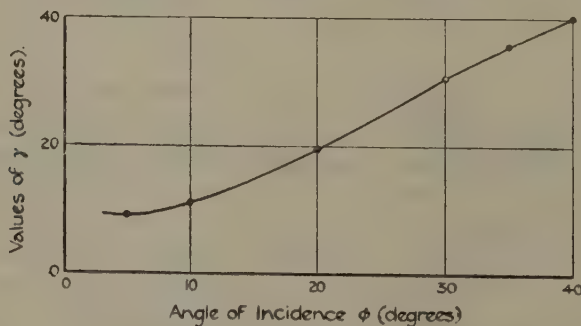


FIG. 52.

G. Eiffel's experimental results ⁶ for small square flat plates are shown in the same table and are plotted in Fig. 52.

TABLE 14

Angle of Incidence Degrees	K	γ Degrees
5	0.0123	9.0
10	0.0139	11.0
20	0.0174	19.5
30	0.0172	30.7
35	0.0167	35.8
40	0.0160	40.3
45	0.0151	—

Application of Flat-Plate Characteristics to the Anemometer.—We may now consider the aerodynamic characteristics of flat plates in relation to their application to the vane anemometer. The following notation will be used :—

F = resultant wind force on one blade,

T = frictional resisting torque,

Q = wind torque on blades,

V = wind velocity,

v = linear tangential velocity of centre of pressure of blade,

$\bar{V} = \sqrt{(V^2 + v^2)}$ = velocity of wind relative to blade,

K = wind force coefficient (see above),

ρ = air density = weight per unit volume/ g ,

θ = inclination of blade (when at rest) to wind direction,

ϕ = inclination of blade (when in motion) to relative wind
= effective incidence of blade,

γ = inclination of resultant wind force on blade to perpendicular to direction of relative wind,

n = rotational speed of blades,

N = rotational speed of pointer,

C = instrument constant, such that $N = Cv$,

A = area of one blade,

D = diameter over blade tips,

m = number of blades,

r = distance of centre of pressure of blades from axis of rotation,

I = total moment of inertia of all blades about axis of rotation,

g = acceleration due to gravity.

Case 1. Mechanical Friction Neglected.—Let PQ (Fig. 53) represent one blade inclined at an angle θ to the wind direction RO, and let O be the centre of pressure of the blade.

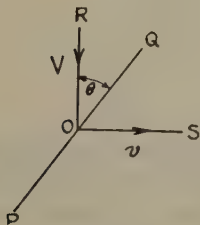


FIG. 53.

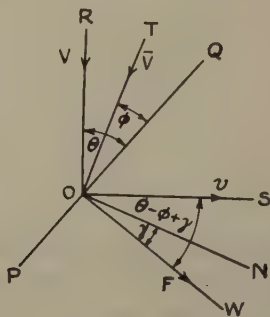


FIG. 54.

It should be noted that the blade is rotating in a circular path perpendicular to the plane of the paper, and that, at the instant under consideration, the blade is at the top of its path and moving to the right with a velocity v along OS, perpendicular to the wind direction. The blade speed v will be such that the resultant of V and v lies along QP, and there will be no resultant force perpendicular to the plane of the blade,

i.e.,
and

$$\left. \begin{aligned} v &= V \tan \theta \\ N &= Cv = CV \tan \theta \end{aligned} \right\} \cdot \cdot \cdot \cdot (3)$$

Case 2. Mechanical Friction Included.—The velocity diagram when mechanical friction is taken into account becomes more complicated; and it is necessary to consider the forces acting on the blade. PQ (Fig. 54) will still move along OS, perpendicular to the wind direction, but at a lower speed v , such that the resultant of V and v , i.e., the wind speed $\bar{V}$ relative to the blade, will now, when the motion has become steady, be inclined at an angle ϕ to the plane of the blade. The blade thus becomes a flat plate inclined to the wind at an effective incidence ϕ ; and there will be a resultant force F on the blade

acting along OW, inclined at an angle γ to ON, the normal to the relative wind direction. The component of F along OS, multiplied by the distance of O from the axis of rotation and by the number of blades, provides the torque which overcomes the resisting torque due to mechanical friction.

If, therefore, we neglect any change in velocity of the air during its passage through the vane circle, and any interference effect that one blade may exercise on its neighbours, we may use equation (1) to give us, for the blade under consideration, the following equation for F :

$$F = K\rho\bar{V}^2A\phi.$$

Resolving along OS, putting $\bar{V}^2 = V^2 + v^2$, and equating the wind torque to the frictional resisting torque, we therefore have for steady motion,

$$T = K\rho\phi A(V^2 + v^2)mr \cos(\theta - \phi + \gamma). \quad (4)$$

Also, from the velocity diagram,

$$v = V \tan(\theta - \phi). \quad (5)$$

We shall find it convenient at this stage to consider the practical aspects of the equations so far derived. It will be apparent that the speed of rotation of a given set of vanes will be determined not only by the wind speed, but also by the mechanical friction introduced by the spindle bearings and by the gearing. In the ideal case where friction is entirely absent, the calibration curve of the instrument, i.e., the curve of indicated wind speed against true wind speed, would be a straight line passing through the origin—see equations (3)—whose slope would be determined (once the gearing ratio is fixed) by the inclination of the vanes to the wind direction. In practice, however, friction exercises an effect which is felt mainly at low speeds when the aerodynamic forces on the vanes are small. At high speeds, when the relative effect of friction is small, ϕ becomes small compared with θ , and we see from equation (5) that the calibration curve of the instrument will be practically linear. At low speeds ϕ is comparable in magnitude with θ , and v will be zero at a certain low value of V when ϕ becomes equal to θ .

The behaviour of all well-made anemometers fully substantiates the above reasoning. The usual type of calibration curve

for such instruments is illustrated in Fig. 55. At high speeds, the curve exhibits no sensible departure from linearity; but at low speeds it bends over somewhat in the manner shown, the abscissa OA representing the true wind speed at which the vanes just begin to turn, that is, the limiting speed at which ϕ becomes equal to θ . The magnitudes of the wind speeds corresponding to OA and to OB (above which the curve is linear) vary, as is naturally to be expected, with different instruments; in the author's sensitive low-speed anemometer, to which reference has been made above, OA was found to be about 0.6 and OB about 1.0 feet per second.

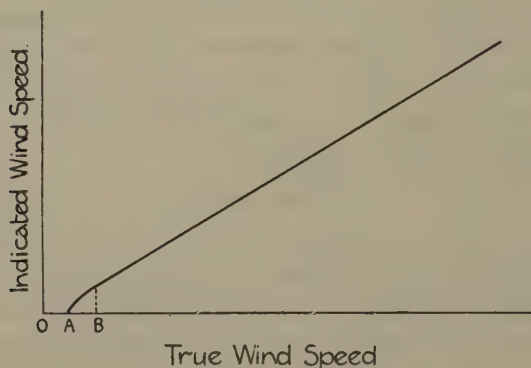


FIG. 55.—Typical calibration curve for vane anemometer.

The friction torque T in equation (4) cannot be computed from first principles; it will obviously depend upon the construction of the instrument, and so will have to be determined separately for each anemometer by experiment if a knowledge of its magnitude is required. The necessary tests, however, are simple, consisting, as they do, only of a calibration to establish the relation between v and V for a number of values of the wind speed V . The torque T for a given speed V can then be calculated by the use of equations (4) and (5), v being taken from the calibration curve, and the values of K and γ being obtained from Figs. 51 and 52 at the values of ϕ calculated from equation (5).

Now the torque T is made up of two components, one due to the weight W of the moving parts, and one due to the end-thrust of the spindle on the rear bearing, arising from the

component P of the resultant wind force acting along the wind direction. Hence we may write

$$T = \mu W r_1 + \mu P r_2, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where μ is the coefficient of friction, and r_1 and r_2 are the effective radii, whose actual values do not concern us, at which the forces μW and μP act.

If, as is generally the case, the bearings are cylindrical, it is justifiable to regard r_1 and r_2 as constant. If, further, we assume that μ is constant, we may, since W is constant, write (6) in the form

$$T = k + hP, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where k and h are constants.

T is given by equation (4) and can be calculated from the calibration curve, as already shown. Similarly, P will be given by

$$P = K \varphi \rho A (V^2 + v^2) m \sin (\theta - \varphi + \gamma), \quad . \quad . \quad (8)$$

and is calculable in an analogous manner. From (7), we may expect a curve of T against P to be a straight line, from which

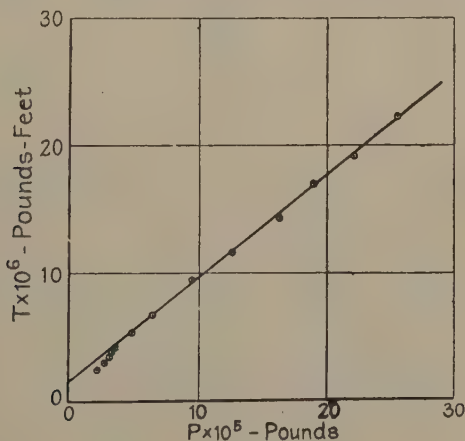


FIG. 56.

the values of k and h can be determined. Values of T and P for the low-speed anemometer previously mentioned have been calculated, and are plotted in Fig. 56. It will be seen that, except for the small values near the origin (corresponding to

values of V below 1 foot per second), the points lie well on a straight line, which gives values of k and h of 1.6×10^{-6} and 8.0×10^{-2} respectively. The departure from linearity at low speeds may be due to a variety of causes, which are discussed more fully below.

The values of T and P plotted in Fig. 56 are given in Table 15. The mechanical constants of the instrument to which they relate are as follows :—

$A = 0.0068$ sq. ft. ; $r = 0.115$ ft. ; $m = 8$; $D = 0.312$ ft.

$\theta = 45^\circ$; gear reduction (vanes to pointer) = $\frac{1}{60}$.

$I = 192 \times 10^{-8}$ slugs ft.²

TABLE 15

Wind Speed V Feet per Second	$T \times 10^6$ Pounds-Feet	$P \times 10^5$ Pounds
0.59	2.46	2.17
0.7	2.98	2.67
0.8	3.42	3.02
0.9	3.85	3.34
1.0	4.18	3.57
1.5	5.39	4.86
2.0	6.69	6.40
4.0	11.82	12.60
6.0	16.95	18.98
8.0	22.20	25.60

Effect of Variations of Air Density.—We may turn now to a consideration of the effects of variations of atmospheric density on the readings of a vane anemometer. We have already seen that these effects are likely to be mainly felt at low wind speeds ; so that if an anemometer is employed for the measurement of such speeds, for which purpose it has an eminent superiority over other types of air-speed measuring instruments, errors may arise on account of the fact that when measurements are being taken the air density may differ from that existing at the time when the instrument was calibrated, for which conditions only the calibration curve is strictly valid. The calibration curve or certificate accompanying the anemometer should, in the case of an instrument intended for low-speed work, bear a note recording the atmospheric density at the time of calibration.

The problem we are now considering is the determination of the effect of a variation in ρ on the value of the blade speed v . We shall assume for the purpose of the analysis that the linear speed of the vanes v is known for each value of the wind speed V ; although the calibration will be given in terms of revolutions per second for a series of values of V , it is a simple matter to deduce v from these figures and the gearing ratio and the dimensions of the instrument. Thus, if N is the rotational speed of the pointer at a wind speed V ,

$$v = N/C = 2\pi r G n, \quad . \quad . \quad . \quad (9)$$

where G is the gear ratio of reduction between the vanes and the pointer.

The radius r (see list of symbols, page 157) may be taken with sufficient accuracy as the radius of the centre of area of a blade from the axis of rotation.

Let suffixes 0 refer to the conditions under which the instrument was calibrated, and suffixes 1 to the conditions under which measurements are being made, i.e., when the air density has changed from ρ_0 to ρ_1 . For a given wind speed V_0 , the linear blade speed v_0 can be obtained from the calibration curve when the density is ρ_0 , and the resisting torque is given by

$$T_0 = K_0 \varphi_0 \rho_0 A (V_0^2 + v_0^2) m r \cos (\theta - \varphi_0 + \gamma_0),$$

and the end thrust by

$$P_0 = K_0 \varphi_0 \rho_0 A (V_0^2 + v_0^2) m \sin (\theta - \varphi_0 + \gamma_0).$$

Also, from (7),

$$T_0 = k + h P_0.$$

Now suppose the air density alters to ρ_1 . At a certain value V_1 of the wind speed and a corresponding value v_1 of the blade speed, the torque T_1 will be the same as before.

That is,

$$T_0 = k + h P_0 = T_1 = k + h P_1,$$

from which it follows that

$$P_0 = P_1.$$

Hence

$$\begin{aligned} K_0 \varphi_0 \rho_0 A (V_0^2 + v_0^2) m r \cos (\theta - \varphi_0 + \gamma_0) \\ = K_1 \varphi_1 \rho_1 A (V_1^2 + v_1^2) m r \cos (\theta - \varphi_1 + \gamma_1) \end{aligned} \quad (10)$$

and

$$K_0 \varphi_0 \rho_0 A (V_0^2 + v_0^2) m \sin (\theta - \varphi_0 + \gamma_0) \\ = K_1 \varphi_1 \rho_1 A (V_1^2 + v_1^2) m \sin (\theta - \varphi_1 + \gamma_1). \quad (11)$$

Dividing (11) by (10), we obtain

$$\tan (\theta - \varphi_0 + \gamma_0) = \tan (\theta - \varphi_1 + \gamma_1), \\ \text{so that} \quad \varphi_0 - \gamma_0 = \varphi_1 - \gamma_1. \quad (12)$$

Now γ is a definite function of φ , so that (12) can only be true if $\varphi_0 = \varphi_1$; and since

$$\frac{v_0}{V_0} = \tan (\theta - \varphi_0),$$

and

$$\frac{v_1}{V_1} = \tan (\theta - \varphi_1),$$

the equality of φ_0 and φ_1 leads to the relation

$$\frac{v_0}{V_0} = \frac{v_1}{V_1}. \quad (13)$$

Also, since K is a function of φ , $K_0 = K_1$. Hence equation (10) reduces to

$$\rho_0 (V_0^2 + v_0^2) = \rho_1 (V_1^2 + v_1^2),$$

and, putting in the condition represented by (13), it is easy to see that

$$\frac{v_1}{v_0} = \frac{V_1}{V_0} = \sqrt{\frac{\rho_0}{\rho_1}}. \quad (14)$$

This equation indicates a ready means for deriving the calibration curve of the anemometer at density ρ_1 from that given at density ρ_0 , since the interpretation of (14) is that a point (v_0, V_0) on the ρ_0 curve becomes the point $\left(v_0 \sqrt{\frac{\rho_0}{\rho_1}}, V_0 \sqrt{\frac{\rho_0}{\rho_1}}\right)$ on the ρ_1 curve. Hence a point P' on the new

curve can be obtained from any point P on the original curve by joining P to the origin O and producing the line OP to P' , so

that $\frac{OP'}{OP} = \sqrt{\frac{\rho_0}{\rho_1}}$ (see Fig. 57).

Except at the lowest speeds, the original calibration curve will generally be a straight line whose equation may be expressed in the form

$$v = pV + q, \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where p and q are constants.

It is easy then to show that the corresponding portion of the calibration curve for density ρ_1 will be a parallel straight line whose equation will be

$$v = pV + q\sqrt{\frac{\rho_0}{\rho_1}}. \quad . \quad . \quad . \quad . \quad (16)$$

Hence the linear portion of the new curve can be derived from that of the original curve by obtaining one point in the manner indicated above, and drawing a parallel line through this point.

Generally, the calibration curve will not be expressed in terms of v , the linear blade speed, but of N , the rotational speed of the pointer. This will not, however, invalidate the above argument; its only effect will be to alter the values of the constants p and q in (15) and (16).

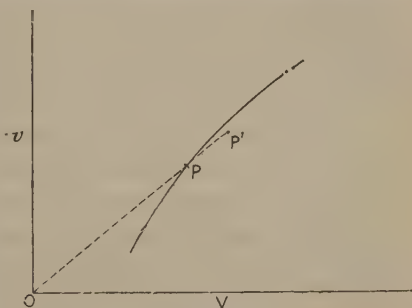


FIG. 57.

In practice, it is not necessary always to draw a new calibration curve each time the value of the density changes. The air speed corresponding to an observed rotational speed can be obtained by calculation as follows. The calibration curve, obtained at density ρ_0 , will be given in terms of the true wind speed V and an observed quantity, which we may denote by V_i , which will be the rotational speed of the pointer, or the "indicated air speed" if the dial is graduated in feet. In the latter case, V_i will simply bear a constant ratio to the rotational speed, so that the two cases are in effect identical. Suppose now that the quantity V_i is observed when the density is ρ_1 . The corresponding value of V_i when the density is ρ_0 will be $V_i\sqrt{(\rho_1/\rho_0)}$, and this will correspond to a value V of the wind speed, as determined from the calibration curve for density ρ_0 .

The value of the wind speed required can then be shown to be equal to $V\sqrt{(\rho_0/\rho_1)}$. To summarise, therefore, the procedure is to multiply the observed value of V_i by $\sqrt{(\rho_1/\rho_0)}$; to read off the value of V corresponding to this from the calibration curve; and finally to multiply this value of V by $\sqrt{(\rho_0/\rho_1)}$.

The above methods of correction for density changes have been derived on the assumption that the friction torque can be expressed in the linear form of equation (7). This is probably not quite true at the lowest speeds, but the method of correction will be found, even in this region, to give sufficiently close approximations to actual conditions.

Practical Importance of the Density Correction.—On the assumption that the calibration curve is a straight line, which appears to be generally true for vane anemometers if we disregard that portion of the curve in the vicinity of the lowest wind speed at which the vanes begin to turn, we can examine analytically the practical importance of the density correction. As already remarked, the calibration curves for different air densities are parallel straight lines, and it is easy to show that the displacement along the V axis of two such curves, one relating to density ρ_0 and the other to density ρ_1 , is equal to

$\frac{q}{p} \left(\sqrt{\frac{\rho_0}{\rho_1}} - 1 \right)$, where p and q are the constants of the

calibration curve for density ρ_0 (see equation (15)). It follows from this that, if ρ_0 is the air density for which the anemometer was calibrated and we use that calibration curve (instead of the ρ_1 curve) to determine the value of the wind speed V corresponding to an indicated wind speed V_i on a day when the density is ρ_1 , the error on V will be the displacement of the two

curves along the V axis, i.e., $\frac{q}{p} \left(\sqrt{\frac{\rho_0}{\rho_1}} - 1 \right)$.

Now an examination of the characteristics of numerous vane anemometers indicates that the value of p is nearly always of the order of 1 while q only rarely exceeds 2 numerically, although its sign may be positive or negative. These values of p and q relate to calibration curves in which V and V_i are expressed in feet per second. Thus we may say that the error on true speed

V due to neglect of a change in density from ρ_0 to ρ_1 is of the order of $2 \left(\sqrt{\frac{\rho_0}{\rho_1}} - 1 \right)$. The percentage error on true speed is, of course, $\frac{200}{V} \left(\sqrt{\frac{\rho_0}{\rho_1}} - 1 \right)$; and if this error is not to exceed 1 per cent., we have

$$\frac{200}{V} \left(\sqrt{\frac{\rho_0}{\rho_1}} - 1 \right) < 1,$$

i.e.,
$$\frac{\rho_0}{\rho_1} < \left(\frac{V}{200} + 1 \right)^2.$$

From this relation, the following table has been prepared, which shows, for a series of values of the true air speed V , the maximum value of the density ratio ρ_0/ρ_1 that can be allowed without incurring an error of more than 1 per cent. on V .

TABLE 16

V feet per sec.	Max. value of ρ_0/ρ_1
5	1.05
10	1.10
20	1.21
30	1.32

From this table, it is evident that the higher the wind speed the greater is the variation of density that can be neglected. Thus, when the wind speed is 5 feet per second, a 5 per cent. density change can be neglected without causing an error of more than 1 per cent. on V for an average vane anemometer. But at 30 feet per second a 32 per cent. density change can be neglected for a 1 per cent. limiting error. Now density changes of the order of 5 per cent. will be extremely rare under ordinary conditions of use, so that we may say that usually no attention need be paid to density corrections at speeds above 5 feet per second. But there are circumstances, as for example in the

mines in South Africa, where a combination of high temperature and elevation above sea level gives values of the air density that may differ by as much as 20 per cent. from the average value in England. In that case, the density correction would have to be taken into account at all speeds below about 20 feet per second if the anemometer in use had been calibrated in this country. Other instances of large density changes might occur if the anemometer were to be used for measuring the flow of hot air.

It should be remarked that the figures given in Table 16 are based on the values of p and q stated above, namely, 1 and ± 2 respectively. It will usually be found that p is within 10 per cent. of unity, but the value given for q seems to be rather higher than the average. The values given in Table 16 are therefore rather conservative; with many vane anemometers somewhat larger density variations may be neglected than the table indicates. However, the values given furnish a useful guide on the order of importance of the density correction.

Optimum Blade Angle.*—We may now consider another aspect of the relation between the frictional and aerodynamic forces acting on the vanes of an anemometer. When the wind speed is such that the vanes are on the point of beginning to rotate, the angle ϕ will become equal to the blade angle θ , and v will be zero. Hence equations (4) and (8) become

$$T = K\theta\rho AV^2mr \cos \gamma,$$

and

$$P = K\theta\rho AV^2m \sin \gamma,$$

so that, writing T in the form of equation (7),

$$K\theta\rho AV^2mr \cos \gamma = k + hK\theta\rho AV^2m \sin \gamma,$$

or

$$\rho AmV^2(K\theta r \cos \gamma - hK\theta \sin \gamma) = k. \quad (17)$$

* In this section, as in those that precede it, the theory has been simplified, as mentioned on page 159, by the neglect of the inflow velocity, that is, the change in speed of the air as it approaches the vane circle. When this effect is included, the agreement between the theoretical and the experimental values of the optimum blade angle is much closer than that obtained with this simplified theory (see page 172). Mr. B. G. van der Hegge Zijnen pointed this out in a communication to the author in 1932: taking account of the inflow velocity, he obtained a value of the optimum blade angle of 40° , which is the same as that found experimentally by the author. In spite of this defect, however, the simplified theory given in this section presents points of interest and is worth inclusion. The complete theory is too complicated to be given here; it will be found in Reference 7 (page 184).

We may use this equation to determine theoretically the angle at which the vanes of an anemometer must be set in order that they may begin to rotate at as low a speed as possible. For, since k is constant, the left-hand side of (17) is constant, and if V is to be a minimum it follows that $K\theta(r \cos \gamma - h \sin \gamma)$ must be a maximum.

A complication is introduced by the fact that the departure from linearity of the curve of Fig. 56 at low speeds shows that h cannot be taken as the slope of the linear portion when we are considering, as is the case at present, conditions in the neighbourhood of zero rotational speed. The variation from the straight-line law of equation (7) between friction torque and end-thrust at very low rates of rotation may be due to a change of the coefficient of friction μ in this region. Little is known about the coefficients of friction between solid surfaces moving at very low relative speeds, but there seems to be some evidence, slight though it is, that the value of μ between surfaces at rest may be different from that when the surfaces are moving relatively to one another. Further, for all except very slow relative motions, μ is found to be approximately constant, and independent of the speed. It is, then, possible that in the transition stage from rest to motion μ changes rapidly but continuously from its constant static value to its constant dynamic value. If this is so, when the vanes are just beginning to rotate, μ is probably not the same as it is when they are in steady motion; and this will furnish one possible cause for the deviation of the low-speed points in Fig. 56 from the line through the other points. Other causes that may conceivably contribute to this departure are variations in the type of flow of the air past the blades that may occur at very low wind speeds, and the interference between the blades that may tend to become more pronounced as the rotational speed becomes smaller in relation to the wind speed, that is, as the effective incidence ϕ of the blades becomes larger.

Although, therefore, any application of equation (17) to determine the optimum blade angle cannot, in the present state of knowledge, be regarded as rigorous, some useful indications are nevertheless supplied. If we attribute the variation in the straight-line law between T and P at low wind speeds entirely to changes in μ , we may proceed as follows. Let us write $\mu_{\max}$

for the constant value of μ at the higher speeds, and $\mu_{\min}$ for its value when the vanes are just beginning to rotate. Since μ is no longer to be taken as constant, equation (7) must now be written in the form,

$$T = \mu(k' + h'P), \quad . \quad . \quad . \quad (18)$$

where k' and h' are constants.

If $T_{\min}$ and $P_{\min}$ are the values of T and P when the vanes are just on the point of beginning to rotate, equation (18) becomes

$$T_{\min} = \mu_{\min}(k' + h'P_{\min}), \quad . \quad . \quad . \quad (19)$$

and, at the higher speeds, we have

$$T = \mu_{\max}(k' + h'P),$$

where $k'\mu_{\max} = k$ in equation (7),

and $h'\mu_{\max} = h$ in equation (7).

From (19),
$$\mu_{\min} = \frac{T_{\min}}{k' + h'P_{\min}},$$

so that
$$h'\mu_{\min} = \frac{h'T_{\min}}{k' + h'P_{\min}}.$$

Multiplying and dividing by $\mu_{\max}$, we have, therefore,

$$h'\mu_{\min} = \frac{h'\mu_{\max}T_{\min}}{k'\mu_{\max} + h'\mu_{\max}P_{\min}}. \quad . \quad . \quad . \quad (20)$$

Now $h'\mu_{\max}$ and $k'\mu_{\max}$ are, as we have seen, equal to h and k respectively in equation (7); that is, they can be obtained by plotting T against P , as shown in Fig. 56. Also $T_{\min}$ and $P_{\min}$ are the values of the torque and end-thrust when the vanes are just beginning to rotate, and can be calculated by putting, in equations (4) and (8), $\phi = 0$ and $v = 0$, and reading off the value of V at which the calibration curve between v and V cuts the V -axis. Hence $h'\mu_{\min}$ can be calculated by the use of (20).

Returning now to equation (17), we see that, for the best blade angle, $K\theta(r \cos \gamma - h'\mu_{\min} \sin \gamma)$ must be a maximum, the quantity $h'\mu_{\min}$ being equal to the value of h when the vanes are on the point of rotating. At this value of the rotational speed, namely zero, the angle of incidence of the blades to the wind is

given by θ , their geometrical incidence, so that K and γ can be obtained directly for a given value of θ from the data of Figs. 51 and 52. The quantity

$$K\theta(r \cos \gamma - h'\mu_{\min} \sin \gamma)$$

can thus be evaluated for different assumed values of θ , and will be found to exhibit a maximum value for a definite value of θ . This angle will therefore be the angle at which the vanes should be set in order that they may rotate at the lowest possible speed.

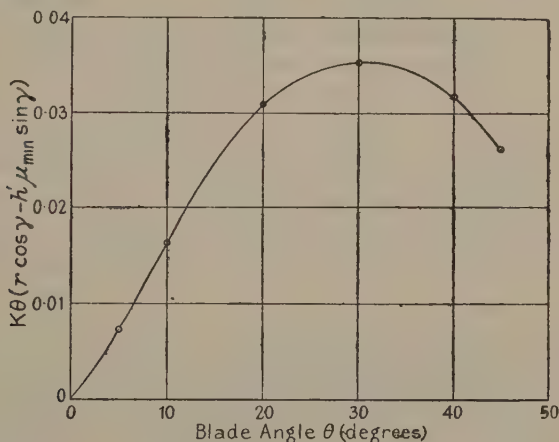


FIG. 58.

For the author's low-speed anemometer, we have already seen that $h'\mu_{\max} = h = 8.0 \times 10^{-2}$ and $h'\mu_{\max} = k = 1.6 \times 10^{-6}$. Also, from Table 15, $T_{\min} = 2.46 \times 10^{-6}$ and $P_{\min} = 2.17 \times 10^{-5}$. Hence, from (20), $h'\mu_{\min}$ will be found to be equal to 0.059. For this instrument, also, $r = 0.115$ feet, so that the best blade angle for this type of anemometer is that at which

$$K\theta(0.115 \cos \gamma - 0.059 \sin \gamma)$$

is a maximum. In Fig. 58 this quantity is plotted against θ , and it will be seen that the maximum value occurs at $\theta = 31^\circ$, which is therefore the best blade angle as indicated by the theory.

Actually, from experiments made by the author, the best

blade angle for an anemometer appears to be about 40° . In these experiments, a series of observations of the minimum wind speed at which the vanes would rotate was made with an instrument whose vanes were successively adjusted to each of a number of angles of incidence. In view of the remarks made above on the uncertainty regarding the variation of frictional and interference effects that exists at these very low speeds, and of the fact that the inflow velocity has been neglected,* the discrepancy of about nine degrees between the theoretical and experimental optimum values of θ is not regarded as excessive. The instrument on which the tests were made was fairly representative of vane anemometers in general, and it may be taken that in all such instruments the blades should be set at forty degrees. The author's low-speed anemometer was designed before the theory was developed, and its blade angle of forty-five degrees was fixed arbitrarily. A similar instrument, subsequently constructed with a forty-degree vane angle, was found to begin rotation at a wind speed slightly but definitely lower than that at which motion of the original anemometer began.

Effect of a Fluctuating Wind.—We have tacitly assumed in the preceding analysis that the anemometer is placed in a steady wind whose speed is constant and uniform across the whole vane circle. In practice, however, the air speed is usually both more or less unsteady in magnitude and variable from point to point across the vanes ; and it is necessary to consider what errors may be incurred from each of these causes. We shall deal firstly with the effect of a fluctuating wind speed, which we shall assume at any instant to be the same at all points over the vane circle, but to vary with time over the whole circle. We must exclude from consideration natural winds, and confine our attention to those cases in which an anemometer is used to measure the speed of an artificially produced air current. Such a current will usually be obtained by means of a fan running at a speed which is intended to be constant, but which will probably fluctuate about a mean value. As a consequence, the wind speed will also fluctuate in a more or less periodic manner about a mean value ; and for the purpose

* See footnote on page 168.

of analysis we will assume that the wind speed V at any time t is given by the simple harmonic relation,

$$V = V_0(1 + \lambda \sin pt), \quad . \quad . \quad . \quad (21)$$

where V_0 is the value of V when $t = 0$.

It should be noted that λ is the amplitude of the fluctuations, V_0 is the mean speed about which fluctuations occur, and $2\pi/p$ is the time for one complete cycle.

Now the aerodynamic theory of airscrews has established the fact that, both for an airscrew producing a wind and for a windmill driven by the wind, the wind torque Q is given by an equation of the form

$$Q/\rho V^2 D^3 = f(nD/V), \quad . \quad . \quad . \quad (22)$$

where $f(nD/V)$ represents some function of (nD/V) . We may therefore assume that an equation of this type holds for a vane anemometer. When the anemometer is in steady motion $Q = -T$;^{*} and therefore, by plotting $-T/\rho V^2 D^3$ against nD/V , the form of the function $f(nD/V)$ can be determined. To take a concrete case, we may consider once more the low-speed anemometer,[†] for which the values of T have been calculated (see Table 15). Values of Q are plotted against nD/V in Fig. 59, and it will be seen that all the points for values of $V = 1.0$ and upwards lie on a straight line. It should be noted that this range of V corresponds with the linear portion of the calibration curve of the instrument, so that over this region we may write

$$\frac{Q}{\rho V^2 D^3} = a + b \frac{nD}{V}, \quad . \quad . \quad . \quad (23)$$

where a and b are constants; and this result may be expected to apply, over the linear range, to all vane anemometers of normal design, the values of a and b varying from one to another. For the particular instrument under consideration, the values of a and b taken from Fig. 59 are 0.118 and -0.283 respectively.

^{*} In the following analysis the usual convention is adopted by which the driving torque Q , which acts in the same sense as the rotation, is taken as positive and the frictional resisting torque T as negative.

[†] Continual reference to this anemometer is unfortunately unavoidable, since it is the only instrument whose complete characteristics are available to the author.

In a fluctuating wind, the driving torque Q due to the air no longer necessarily balances the resisting frictional torque T . The former depends on the instantaneous value of nD/V , in

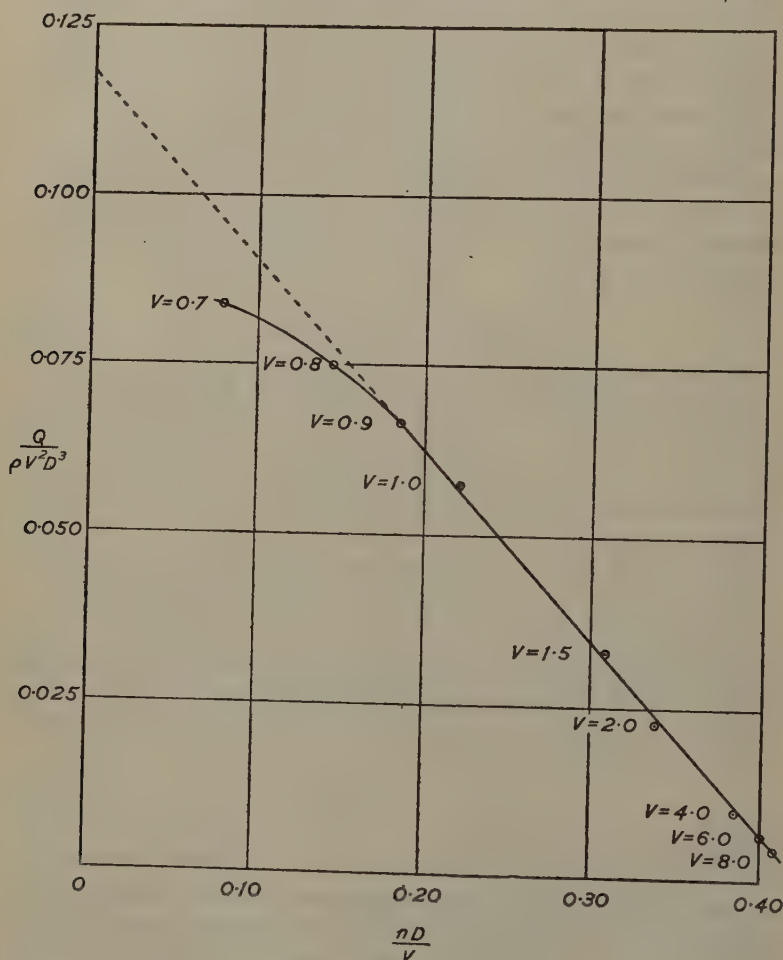


FIG. 59.

accordance with equation (23), while we may expect T to depend only on the relative rubbing velocity at the bearings. This expectation is borne out by the values given for T in Table 15; if these are plotted against V , to which n , and consequently also the rubbing velocity, bears a linear relation over most of the

speed range, the points will be found to lie on a straight line over the linear portion of the calibration curve. Although this line does not pass through the origin, a good approximation to the value of T at any value of V is obtained by assuming the linear relation

$$T = KV, \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where $K = 28 \times 10^{-7}$ lb. sec.

We now imagine the anemometer subjected to a wind fluctuating in speed, according to equation (21), about a mean speed V_0 . At any time t , let the rotational speed of the anemometer be n , and the corresponding velocity of the air be V . Then, equating the rate of change of angular momentum of the vanes to the total torque (air minus friction), we get for the equation of motion of the vanes :

$$2\pi I \cdot dn/dt - Q + T = 0, \quad . \quad . \quad . \quad . \quad (25)$$

where I is the moment of inertia of the vanes and their supporting arms and spindle. Substituting for V , Q , and T their equivalents from (21), (23), and (24), we get from (25), after a little simplification,

$$\frac{dn}{dt} + \alpha(1 + \lambda \sin pt)n = \beta(1 + \lambda \sin pt) + \gamma(1 + \lambda \sin pt)^2, \quad . \quad (26)$$

$$\text{where} \quad \left. \begin{aligned} \alpha &= -\frac{b\rho D^4}{2\pi I} V_0; \\ \beta &= -\frac{K}{2\pi I} V_0; \\ \gamma &= \frac{a\rho D^3}{2\pi I} V_0^2. \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad (27)$$

Now (26) is a differential equation of the standard form $dy/dx + Py = Q$, where P and Q are functions of x . Applying the well-known solution, we therefore obtain from (26)

$$ne^{\int \alpha(1+\lambda \sin pt)dt} = \int [\beta(1+\lambda \sin pt) + \gamma(1+\lambda \sin pt)^2] e^{\int \alpha(1+\lambda \sin pt)dt} dt + C,$$

or

$$ne^{\alpha(t - \frac{\lambda}{p} \cos pt)} = \int [(\beta + \gamma) + (\beta + 2\gamma)\lambda \sin pt + \gamma\lambda^2 \sin^2 pt] e^{\alpha(t - \frac{\lambda}{p} \cos pt)} dt + C, \quad . \quad . \quad . \quad (28)$$

As it stands, this solution can be carried no further ; we can, however, reduce it to a manageable form if we reflect upon the practical aspects of the problem. It seems reasonable to suppose that, in most cases of practical importance, the amplitude of the fluctuations will not exceed 10 or 20 per cent.,* and that the frequency of the pulsations will be faster than one or two per second. Let us therefore assume for the moment a 10 per cent. fluctuation in speed and a frequency of 2 per second. Then $\lambda = 0.1$ and $p = 2\pi/\frac{1}{2} = 4\pi$, so that $\lambda/p = 0.008$. If we insert this value of λ/p in (28) we get on both sides the term

$$e^{\alpha(t - 0.008 \cos pt)}.$$

Now the maximum value of $0.008 \cos pt$ is 0.008, so that when $t = 1$, i.e., after the motion has proceeded for only one second, the value of $\frac{\lambda}{p} \cos pt$ will be less than 1 per cent. of t , and as t increases the relative magnitude of this term will become still smaller. This suggests that the term $\frac{\lambda}{p} \cos pt$ can be neglected in the index of e in equation (27) when we are considering the steady state to which the anemometer quickly settles down.

The attractive feature about this assumption is that it enables us to integrate (28), as we shall shortly show ; but admittedly the practical considerations on which it is based are not sufficient to justify it. After the second edition of this book was published, the author carried out a special investigation which established the validity of the assumption.⁵ In the first place, the integration of the complete equation (28), including the term $\frac{\lambda}{p} \cos pt$, was carried out graphically for two cases, and the results agreed closely with those obtained by means of the approximate analytical solution, given below, in which $\frac{\lambda}{p} \cos pt$ is neglected. The calculations were done with an assumed value of λ of 0.5 and for periods of $\frac{1}{2}$ second and $2\frac{1}{2}$ seconds. This gave values of λ/p of 0.04 and 0.2, i.e., five and twenty-five times respectively that on which the original argument in

* See below, however, where it is shown that the ensuing argument is valid for considerably larger fluctuations.

favour of the neglect of the term was based. In the first case, the complete solution agreed with the approximate one to less than 1 in 2,000, and in the second the difference was less than 2 per cent.

In addition, experimental confirmation was obtained from experiments in a pulsating stream whose speed was independently recorded by a fine hot-wire anemometer (see Chapter X), which was known to be capable of following the pulsations accurately. For details of these experiments the reader is referred to the original paper.⁵

It appears, then, that we can safely neglect $\frac{\lambda}{p} \cos pt$, even when the velocity is subject to a 50 per cent. pulsation; and it also follows from the form of the term that, for a given λ , the error due to its neglect decreases the faster the pulsations. There is therefore little doubt that the term can be neglected in the great majority of practical cases.*

We are thus justified in reducing (28) to

$$ne^{\alpha t} = \int [(\beta + \gamma) + (\beta + 2\gamma)\lambda \sin pt + \gamma\lambda^2 \sin^2 pt] e^{\alpha t} dt + C, \quad (29)$$

which can be integrated, giving, after reduction,

$$\begin{aligned} n = & \frac{\beta + \gamma}{\alpha} + \frac{\lambda(\beta + 2\gamma)}{\alpha^2 + p^2} (\alpha \sin pt - p \cos pt) \\ & + \frac{\gamma\lambda^2}{2\alpha(\alpha^2 + 4p^2)} (\alpha^2 + 4p^2 - \alpha^2 \cos 2pt - 2\alpha p \sin pt) + Ce^{-\alpha t}. \end{aligned} \quad (30)$$

The value of C could be obtained by putting $n = n_0$ when $t = 0$, but this is not necessary. Examination of the solution shows that the motion consists of two components, one a steady periodic motion represented by the sum of the first three main terms, and the other, represented by the term $Ce^{-\alpha t}$, a damped motion, which has its maximum value when $t = 0$ and decays as the time proceeds. Substituting for α from (27) and using the values of the mechanical constants given on page 162, we find that the term $Ce^{-\alpha t}$ is reduced to one-tenth of its initial ampli-

* This is not necessarily true for the measurement of natural winds, in which there may be several natural frequencies, instead of a predominant one as in the case now under consideration.

tude in under 5 seconds, so that we can ignore it after the motion has settled down a minute or so from the start. What we are concerned with is the average value of n over a complete cycle of the pulsation once the flow has settled down. Denoting this average value of n by $\bar{n}$, we deduce from (30), which gives the *instantaneous* value of n at any specified moment, that

$$\bar{n} = \frac{\beta + \gamma}{\alpha} + \frac{\gamma\lambda^2}{2\alpha} \quad . \quad . \quad . \quad (31)$$

Substitute for α , β , and γ from equation (27) ; then

$$\bar{n} = \frac{K}{\rho b D^4} - \frac{a}{bD} V_0 - \frac{a\lambda^2}{2bD} V_0 \quad . \quad . \quad . \quad (32)$$

In a steady stream $\lambda = 0$, so that the third term on the right-hand side of (32) vanishes, and the residual part of (32) becomes the equation of the linear portion of the calibration curve of the anemometer.* In a pulsating stream, λ is not zero, and the

term $-\frac{a\lambda^2}{2bD} V_0$ represents the amount by which the

anemometer overestimates (overestimates because b is negative) the mean air speed V_0 . We can calculate the amount of this overestimation for any assumed conditions and for any anemometer for which we are given the calibration curve, but this is not the form in which we shall generally need to use the result. What will happen is that we shall use an anemometer (for which we have the calibration curve) in the ordinary way in a pulsating stream, measuring the time required to complete a given number of revolutions of the pointer. What we shall then wish to know is what error we shall incur if we use the value of the rate of rotation of the pointer thus obtained, in conjunction with the calibration curve supplied, to get a value of the air speed, neglecting any effect of the pulsation. This error is easily calculated thus :—

As before, let $\bar{n}$ be the observed rate of rotation of the vanes (or of the pointer ; it is immaterial which, since they are related merely by the gearing constant). Then if V_0 is the mean about

* We may note in passing that, for the author's low-speed anemometer, the calibration curve so obtained, which became $n = -0.444 + 1.335 V_0$ when the values (already given) of the various constants were inserted, agreed satisfactorily with that obtained by direct observation.

which the air speed is pulsating, and if n is the speed of the vanes corresponding to this air speed *under steady conditions*, we know from (32) that

$$\bar{n} = n - \frac{a}{bD} \frac{\lambda^2}{2} V_0. \quad (33)$$

By hypothesis, we use the calibration curve for steady conditions, the equation for which is (32) with the last term zero, to determine from the observed $\bar{n}$ a value of the "indicated speed," V_i , in the unsteady conditions. Thus

$$\bar{n} = \frac{K}{\rho b D^4} - \frac{a}{bD} V_i. \quad (34)$$

Also, we know that n in (33) is given by

$$n = \frac{K}{\rho b D^4} - \frac{a}{bD} V_0. \quad (35)$$

Hence, substituting (34) and (35) in (33), we get

$$\begin{aligned} \frac{a}{bD} V_i &= \frac{a}{bD} V_0 + \frac{a}{bD} \frac{\lambda^2}{2} V_0 \\ \text{or } V_i &= V_0 \left(1 + \frac{\lambda^2}{2} \right). \end{aligned} \quad (36)$$

The true mean speed is, of course, V_0 , so that the percentage error incurred by this use of the calibration curve is

$$E = 100 \lambda^2 / 2.$$

We see that this error is negligible unless λ is large; thus, it is just under 1 per cent. for a wind fluctuating ± 14 per cent. on either side of the mean value. But since the error is proportional to the square of the amplitude of the fluctuations, it increases rapidly with amplitude: when the latter is 50 per cent., for example, the error is $12\frac{1}{2}$ per cent. In very unsteady conditions, therefore, such as may occur sometimes in mine-drifts, where anemometers are often used for speed measurements, the error becomes important. It can be allowed for in accordance with equation (36) if λ is known, but if this is not the case, and if λ cannot be measured even approximately, some other method of measurement should be used.

Effect of a Non-uniform Distribution of Air Speed Across the Vane Circle.—As we have already learned in Chapter III, the velocities at different points in a section of a pipe are not, as a rule, the same, being generally higher towards the centre than near the walls. For this reason, as we saw in Chapter V, it is necessary to determine velocities at a number of points across a section if we wish to measure the total quantity flowing. The question therefore arises whether, when measurements are made with an anemometer with its axis at the proper radii, the blades will rotate at the speed appropriate to the velocity of the filament of air passing the centre of the anemometer disc, or at a rate corresponding to some other air speed.

To define the problem, we shall assume that at the section of measurement there is normal smooth-pipe velocity distribution which, as shown on page 32, can be represented sufficiently closely by the relation

$$V_r = V_0(1 - Ar^2), \quad . \quad . \quad . \quad (37)$$

where V_r is the velocity at any radius r less than the full pipe radius R , V_0 is the velocity at the axis, and A is a constant having the value $0.35/R^2$.

Consider an eight-bladed anemometer (see Fig. 60), with the blades uniformly spaced, placed in a pipe of radius R , the centre of the vane circle being at a point O , whose polar co-ordinates referred to the centre P of the pipe are (R_0, ψ) . Let $01, 02, 03$, etc., represent the median lines of the blades and let r be the radius to the centre of pressure of a blade. Let the blade 01 be instantaneously at an angle α to the radius of reference PQ .

We have firstly to find the radii R_1, R_2, R_3 , etc., of the centres of pressure of the blades referred to the centre of the pipe.

For R_1 we have, by projection along and perpendicular to PQ ,

$$R_0 \cos \psi + r \cos \alpha = R_1 \cos \chi,$$

$$R_0 \sin \psi + r \sin \alpha = R_1 \sin \chi.$$

Hence, by squaring and adding,

$$\left. \begin{aligned} R_1^2 &= R_0^2 + r^2 + 2R_0 r \cos(\psi - \alpha). \\ \text{Similarly, } R_2^2 &= R_0^2 + r^2 + 2R_0 r \cos(\psi - \alpha - (\pi/4)); \\ &\quad \text{etc.} \\ R_3^2 &= R_0^2 + r^2 + 2R_0 r \cos(\psi - \alpha - (7\pi/4)). \end{aligned} \right\} \quad (38)$$

In accordance with equation (37), we therefore have for the velocities of the air impinging on the centres of pressure of the blades

$$\left. \begin{aligned} V_1 &= V_0 [1 - A \{ R_0^2 + r^2 + 2R_0 r \cos (\psi - \alpha) \}] \\ V_2 &= V_0 [1 - A \{ R_0^2 + r^2 + 2R_0 r \cos (\psi - \alpha - (\pi/4)) \}] \\ &\quad \text{etc.} \\ V_8 &= V_0 [1 - A \{ R_0^2 + r^2 + 2R_0 r \cos (\psi - \alpha - (7\pi/4)) \}] \end{aligned} \right\} \cdot (39)$$

Let the centres of pressure of the blades move under the effect of this variable velocity at a tangential linear speed $\bar{v}$,

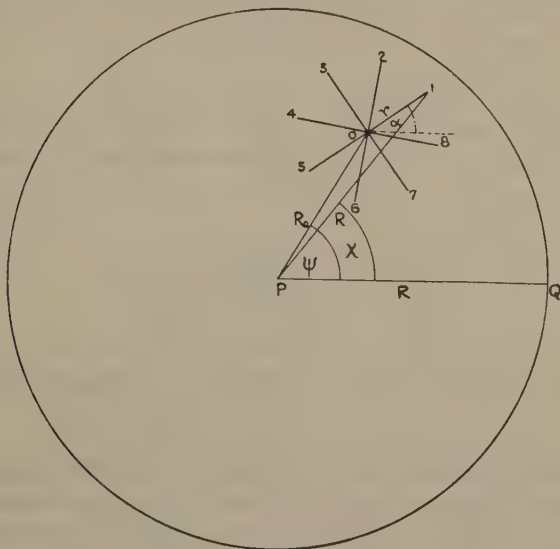


FIG. 60.

which would correspond, according to the calibration curve, to an air speed $\bar{V}$. Then $\bar{V}$ and $\bar{v}$ will be such that the torque on the whole instrument rotating at this speed will be equal to the sum of the individual torques contributed by each blade.

Now we have already shown (equation (4)) that

$$\text{Torque} = K\rho\phi A(V^2 + v^2)mr \cos (\theta - \phi + \gamma).$$

If the variation of air speed across the vane circle is not too large, we may, to a first approximation, assume that ϕ , and hence also K and γ , are constant for all the blades, so that we may write

$$\text{Torque} = K'm(V^2 + v^2).$$

The condition that the total torque is equal to the sum of the torques of all the blades is therefore expressed by the equation

$$8K'(\bar{V}^2 + \bar{v}^2) = K'[(V_1^2 + \bar{v}^2) + (V_2^2 + \bar{v}^2) + \dots + (V_8^2 + \bar{v}^2)],$$

i.e.,

$$8\bar{V}^2 = V_1^2 + V_2^2 + V_3^2 + \dots + V_8^2 = \Sigma V_1^2.$$

From equations (39) it will be found, after reduction, that

$$\Sigma V_1^2 = 8V_0^2[(1 - AR_0^2)^2 + Ar^2(4AR_0^2 + Ar^2 - 2)],$$

which is independent of α , and hence constant for all positions of the blades; so that

$$\bar{V} = V_0 \sqrt{(1 - AR_0^2)^2 + Ar^2(4AR_0^2 + Ar^2 - 2)}. \quad (40)$$

In this equation $\bar{V}$ is, as we have seen, the air speed indicated by the anemometer. If the instrument indicated the speed of the air V_c passing through the centre of its disc, the value of $\bar{V}$ would be $V_0(1 - AR_0^2)$.

Hence
$$\frac{\bar{V}}{V_c} = \sqrt{1 + \frac{Ar^2(4AR_0^2 + Ar^2 - 2)}{(1 - AR_0^2)^2}},$$

and, putting $A = 0.35/R^2$, we have, after reduction,

$$\frac{\bar{V}}{V_c} = \sqrt{1 + \frac{0.35r^2(1.40R_0^2 + 0.35r^2 - 2R^2)}{(R^2 - 0.35R_0^2)^2}}. \quad (41)$$

Obviously, neither R_0 nor r can be greater than R , the radius of the pipe, so that $1.40 R_0^2 + 0.35 r^2$ cannot exceed $1.75 R^2$. Hence the term $1.40 R_0^2 + 0.35 r^2 - 2R^2$ is always negative; and therefore it follows from equation (41) that $\bar{V}$ is always less than V_c , i.e., the speed indicated by the anemometer is always less than the speed of the air passing the centre of its disc. Now suppose we wish to limit the error to 1 per cent.; i.e., we make $\bar{V} = 0.99 V_c$. Then equation (41) becomes

$$0.98 = 1 + \frac{0.35r^2(1.40R_0^2 + 0.35r^2 - 2R^2)}{(R^2 - 0.35R_0^2)^2}. \quad (42)$$

Writing μ for the ratio R_0/R , i.e., the ratio of the distance from the centre of the pipe at which the anemometer is placed to the pipe radius, we obtain from (42)

$$\frac{r^2}{R^2} = \frac{(1 - 0.7\mu^2) - \sqrt{(1 - 0.7\mu^2)^2 - 0.02(1 - 0.35\mu^2)^2}}{0.35}. \quad (43)$$

The second term under the radical may be considered as small compared with the first up to a value of μ of about 0.7. Hence over the range of μ from 0 to 0.7, equation (43) reduces to the following approximate form :

$$\frac{r^2}{R^2} = \left[(1 - 0.7\mu^2) - (1 - 0.7\mu^2) \left\{ 1 - 0.01 \frac{(1 - 0.35\mu^2)^2}{(1 - 0.7\mu^2)^2} \right\} \right] \frac{1}{0.35}$$

$$= 0.0286 \frac{(1 - 0.35\mu^2)^2}{(1 - 0.7\mu^2)^2}$$

The following table gives values of r/R calculated from this equation for different values of μ .

TABLE 17

μ	r^2/R^2	r/R
0	0.0286	0.169
0.1	0.0286	0.169
0.2	0.0286	0.169
0.3	0.0286	0.169
0.4	0.0287	0.169
0.5	0.0289	0.170
0.6	0.0291	0.171

The practical interpretation of this table is as follows. In order to limit the error on indicated air speed to 1 per cent. of the speed of the air at the centre of the vane circle, the radius of the anemometer vane circle must not exceed 0.169 times the radius of the pipe when the anemometer is concentric with the pipe. As the anemometer is moved along the radius of the pipe a slightly larger diameter of the anemometer is permissible. The minimum value—0.169—is, however, the important one, since in practice the same instrument will have to be used at various points in the section at which measurements of velocity are required.

The practical rule to be observed, therefore, is that an anemometer should not be used in a pipe less than six times the diameter of the anemometer. According to the above analysis, a larger size of anemometer is permissible if the velocity distribution at the pipe section is flatter than the standard smooth-pipe distribution; but there is evidence from experiments that errors arise from the blocking effect of the anemometer on the

flow if the anemometer is made appreciably larger than one-sixth of the pipe diameter. No method has been proposed for calculating this blocking effect theoretically, but experiments show that it is negligible with the diameter ratio of anemometer to pipe of one-sixth. This ratio should not, therefore, be made larger than one-sixth, even if the velocity distribution curve is flat. On the other hand, if the curve is appreciably more peaked than the standard type assumed in the calculations, it may be inadvisable to use an anemometer as large as one-sixth the pipe diameter. A judgment on this point can be formed from a preliminary rough survey of the distribution with the anemometer itself. This will show sufficiently well whether it is advisable to use a smaller instrument for better accuracy.

REFERENCES

1. Ower, *Journ. Sci. Insts.*, **3** (1926), p. 109.
2. Rees, *Journ. Sci. Insts.*, **4** (1927), p. 311. See also *Trans. Inst. Mining Engrs.*, **74** (1927-28), p. 359.
3. Ower, *Phil. Mag.* (Series 7), **2** (1926), p. 881.
4. Ower and Duncan, *Journ. Sci. Insts.*, **4** (1927), p. 470.
5. Ower, *Phil. Mag.* (Series 7), **23** (1937), p. 992.
6. Eiffel, *La Resistance de l'Air et l'Aviation* (1910), p. 134.
7. van der H. Zijnen, *Proc. K. Akademie v. Wetenschappen te Amsterdam*, **35** (1932), p. 1004.

CHAPTER VIII

MISCELLANEOUS METHODS OF FLOW MEASUREMENT

Measurement of Flow in Small Pipes.—The pitot-static tube of the standard type is not suitable for traversing a pipe of less than about 8 inches diameter. It will be recalled that when this instrument is used to determine the volume, observations are usually made at points whose positions are fixed in accordance with certain rules; for example, if ten readings are taken along a diameter, one observation has to be made at a distance of 0.026 pipe diameters from each wall (see Chapter V). In a 9-inch pipe, this means that the axis of the pitot tube must be placed at a distance from the wall of just under $\frac{1}{4}$ inch, a position in which the outside of the standard pitot head will only be about $\frac{1}{10}$ inch from the pipe wall; in a 6-inch pipe the outside of the standard pitot tube will just touch the wall of the pipe, and the flow there is likely to be influenced by the presence of the pitot, so that errors, probably mainly on the static-pressure reading, may be expected.

Some alternative to the standard pitot tube is therefore required for small pipes. The obvious method is to use a pitot tube of considerably smaller dimensions; and satisfactory results have been obtained in this manner by the use of an instrument of the form shown in Fig. 61.

Since the concentric construction of the total-head and static tubes presents some difficulty with small tubing, the two tubes are

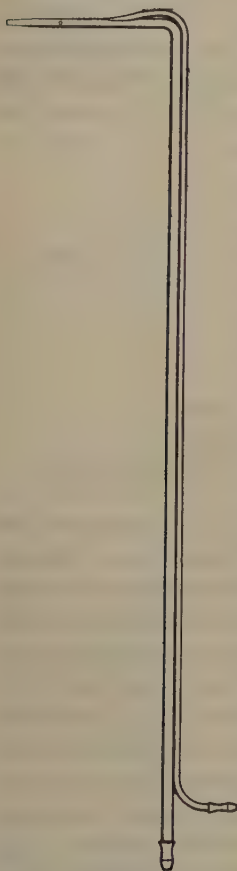


FIG. 61.—Small pitot-static tube.

here arranged side by side, except for a short length in the head itself where the total-head tube enters the static tube through the side, the junction being made airtight by soldering. The mouth of the total-head tube is about 0.03 inch in diameter, the bore of the static tube is about 0.06 inch, and the diameter of the static holes may be about 0.01 inch. An instrument of this kind must be calibrated before use, since it can not easily be made geometrically similar to the standard pitot-static tube.

The Total-Head Tube.—The pitot-static tube shown in Fig. 61 is an instrument requiring somewhat delicate workmanship in its construction. An alternative method of measuring the flow is to use a total-head tube only, and to measure the

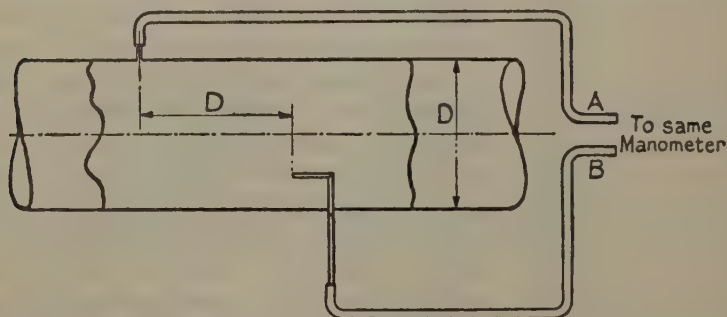


FIG. 62.—Measurement of flow with total-head tube and side hole.

static pressure separately. The advantage of this method lies in the fact that any tube with an open end facing the air current will measure total head accurately, no special shaping of the end in the stream being necessary. The outside diameter of this tube should not be greater than $\frac{1}{16}$ inch for measurements in a 6-inch diameter pipe, and should be even less for pipes below 3-inch diameter. As regards the measurement of the static pressure, if the flow in the pipe is everywhere parallel to the walls, the static pressure is equal at all points in a section; further, it is found that this pressure can be accurately measured by means of a manometer connected to a small tube let in to the pipe, the inner end of this tube being flush with the inside surface of the pipe. Therefore, when the flow is sensibly parallel, the velocity at a point in a pipe section can be determined by the arrangement shown in Fig. 62, in which the

pressure leads A and B are connected to opposite sides of the same manometer, one side of which will be under the static pressure only, whilst acting upon the other there will be the sum of the velocity head and the static pressure. The resultant reading of the manometer will therefore be the velocity head ; and in order to measure the mean rate of flow, the total-head tube is traversed exactly as if it were a combined pitot-static instrument. It will be observed that the total-head tube in the diagram is traversed across a section situated about 1 pipe diameter downstream of the side tube, in order that the readings of static pressure may not be affected by the presence of the mouth of the total-head tube when the observations are being taken near the pipe wall. The difference between the static pressures at these two sections 1 pipe diameter apart will never

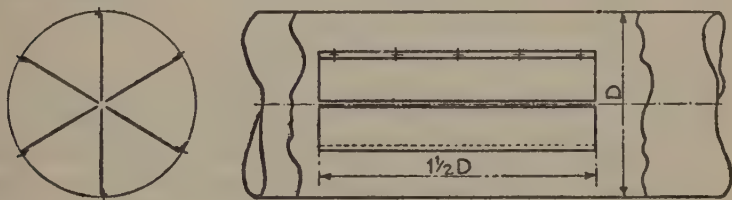


FIG. 63.—Flow-straightening device.

be sufficiently great to lead to any perceptible error in the measurement of velocity. It must be emphasised that the static-pressure side-connection to the pipe must be flush and smooth on the inside surface ; all burrs made in drilling the hole in the pipe wall to take the small side tube must be carefully removed. The bore of the side tube should not exceed about $\frac{1}{16}$ inch.

If the flow at the section is not everywhere parallel to the axis of the pipe, the static pressure at the wall as measured by the side tube will not necessarily be the same as the mean static pressure across the section. Any helical motion of the air-stream, for example, will, on account of radial accelerations, cause the static pressure at the side hole to be greater than it is at the axis. A simple method of straightening helical flow sufficiently to make the static pressure at the section sensibly uniform is to introduce, some 3 or 4, or preferably 6, diameters upstream of the section, a flow-straightening device, which may

consist of six thin flat radial vanes reaching from the centre of the pipe to its circumference—see Fig. 63—and about 1 to 2 pipe diameters in length.

This method will be found effective in most cases, except possibly if a sharp bend precedes the straightener. If this is so, and it is not possible to make measurements at some other section where conditions are more favourable, or to use the small pitot and static tube, it is preferable to use, in place of the single side tube for measuring static pressure, four or six such tubes equally spaced round the pipe circumference, and to lead the connections from these to a common airtight reservoir, in which the static pressure will probably be approximately the mean static pressure at the section. This reservoir may consist of a glass bottle with a neck sufficiently wide to take a large cork through which the necessary number of pressure connections pass; the pressure in the bottle is conducted through an additional connection to the manometer.*

It will be evident that this method does not give the true velocity at each point at which the mouth of the total-head tube is placed, even if the pressure in the reservoir is equal to the mean of the static pressures transmitted from the side tubes in the pipe. As, in addition, there seems to be some reason to doubt whether the pressure in the reservoir is exactly equal to this mean static pressure, it will be clear that this method of measurement is not one of the highest accuracy, although for many practical purposes it will give a sufficiently close approximation to the true rate of flow. When the velocity and static pressure both vary considerably across the section, better precision can be obtained by a thorough exploration by means of a small pitot-static tube such as that already described.

The Smooth Calibrating Pipe and Micrometer Pitot.—

The method described in the preceding section forms the basis of the most accurate method available of measuring the flow of air in pipes when conditions are such that it can be used. It involves the insertion into the pipe line of a length of smooth-bore pipe, at a suitable section of which a special total-head tube can be traversed by means of a micrometer screw. The smooth pipe is most conveniently attached to an open end

* For the method of rendering this cork airtight, see page 235.

of the pipe line if this is available ; otherwise it may be inserted at some suitable place in the line. A sketch of the calibrating pipe is given in Fig. 64. The traversing pitot is placed at P, and A, B, C are three static-pressure side holes, similar to that described in connection with Fig. 62. In this case, however, C is opposite the mouth of the pitot, and A and B are upstream, B being some 4 or 5 pipe diameters forward of the pitot, to be quite clear of interference. The velocity head at the mouth of the pitot is obtained by connecting the pitot and B to opposite sides of a differential manometer, and by adding to the reading obtained the drop in static pressure between B and C. In most cases, the pressure drop along the pipe is linear, so that this correction will be simply $\frac{l_2}{l_1} (S_A - S_B)$, where l_1 is the length

AB, l_2 is the length BC, and S_A and S_B are the static pressures at A and B. It is advisable, however, always to make a few preliminary measurements of the static pressures at A, B, and C, with the pitot removed or screwed back against the far pipe wall, to check this linearity of the pressure drop and, if it is found not to be true owing to special circumstances, to establish the relation between the static pressure at C and the pressure drop between A and B.

When the correction to the pitot pressure at C minus the static at B has been established, the determination of the quantity of air flowing is made by taking readings with the pitot at various points in the section, whose radii can be determined from the micrometer readings (see below). Readings are not now taken in zones of equal area, which is, after all, only a convenient practical approximation, but must be sufficient in number to define the curve of velocity distribution across the diameter. Then, if v is the velocity at a radius r , and R is the full internal

pipe radius (see Fig. 11), the volume of air flowing is $2\pi \int_0^R vrdr$;

and the integral is evaluated graphically by plotting vr against r and determining the area of the resulting curve. The results have then to be corrected for the effect of turbulence and size of pitot in the light of Fage's work (see page 72). A numerical example is worked out in Chapter XI.

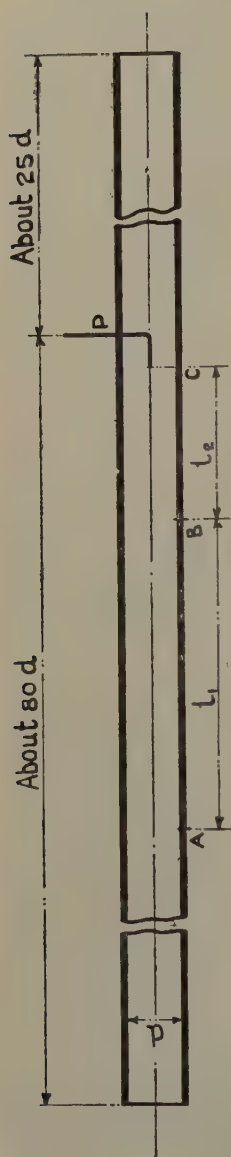


FIG. 64.—Sketch of calibrating pipe.

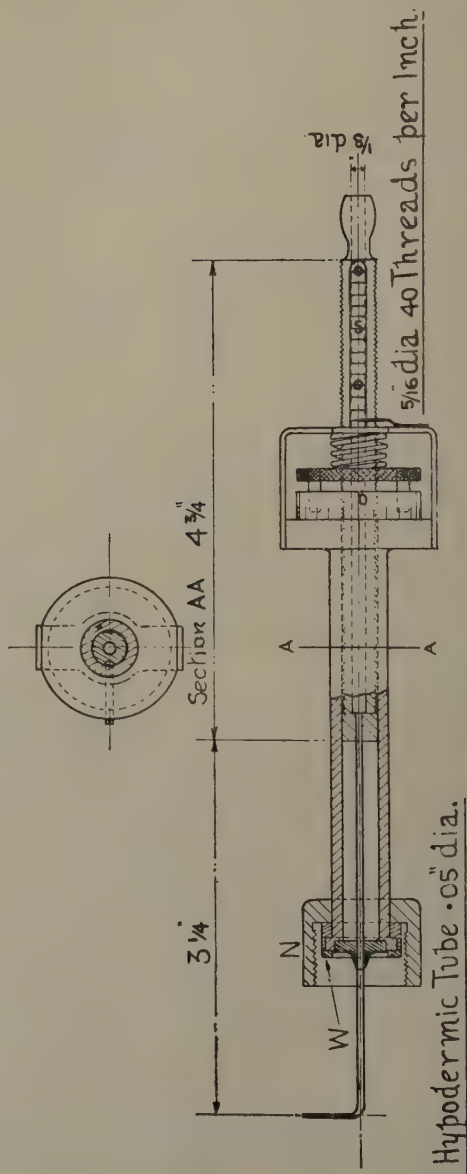


FIG. 65.—Micrometer traversing pitot.

This method is usually used for the measurement of flow in small pipes, but there is no reason why it should not be adapted to larger pipes if desired. A drawing of a form of traversing pitot suitable for small pipes is given in Fig. 65. The pitot itself is in this case made of hypodermic tubing, which can be obtained in very small diameters. For a $\frac{1}{2}$ -inch bore calibrating pipe, the outside diameter of the hypodermic tube should not exceed 0.03 inch. The head of the pitot is bent at right angles to face the stream in the usual way; and this portion should be flattened, both to reduce the correction due to the relative size of pipe and the pitot (Fage's correction—see page 74), and to enable readings to be taken as close to the far wall of the pipe as possible. By measuring the pipe diameter and the thickness of the mouth of the pitot, and observing the micrometer reading when the pitot head is just in contact with the far wall of the pipe, we can easily deduce the radius of the axis of the head for any micrometer reading. Contact with the far wall of the pipe can be simply indicated if the pitot itself, when mounted in position in the pipe, is electrically insulated from it. If then a small flash-lamp bulb and battery are connected in series with the pipe itself and the pitot, the lamp will light up when the pitot head touches the interior of the pipe. Insulation is ensured by means of the ebonite nut N and washer W which secure the pitot to the boss fixed to the outside of the pipe (see Fig. 65). The hole in the wall of the pipe through which the pitot passes should not be larger than necessary to allow the pitot to be introduced without straining it. Leakage through the passages between parts of the pitot gear having relative movement can be reduced to a negligible amount by the use of vaseline.

Calibrating pipes of this kind, of smooth internal bore and equipped with static side holes and traversing pitot, are extremely useful for many kinds of air flow measurements and calibrations. If there is room available to make the pipe of such length that steady conditions exist at the section of measurement, the quantity of air flowing can be deduced from a single velocity measurement at the axis of the pipe, once the relation between the axial velocity and the mean velocity has been established experimentally. (See page 193 and example worked out in Chapter XI.) In this way, the amount of experimental work is greatly shortened.

The Shaped Nozzle.—Perhaps the most convenient practical method of measuring the mean flow in a small pipe is to introduce into it a shaped nozzle of the standard type, or any other nozzle or orifice plate for which the coefficient is accurately known, and to measure the resultant pressure drop. The quantity flowing can then be calculated from the equations already given. This method is particularly suitable when there is a sufficient length of parallel pipe existing upstream of the section at which the nozzle or orifice is inserted for the flow at that section to have assumed the normal distribution for pipe flow ; and when, in all other respects, the conditions comply with those specified for the application of the standard values of the discharge coefficient.

Very often it is not possible to meet these requirements, but the same method may still be used with a slight modification, whereby advantage is taken of the uniform velocity of the jet leaving a shaped nozzle—a feature to which attention has already been drawn on page 77. A single observation of velocity head, taken with a pitot-static tube arranged coaxially with the nozzle in the outlet plane, will give a good approximation to the mean velocity of the jet leaving the nozzle ; and this velocity, multiplied by the area of the nozzle throat, will give the volume flowing. When the highest accuracy is required, a preliminary exploration of the jet by means of the pitot-static tube may be desirable in order to verify the constancy of velocity across the jet. A uniform distribution of this nature was only observed when there was normal smooth-pipe distribution of velocity upstream of the nozzle ; and it is conceivable that if conditions here are very widely different from the normal, a lack of uniformity may occur in the jet. For most purposes, however, this refinement will be found to be unnecessary. If the nozzle can be conveniently placed at the outlet end of the pipe, a further simplification is possible, since the static pressure in the jet will now be equal to that of the surrounding atmosphere, that is to say, zero, according to the convention by which atmospheric pressure is taken as the datum.* The total head at a point in the jet is therefore equal to the velocity head only ; so that all

* To avoid confusion, the reader is reminded that this convention is followed throughout this chapter.

that is required in these circumstances is a single observation of total head at the axis of the jet in the outlet plane of the nozzle. As we have already seen, this total head can be measured by any open-ended tube bent to face the air stream, and no specially shaped tube is now required. A thin-walled tube of about $\frac{1}{16}$ -inch bore may conveniently be used. For the reasons mentioned above, a preliminary exploration of the jet may sometimes be advisable in this case also.

Mean Rate of Flow from a Single Observation.—When the distribution of velocity and static pressure at the section of measurement are fairly symmetrical, and conditions there are reasonably steady, a rapid and simple method of determining the mean rate of flow is to establish, by means of a preliminary exploration, the relation between the mean and the axial velocity; and thereafter to take single readings of velocity at the axis of the pipe. If conditions at the section of measurement are such that normal smooth-pipe flow is established, the preliminary exploration would be unnecessary were it not for the unfortunate, hitherto unexplained discrepancy between the results of Stanton and Panell and of Nikuradse to which we have already referred in Chapter III. This discrepancy prevents us from using a standard curve of the form given in Fig. 9 for the relation between mean and axial velocity in normal pipe, flow; and a preliminary exploration must therefore always be made if this method is used, unless a possible error of perhaps 5 per cent. at high rates of flow can be tolerated.

The axial velocity may be measured either by means of a pitot-static tube of suitable size, or by means of a total-head tube in conjunction with a static-pressure side hole in the pipe. The initial calibration to establish the ratio of mean to axial velocity can be determined by means of a traverse across the section followed by an integration, using one of the methods already described. If the rates of flow that it is subsequently desired to measure extend over a considerable range, it will be advisable to conduct enough preliminary explorations to cover the range adequately. An indication of the variation likely to occur in the ratio of mean to axial velocity within the range can be gained from Fig. 9; the amount of preliminary exploration will be governed by this

variation and by the degree of accuracy required. Sometimes, a single exploration at a rate of flow about half-way between the minimum and maximum likely to be experienced in practice may suffice.

This method may be used for large as well as for small pipes, provided that favourable velocity and static pressure distributions exist. It is because a condition making for such distributions, namely, a portion of straight pipe some 20 diameters or so in length, is more usually present in pipes of small diameter that the method finds its largest field of application for such pipes. This condition of an adequate length of pipe upstream of the section of measurement is essential for a symmetrical distribution of velocity and static pressure in a pipe system on the blowing side of a fan, since, in general, all fans deliver the air in a very disturbed turbulent condition, and the flow close to the fan outlet very often does not fill the pipe symmetrically but is, as it were, piled up to one side or another. On the suction side, however, 3 or 4 diameters of pipe upstream of the section may suffice, with a further 6 diameters or so between it and the fan inlet. Although the distribution of velocity at such a section will naturally not be of the type we have termed the normal distribution, it will nevertheless usually be symmetrical and have a definite value for the ratio of mean to axial velocity, which can be determined by exploration as already described. A word of caution is necessary for this case. It will probably be found that the open

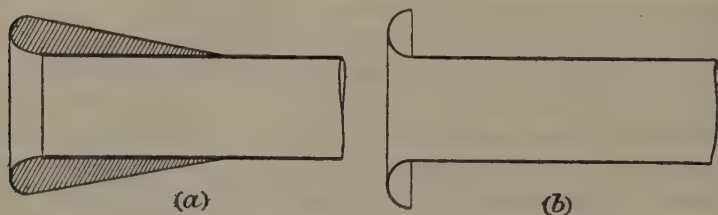


FIG. 66.—Types of shaped inlet to a pipe.

end of the pipe acts to some extent as a sharp-edged orifice, that is, the air stream will contract and not fill the pipe completely for a short distance from the end. This effect can be very largely reduced by fitting the pipe with a shaped inlet, having an easy entry, as shown in Fig. 66 (a) and (b). The type illustrated

in Fig. 66 (a) gives better conditions, but is somewhat more expensive to construct ; either type may be made in wood and fitted to the pipe.

A modification of this method may often be found convenient. Instead of determining the mean rate of flow from the axial velocity, we may use the total head at some fixed point in the section, not necessarily on the axis, or the static pressure at a side hole. With steady, parallel flow at the section, and if the air density is constant, each of these quantities will be a function only of the rate of flow. The relation between the pressure and the mean rate of flow must be established by a preliminary calibration. Thus, if P is the pressure (either total or static), we have to measure P for various values of v_m covering the range that will subsequently be required, v_m being measured by one of the methods already described. We then draw a calibration curve of v_m against P , which is afterwards used to determine v_m for any measured value of P . Usually, as shown below, P will vary nearly as the square of the speed for constant air density, so that the relation between the two will be of the form

$$P = K\rho v_m^2 \text{ (approx.)}, \quad \quad (1)$$

where K is a constant.

It will be useful to examine this method in greater detail. Let us confine our attention to a section X in a given length of

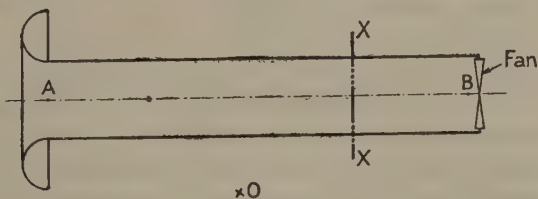


FIG. 67.

pipe AB (Fig. 67) having a fan at B, which may either be made to blow from B to A or to exhaust from A to B. For simplicity, imagine also that in the latter case there is a shaped entry at A. We shall assume initially that the velocity across the section is uniform and equal to v_m . Let O be a point in the undisturbed atmosphere where the air is at rest.

If we neglect the beginning of the motion when the fan is

starting up, and confine our attention to steady conditions when the flow is established and proceeding at a constant rate, we see that the only work done is that against friction between the moving air current and the pipe walls, and the energy equivalent of the internal losses in the current itself due to turbulence. These may be grouped with the frictional losses, since both are inseparable from the flow. We may regard the flow as a complete circuit, which begins and ends at the point O in the external atmosphere: the air starts from rest, acquires a velocity v_m in the pipe, and is returned to a state of rest. (We shall neglect internal losses in the fan itself.) The change of total head between any two sections of the circuit is equal to the energy loss per unit volume between those two sections (see page 24).

(a) *Fan Sucking Air from A to B.*—Assume that at A, just inside the pipe inlet, the air has acquired its velocity v_m . At O, the total head, velocity head, and static pressure are all zero (the atmospheric pressure is taken as the datum according to the usual convention). Between O and A, there is a certain loss of head f_1 due to friction between moving and still air, and to inlet losses. With a shaped entry, however, f_1 is very small. At A, the air has acquired its velocity v_m ; and since f_1 is the only loss of head between O and A, the total head at A differs from that at O by this amount only. That is, the total head at A is $-f_1$. Now we know from Bernouilli's equation (page 13) that the static pressure is equal to the total head minus the velocity head, and since the velocity head at A is $\frac{1}{2}\rho v_m^2$, it follows that the static pressure at A is $-\frac{1}{2}\rho v_m^2 - f_1$. Between A and X there is an additional energy loss f_2 , which is, in fact, the resistance of the length AX of the pipe. The total head at X is therefore less than that at A by an amount f_2 ; and the static pressure is again the total head less the velocity head, the latter being again $\frac{1}{2}\rho v_m^2$ since we are assuming a parallel pipe. Thus we have

$$\text{Total head at X} = -(f_1 + f_2),$$

$$\text{and} \quad \text{Static pressure at X} = -(f_1 + f_2 + \frac{1}{2}\rho v_m^2),$$

the negative signs indicating that both these pressures are below atmospheric.

We have already remarked that f_1 is small and, on the

assumption that it can be neglected in relation to the other pressures (which seems to be generally true), we may say that, approximately

$$\left. \begin{array}{l} \text{Total head at X} = -f_2, \\ \text{and Static pressure at X} = -(f_2 + \frac{1}{2}\rho v_m^2). \end{array} \right\} \quad (2)$$

The quantity f_2 is the frictional resistance of the length AX of the pipe, and is related to the mean velocity v_m in the manner described in Chapter III. Thus, for smooth pipes, the relation between f_2 and v_m is given by the curve of Fig. 16, or by equation (4), Chapter III, and for rough pipes f_2 varies as the square root of v_m . The relation for smooth pipes is not far from a square law, and hence both the total head and the static pressure vary approximately as the square of the speed. Thus, the approximate equation is established for the inlet side of the fan.

Usually f_2 will be found to be considerably smaller than $\frac{1}{2}\rho v_m^2$, the velocity head.* For the rates of flow for which the method of measurement under consideration can be used, that is to say for which the pressures are large enough to be measurable with the necessary accuracy, the section X will have to be something like 50 pipe diameters from A before the frictional resistance of the length AB becomes equal to the velocity head.† As X will usually be considerably nearer than this to A, it follows that on the suction side of a fan the total head is usually small, and the static pressure somewhat greater than the velocity head. Hence, if this method of measurement is adopted, it is preferable, on the suction side, to utilise the static pressure as a measure of the speed.

The conclusions to which the reasoning has led will not be invalidated if there is not shaped entry at A; the effect of this will simply be to alter the value of K in equation (1).

(b) *Fan Blowing Air from B to A.*—We are again concerned only with the length XA of the pipe. Assume that the mean rate of flow is v_m as before.

At A, the air will have a velocity head $\frac{1}{2}\rho v_m^2$ and the static pressure referred to the atmospheric pressure will be zero, so that the total head at A will be $\frac{1}{2}\rho v_m^2$. The resistance of the

* But see remarks on page 198 regarding mine passages.

† This can easily be verified by equating the expression for $p_A - p_B$ in equation (5) on page 29 to $\frac{1}{2}\rho v^2$, and solving for l/d .

length AX will, of course, be f_2 as before, so that the total head and static pressure at X will both be higher by f_2 than they are at A. Thus,

$$\left. \begin{array}{l} \text{Static pressure at X} = f_2, \\ \text{Total head at X} = f_2 + \frac{1}{2}\rho v_m^2. \end{array} \right\} \cdot \cdot \cdot \quad (3)$$

Following a similar line of argument to that previously adopted, we arrive at the conclusion that in this case, i.e., when X is on the delivery side of the fan, the total head, being greater than the static pressure, should be taken as a measure of the speed, and that it varies very nearly as the square of the speed.

In both cases (a) and (b) the assumption has been made that the velocity v_m is uniform across the section. Actually, of course, it is not, but the effect of this will merely be to alter the value of K in equation (1). It has also been implied in both cases (a) and (b) that f_2 is considerably smaller than $\frac{1}{2}\rho v_m^2$. This will be true for a large percentage of pipe installations, but not by any means for such cases as a system of mine ventilation passages, where the total length of air duct to be traversed by the air is so great that f_2 is considerably larger than the mean velocity head. The total head at a point in a mine passage may be from 4 to 6 inches of water, with a velocity head of perhaps $\frac{1}{4}$ inch. Also, the system is so far removed from a simple piping system that one can no longer assume that f_2 varies as v^2 , particularly as f_2 is now so large. However, the method may still be used to measure the flow; all that is necessary is to draw up, by actual test, a calibration curve between the pressure and the air speed in the duct. As the total head and static pressures will be nearly equal (since f_2 is large compared with $\frac{1}{2}\rho v_m^2$), it is immaterial whether the calibration is made on the total head or the static pressure.

A thorough understanding of the changes of static pressure and total head that occur along a circuit of piping, as set out in the foregoing analysis, is of fundamental importance to mining and ventilating engineers, and others concerned with similar problems; and so no apology is offered for discussing it further. In the first place, it may not be clear at first sight why it is the total head that is zero at A* when the fan is sucking air from A to X, and the static pressure that is zero at A when the fan is

* Neglecting the small quantity f_1 .

blowing from X to A. The way in which this happens is as follows : When the fan is sucking, there would be nothing to cause air from the atmosphere to enter the pipe unless the mouth formed a low-pressure region. In fact, one may regard the flow as being started by the formation by the fan of a region of low pressure at A, into which air is forced by the higher atmospheric pressure outside. On the other hand, when A is on the outlet side, the air emerges into the surrounding atmosphere by its own momentum, which the fan has generated. Immediately outside the boundary of the jet, as it issues from A, the pressure is atmospheric, and as the jet is parallel just as it leaves the pipe, the static pressure across it must be equal to the pressure just outside, that is, atmospheric pressure. When A is on the inlet side, the air does not enter as a parallel jet but is sucked in at all angles, and the pressure in the entering stream is not the same as the pressure immediately outside the pipe.

The analysis shows also the origin of the dangerous statement already referred to in Chapter III, to wit " the resistance of a system of piping on the inlet side of a fan is measured by the total-head tube, and on the outlet side by the static tube." In a sense, this statement is true, for equations (2) and (3) show that on the inlet side the frictional resistance f_2 of the length of pipe AX is equal to the total head at X, while on the outlet side f_2 is equal to the static pressure at X. This, however, is only because certain pressures are zero at the open end (and so need not be measured), according to whether the fan is sucking or blowing. It must be remembered also that the analysis has been based on the assumption that the pipe is parallel throughout, and that the velocity is uniform across the section. In practice, the latter condition is rarely fulfilled ; and the true resistance (that is, a measure of the energy required to maintain the flow) can then only be obtained by averaging the total head across the sections in the manner described in Chapters III and V. On the inlet side of the fan, since the total head at A is zero, this means that we need only make a total-head traverse across the section X, but on the outlet side the total head at A is not zero, so we must make a total-head traverse at both X and A.

It is true that in many cases the velocity distribution is fairly uniform, and that one can obtain a measure of the resistance that is sufficiently accurate for practical purposes by a simple total-

head or static-pressure reading at X, according to whether the latter is on the suction or the delivery side. But to accept this procedure unquestioningly as universally true will lead to trouble and confusion. Obviously, the resistance of a system of piping for given conditions of flow cannot depend on whether it is on one side of a fan or the other ; and in the last resort the resistance must be measured in the same way whether the fan is sucking or blowing. It is absurd to suppose that either a facing tube or a static tube is sufficiently discerning to know on which side of the fan it is placed, so that it can exercise discrimination in the kind of physical quantity it measures.

Another source of confusion arises from the fact that, when the system of piping is on the pressure side of the fan, some engineers include in the resistance of the system the velocity head at the outlet, where discharge takes place into the atmosphere. This leads to the anomaly that a given piping system through which a given rate of flow is taking place, has a different resistance according to whether it is on the pressure or the suction side of the fan. It is true that the energy represented by the velocity head is lost at efflux, but this quantity should be included in calculations of the fan duty and not in the resistance of the piping system.

If it is argued that this point has been laboured, the writer can only plead in extenuation that, from his personal experience of the confusion that exists on the meaning and measurement of resistance, a full discussion is amply justified.

Other Useful Methods and Apparatus.—The methods so far discussed in this chapter are all dependent on the measurement of total head, velocity head, or static pressure ; in addition to these, and to methods employing the vane anemometer, which are dealt with in Chapter VII, there are a few other methods deserving of mention.

(a) *The Rotameter.*—For many purposes, particularly when an instantaneous indication of the rate of flow is required, a development of the Ewing ball-and-tube flow-meter is useful. In its original form,¹ this consisted of a vertical glass tube,

slightly tapered in bore, with the diameter decreasing downward. The fluid flow to be measured passed upwards through the conical tube, which was inserted somewhere in the flow circuit. A ball of diameter slightly greater than the minimum bore of the conical tube was carried upwards by the passage of the fluid, until it reached a position in the tube where its weight was balanced by the force due to the fluid flowing past it. The position of the ball in the tube depended on the rate of flow, and was used to measure it.

The main objection to the Ewing flow-meter was unsteadiness of the ball ; this could be partly overcome by inclining the tube, but a better solution is that adopted in the instrument known as the Rotameter (Fig. 68), in which the ball is replaced by a small, specially-shaped float having spiral grooves in its surface, by means of which it is maintained in rotation about a vertical axis as the fluid streams past it. The tapered glass tube has a quantity or speed scale marked on it (see Fig. 69) ; and the position of the float in relation to the scale gives a direct and instantaneous indication of the rate of flow, no calculation being required.

As an indicator in a system in which the flow has to be constantly checked and maintained at a specified rate, this instrument serves a particularly useful purpose ; if it is installed where it can easily be seen by the operator whose duty it is to control the rate of flow, the necessary adjustments to the control valve can readily be made by its aid.

It must be remembered, however, that the calibration of the Rotameter will be affected by the density of the air, so that the indications will be correct only for one temperature and pressure, which are generally marked on the scale. If the instrument is calibrated for air of density ρ_1 and used when the density is ρ_2 , the true rate of flow when the indicated quantity is q will be approximately $q\sqrt{(\rho_1/\rho_2)}$ if the calibration is in terms of volume, and $q\sqrt{(\rho_2/\rho_1)}$ if it is in terms of mass. For accurate work, the instrument should be calibrated over the range of density in which it will be used.

Rotameters are made in a number of sizes, ranging from those for measuring very low rates of flow to others suitable for insertion in pipe lines of the order of 6-inches diameter. The standard range of a single instrument is 10 : 1.



FIG. 68.



FIG. 69.

The theory of this class of flow-meter has been discussed by Awberry and Griffiths.²

(b) *The Velometer*.—Another useful proprietary instrument is the Velometer,³ originally developed by Boyle in America. The most important component of this instrument is a meter (Fig. 70) consisting of a balanced, damped, pivoted vane, which can be deflected by a very light current of air or gas and carries a pointer moving over a graduated scale. The movement is

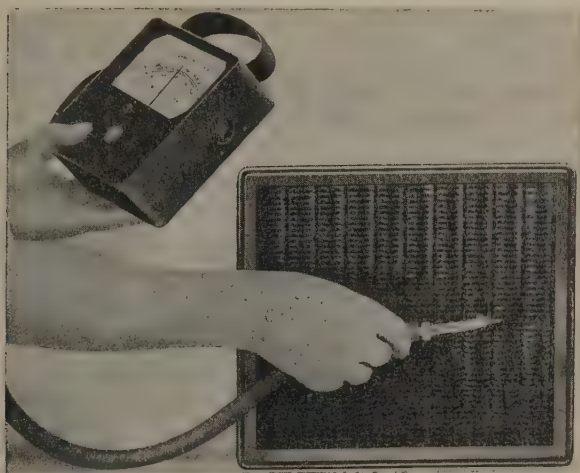


FIG. 70.

enclosed in a case having two orifices, one to admit the air current and the other to provide the exit. In large ducts or passages—mine ventilation passages, for example—this meter can be exposed directly to the current, with the inlet orifice facing the stream; it indicates air velocities as low as 20 feet per minute.

The Velometer can also be used to measure flow in pipes in which the meter is too large to be inserted, by introducing special “jets” into the pipe. These jets are simply pairs of tubes so arranged that the pressure at the mouth of one is greater than that at the other, so that if they are connected, one to each of the meter orifices, a current of air or gas flows through

the meter and actuates the vane. For use in this way, standard lengths of rubber tubing are supplied. The rate of flow through the meter will obviously depend on the resistance of the circuit, i.e., in part on the length and diameter of rubber tubing used ; with the jets it is therefore essential to use the tubing with which the instrument was calibrated, or tubing of identical bore and length.

One of the most useful purposes served by the Velometer is to measure the air discharged through a ventilation grille, which is very difficult to determine in any other way. For this purpose a special jet is used.

(c) *The Soap-Film Method*.—For measuring very small rates of flow, which are perhaps more common in the chemical laboratory than in engineering practice, a method originally suggested by Barr ⁴ and afterwards elaborated by Gooderham ⁵ will often be found useful. The gas or air whose flow is to be measured is passed through a vertical glass tube, such as a wide burette and, on entering, forms a soap film across the tube ; this film travels up the tube at the same velocity as the gas, and the time taken for it to pass between two fixed marks on the tube, or along a given length of a scale of graduations etched on the tube, is observed. The volume between the marks, or the calibration of the scale in terms of volume, is previously determined by direct measurement with liquids ; and therefore the rate of flow of the air or gas can easily be determined from the time measurement. Such instruments, examples of which were shown at the Physical Society's Exhibition, in January, 1946, have been used for rates of flow up to about 1 or $1\frac{1}{2}$ cubic feet per hour ; there seems to be no reason why they should not be used for somewhat higher rates of flow, but they are essentially meters for low-speed work.

(d) *Gas Meters*.—Gas meters are almost exclusively used in the gas industry to measure the volume of gas delivered to gas-holders or to consumers. Their indications are proportional to the volume passing through them, so that the rate of flow can not be determined unless the time interval between observations is also recorded. Two main types are in general use, the dry meter and the wet meter ; the former, which is found in most private premises to measure domestic gas consumption, is not a precision instrument, but the latter, which is generally

used in the works of gas supply undertakings, is capable of a high degree of accuracy—of the order of 0.1 or 0.2 per cent.—when accurately calibrated.

The operation of a wet meter is illustrated diagrammatically in Fig. 71*.

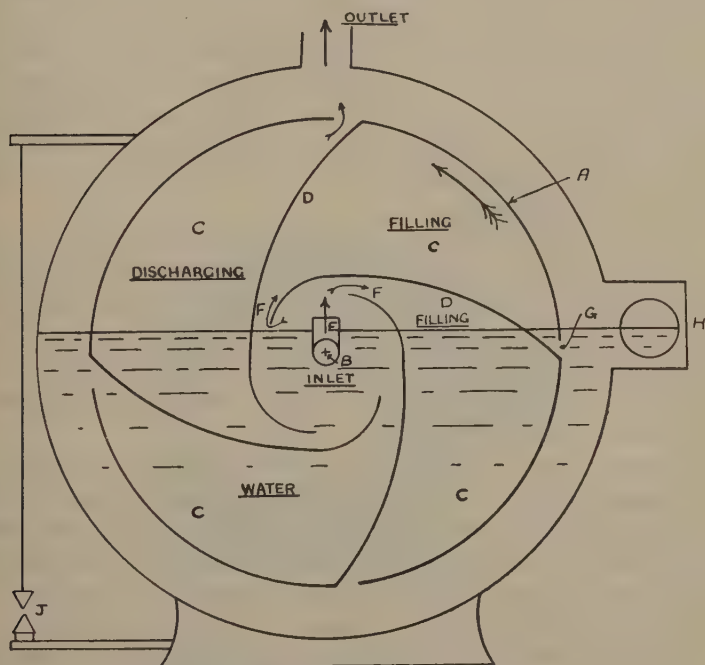


FIG. 71.—Diagram of wet gas meter.

The drum A, revolving on its axis B, is divided into a number of compartments C by means of partitions D, which extend the full length of the drum. Gas is introduced into the centre of the drum from the inlet pipe E, which passes through one end of the drum in the form of a hollow spindle, and flows into the compartment marked "Filling" via port F. In this compartment it becomes a fluid wedge between the surface of the water and the partition D, pushing the partition upwards and turning the drum. When the compartment is filled, the port F is sealed

* For this diagram and for the description that follows of the method of operation of the wet gas meter, the author is indebted to the Gas Light and Coke Company.

by the water ; and soon afterwards the discharge port G rises above the surface of the water and discharge begins. The drum continues to turn owing to the filling of the next compartment, so that water flows into port F, expelling gas from port G into the meter case and thence to the outlet. In turning, the drum rotates the index pointers.

As the drum revolves, each compartment fills alternately with gas and with water, so that it is completely filled and emptied of gas once per revolution. Discharge of gas from a compartment cannot begin before the charging is complete, so that, for every revolution of the drum, an accurately defined volume of gas is delivered.

It will be realised that overfilling of the meter with water beyond the indicated level in the sight box H, or underfilling, will decrease or increase the effective volume of each compartment, and so cause the meter to register more or less gas than that actually flowing. For a similar reason, the meter must always be levelled accurately by means of the plumb-bob J, or by a spirit level.

The drums used in practice contain spiral partitions, arranged in a manner similar to the rifling of a gun barrel, the ends of the partitions corresponding with radial ports cut in the end covers of the drum. Gas passes axially along the drum, from back to front ; but otherwise the operation is as described above.

A typical wet gas meter is illustrated in Fig. 72.

It will be clear from the foregoing description that a wet gas meter needs calibration, since one cannot determine by any convenient means the effective total volume of the compartments. The usual process of fundamental calibration makes use of a standardised apparatus, resembling in principle a cylindrical gasholder, from which a known volume of gas can be passed through the meter. Various difficulties complicate the process ; in particular, the problems of measuring accurately the temperature of the gas, of maintaining it constant during the calibration, and of ensuring that the water used to seal the apparatus is completely saturated with gas before calibration begins.

Although admirably suited for the purpose for which it was primarily designed, the wet gas meter has several disadvantages for general use in engineering practice. Apart from the diffi-

culties of calibration, it is very large in relation to its rating. Thus a meter rated at a maximum delivery of 1,200 cubic feet per hour—not a high rate of flow for the ventilating engineer, for example—is about 4 feet long from back to front, and about 3 feet 6 inches in height and width, and, when filled with water to the datum level, is extremely heavy. The rating cannot be increased appreciably by allowing the drum to rotate faster,

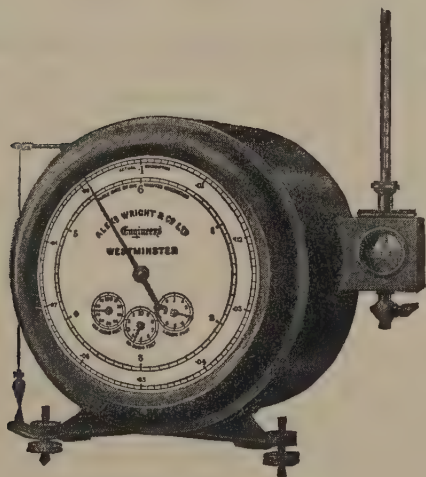


FIG. 72.—Wet gas meter.

since this results in a banking-up of the water level to one side, with a consequent change in effective volume of the compartments. Thus, if we exclude as impracticable for general purposes the very large “station meters” used in gas works to meter large quantities of gas, the use of the gas meter will be restricted to the measurement of low rates of flow. The further difficulty then arises that a head of about $\frac{1}{4}$ to $\frac{1}{2}$ inch of water is needed to operate even a small meter; and static heads of this order are not common with slow rates of flow.

To summarise, therefore, although under suitable conditions the wet gas meter may be one of the most convenient and accurate instruments to use for measuring relatively low rates

of flow, such conditions will occur comparatively rarely in general engineering work.

Further details of the wet and various other types of gas meter are given in Reference 6.

REFERENCES

1. Ewing, *Proc. Roy. Soc. Edinburgh*, **45** (1924-25), p. 308.
2. Awberry and Griffiths, *Proc. Roy. Soc. Edinburgh*, **47** (1926-27), p. 1.
3. Poole and Leadbeater, *Trans. Inst. Mining Eng.*, **97** (1939), p. 225.
4. Barr, *Journ. Sci. Insts.*, **11** (1934), p. 324.
5. Gooderham, *Journ. Soc. Chem. Industry*, **59** (1940), p. 1 ; also *ibidem*, **63** (1944), p. 351.
6. U.S. Bureau of Standards, Circular No. 309, *Gas Measuring Instruments* (1926).

CHAPTER IX

MANOMETERS

WE have already observed that the velocity heads that have to be measured in practice are usually not large: for example, the velocity head, under ordinary atmospheric conditions of temperature and pressure, does not reach 1 inch of water until the air speed is roughly 66 feet per second. Further, since the head is proportional to the square of the speed, it falls off rapidly at the lower speeds; it is, in fact, less than 0.1 inch of water at 20 feet per second, a speed by no means outside the ordinary working range. It follows that, in order to determine air speeds with sufficient accuracy by means of pitot and static observations, sensitive manometers, capable of reading pressures of the order of 0.001 inch of water, or even less, will often be necessary.

Numerous and varied devices have been designed to supply this need. A number of these are, for different reasons, unsuitable for industrial use; and it is not proposed in this chapter to attempt a description of all the types of manometer that have been constructed or suggested. We shall confine our attention only to one or two types of different degrees of sensitivity, which have been proved by experience to be suitable for engineering practice, as distinct from laboratory conditions. The main difficulty in designing such manometers lies in combining high precision with the portability, robustness, and ease and rapidity of manipulation that are essential for their successful application to general practical use.

The Tilting Micromanometer.—This instrument is capable of giving all the accuracy that will be required save in exceptional circumstances. It is, in principle, a sensitive U-tube water gauge, in which provision is made for microscopic observation of the water level in one of the vertical limbs. The U-tube is shaped as shown in Fig. 73; it consists of a long horizontal glass tube A, communicating at each end with one of the glass

vessels BB, whose axes should be parallel and approximately vertical in the zero position. Each vessel is provided with a removable glass cap, ground in to fit, to which rubber tubes leading to the static and pitot heads may be attached. Short vertical glass tubes with sealed ends project from the base of

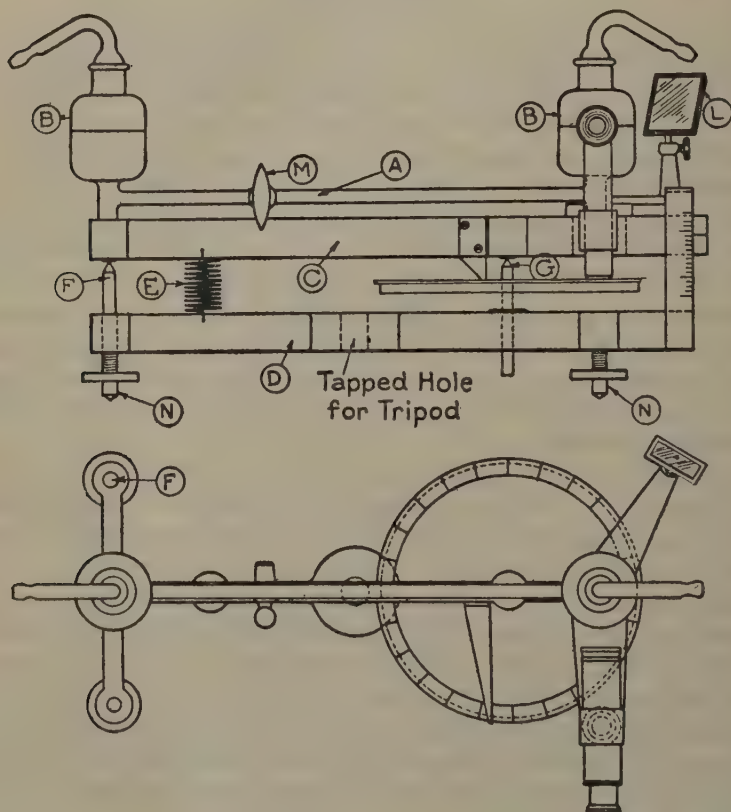


FIG. 73.—Tilting micromanometer.

each cup and fit into corks which, in turn, fit firmly in holes in the upper movable steel frame C. The latter rests, by means of a three-point support, on the lower fixed frame D, and is held firmly thereon by the spring E. The three-point support is arranged with two points FF under the left-hand vessel, the line joining these points being perpendicular to the axis of the tube A. These two supports are fixed, and the upper frame,

with the U-tube, is tilted about the line joining them by means of the third supporting point G, which is formed by the hardened end of a vertical micrometer screw working in the lower frame. Of the two left-hand points, one engages a conical cup on the upper frame and the other a V-groove with its length horizontal and perpendicular to the axis of the tube A ; the end of the micrometer screw bears against a small flat surface on the upper frame. In this way, the tilting frame is given definite location without constraint. All the points and their seatings are of hardened steel. The position of the movable frame with respect to the fixed frame is observed by means of two indicators attached to the former, of which one projects above the graduated rim of the micrometer wheel and records fractions of a turn of the screw, and the other indicates complete turns on the fixed vertical scale on the lower frame. A microscope K, with cross-wires in the eyepiece, is carried by the upper frame for observation of the level of the liquid in the right-hand vessel ; and it is advisable for facilitating observation to have also a small adjustable mirror L, by means of which light from any convenient source can be reflected through the glass vessel into the microscope. The field of vision is then brighter above the water level than below, and the line of demarcation can be sharply focused. A tap M should be provided to prevent spilling of the water whilst the manometer is being moved from place to place.

The instrument is often best used when mounted on a tripod. For this purpose it is desirable to provide the lower frame with a screwed hole to take a tripod, such as that ordinarily used for surveying instruments. It is as well to have also three levelling screws NN on the lower frame, to adjust the zero of the instrument if it should be found more convenient to rest it on a bench or table. A further convenient practical step is to bore eccentric holes to take the two cup supports in the two corks fitting into the holes provided in the tilting frame. In this manner, some adjustment is provided for small variations from the standard dimensions in the glassware, since it is difficult to make this exactly to size in every case.

In using the manometer, the tubes from the pitot and static heads should be connected to the cups before the zero reading is taken, as the instrument may easily be displaced whilst the

tubes are being fitted. The caps fitting into the necks of the cups should be greased with a small quantity of vaseline over the region of contact, in order to ensure the absence of leaks at the joints. The glass tube A should initially be approximately horizontal; and the zero reading is obtained by observing the readings on the fixed scale and the micrometer wheel when the cross-wires of the microscope intersect the image of the water level. The total head and static tubes are then inserted in the pipe in which the air speed is being measured; and the movable frame is tilted by means of the screw until the image of the water level again coincides with the intersection of the cross-wires.

Let d be the difference between the initial and final readings, expressed in turns and fractions of a turn of the wheel;

l_1 be the distance between the cup centres in inches;

l_2 be the perpendicular distance of the axis of the micrometer screw from the line joining the two fixed points;

t be the pitch of the micrometer screw in inches;

and p be the pressure difference in inches of water.

Then, if the angle of tilt be not too great,

$$p = td \frac{l_1}{l_2}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Convenient practical values of l_1 , l_2 , and t are 13 inches, 10 inches, and 0.05 inch respectively. The available travel of the micrometer screw should not exceed 20 turns, giving the instrument a range of about $1\frac{1}{4}$ inches of water, and the diameter of the screw over the thread should be about $\frac{3}{4}$ inch. Smaller sizes lead to "sloppiness" as the thread wears.

If a higher range is required, it should not be obtained by increasing the length of the screw; since the large angles of tilt that then occur at the higher end of the range introduce errors for which it is necessary to make due allowance; and the above equation is no longer valid. It is preferable to increase the distance between the cup centres, a 26-inch U-tube used in place of a 13-inch tube would have double the range, and such a tube could quite conveniently be mounted on a frame designed for a 13-inch tube. In practice, however, when the velocity head exceeds that covered by the range of a 13-inch tilting

manometer, the use of an ordinary U-tube water gauge will probably give sufficient accuracy in most cases.

An important consideration in connection with the tilting manometer is that it is an absolute standard, which does not require calibration against another manometer. Of the dimensions that determine the reading for a given applied pressure difference, two, namely, the pitch of the screw thread and the distance of its axis from the line joining the fixed points, can be made to size or measured sufficiently closely to limit errors in the readings to less than 0.1 per cent. The third important dimension, that is, the distance between the cup centres, cannot be made to a given size with such precision, but can be measured, by calipering, to 0.01 inch without difficulty. With careful workmanship, the overall accuracy of the instrument should be within 0.1 per cent. over the entire range.

If this degree of accuracy is required at the higher pressures, a small correction must be applied to the readings in this part of the range. In equation (1), the product td represents the height h in inches through which the micrometer screw moves, that is the vertical distance by which a certain point on the movable frame is raised or lowered above another point, which does not move vertically. The assumption underlying equation (1) is that the length of the line joining these points is constant and equal to l_2 , the distance between the axis of the micrometer screw and the fixed axis of rotation of the tilting frame. Actually, of course, the length of this line on the moving frame, i.e., the "leverage," is not constant, and has its value assumed in equation (1) only in the initial position. As the micrometer screw moves, its point slides along the hardened steel surface on the moving frame with which it is in contact, so that the leverage changes. It is easy to show that the true leverage for the reading d in equation (1) is $\sqrt{l_2^2 + t^2 d^2}$ instead of l_2 , so that the true pressure difference is obtained from the equation

$$p = td \frac{l_1}{\sqrt{l_2^2 + t^2 d^2}} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The error incurred by using (1) instead of (2) is only 1 per cent. when $td = 1.4$ inches, and decreases as td becomes smaller relatively to l_2 .

The necessity for applying this correction can be entirely

avoided if the upper end of the micrometer screw terminates in a horizontal, hardened, flat surface instead of in a point; and a hardened steel ball, fixed to the moving frame and resting on the flat end of the micrometer screw, is substituted for the plane bearing surface. The leverage will then be constant and equal to l_2 , the distance between the line joining the two fixed left-hand points (FF in Fig. 73) and the centre of the steel ball: the latter will now simply slide on the fixed end of the micrometer screw.

With a microscope having a magnification of 20 or 30 diameters, the tilting manometer as described will be sensitive to pressures of less than 0.001 inch of water; which means that, when this instrument is used with an ordinary pitot-static tube, a wind speed of 10 feet per second can be observed with an accuracy of about 2 per cent. By a modification of the glass-work, the sensitivity can be greatly increased, unfortunately, however, at the expense of ease of manipulation and rapidity of working. The alteration

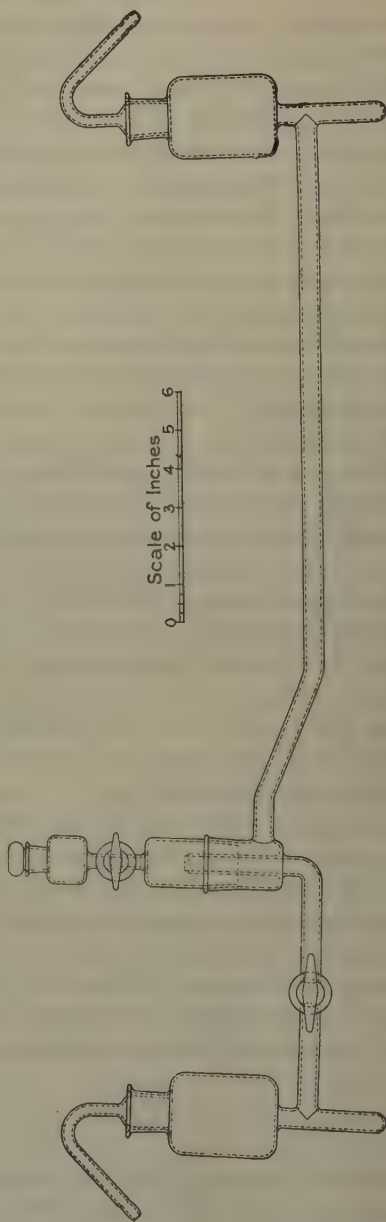


FIG. 74.—Glass-work for 26-inch Chattock manometer.

consists in cutting the horizontal limb of the U-tube and connecting the two parts to a central vessel, one part being connected to the outside of this vessel near its lower end and the other being bent vertically upwards and passing up into the centre of the vessel. The modified arrangement is shown in Fig. 74, which illustrates the glasswork of a manometer constructed on these lines, and from Fig. 75, which shows a complete instrument. It will be seen that the metal frame resembles that previously described.

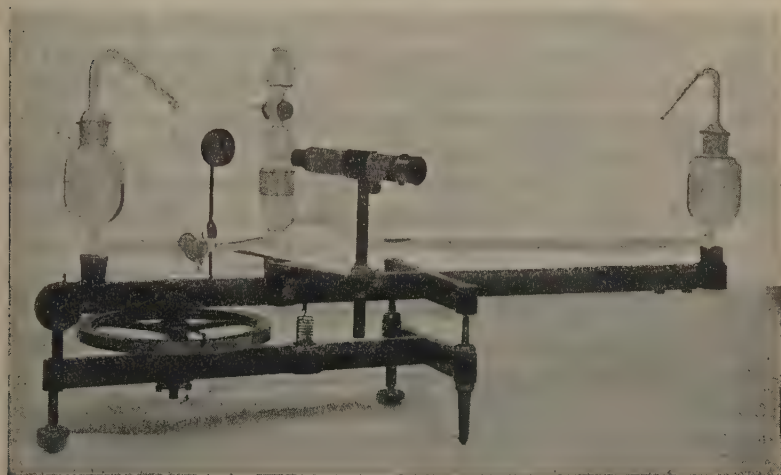


FIG. 75.—Chattock manometer (26-inch cup centre).

This design of glass-work is an elaboration of that first suggested by Chattock.¹ The vertical tube in the central vessel extends upwards for some distance as shown, and is cut off square and ground. Distilled water is poured into the lower parts of the two cups and the central vessel; and into the upper portion of the latter, completely filling it, is admitted a quantity of medicinal paraffin ("Nujol"), by the aid of the filling reservoir and tap shown in the diagram. The paraffin does not mix with water, and two well-marked surfaces of separation are formed, one outside the vertical tube in the interior of the central vessel and one within this tube. The liquid levels are adjusted, by regulating the quantities admitted, so that one surface of separation is formed on the open end of

the central vertical tube, and has the appearance of a bubble resting on the mouth of the tube. If now a pressure difference is applied to the two end-cups, this bubble tends to become larger or smaller ; and its movement, which can be followed by suitable illumination from behind and microscopic observation, is arrested by giving an appropriate tilt to the frame by means of the micrometer screw.

Instruments of this type, having cups whose centres are 13 inches apart, are sensitive to pressure differences of the order of 0.0001 inch of water, the increased sensitivity being attributable to the fact that the motion of the liquid is observed in a narrow tube instead of in the relatively wide end-cups. Apart from surface tension effects, the motion of the bubble for a given pressure difference will be greater than the motion of the surface in one of the end-cups, in the ratio, approximately, of the area of the water surface in the cup to the internal area of the central tube. With a microscope of given magnification, therefore, the sensitivity of a Chattock type of manometer will be greater in this ratio than that of an ordinary tilting manometer. Actually, surface tension effects are by no means negligible, and reduce the sensitivity considerably. Nevertheless, as will be seen from a comparison of the figures quoted, this instrument is considerably more sensitive than the plain tilting manometer. Its chief disadvantage from an engineering standpoint is that, particularly with fluctuating pressures, the bubble is easily broken, in which case measurements have to be interrupted whilst a fresh zero setting is taken. Further, small zero changes may also occur as the temperature alters, because of the different coefficients of thermal expansion of the water and the paraffin. A modification to eliminate this zero creep has been suggested by W. J. Duncan ²; and for further details of the Chattock gauge the reader is referred to this paper, and to a full description that appeared in *Engineering*.³

Inclined-Tube Manometers.—If a small sacrifice of accuracy and the advantage of an instrument that does not require calibration are considered unimportant, a convenient form of gauge suitable for factory use will be found in the Krell manometer, shown diagrammatically in Fig. 76. The principle of this instrument is simple : if one of the arms of a U-tube is

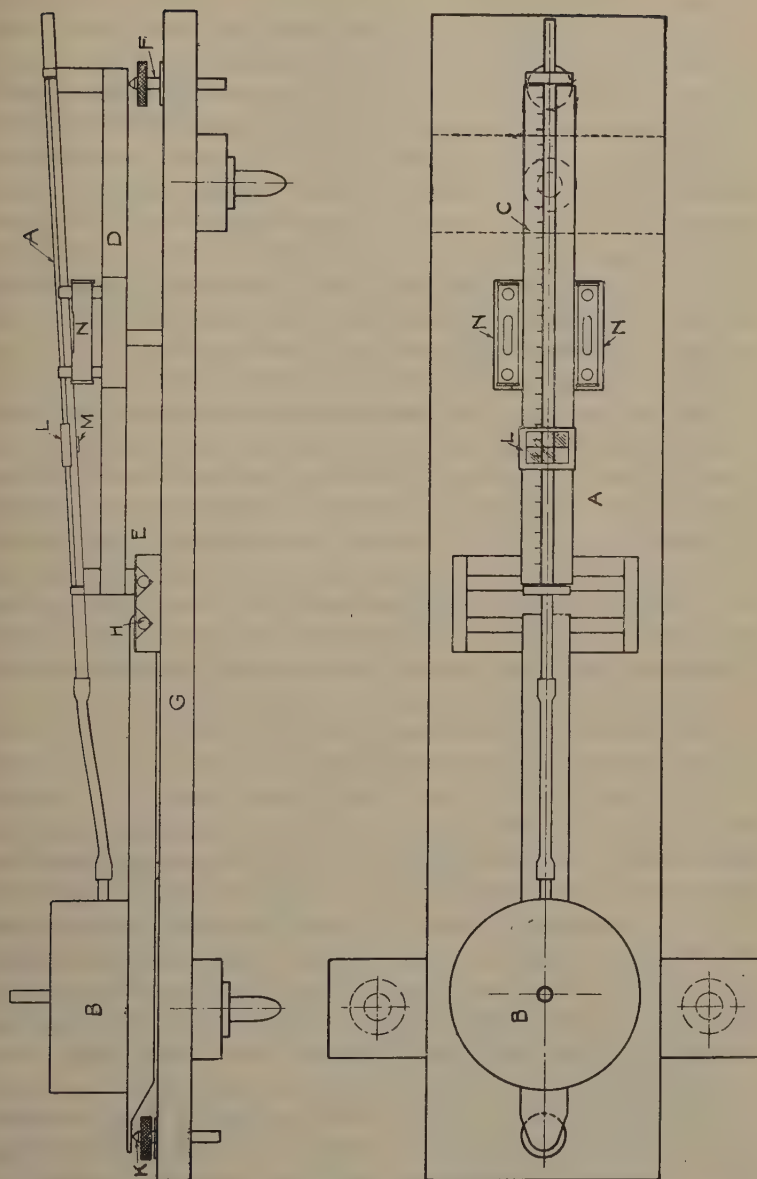


FIG. 76.—Krell type inclined-tube manometer.

inclined at a small angle to the horizontal, the liquid in that arm will move a considerable distance along the tube for a small amount of vertical motion, i.e., for a small applied pressure difference. Reference to the diagram shows that the gauge consists essentially of an inclined tube A, which forms one arm of the U-tube, and a large reservoir B, having an area three or four hundred times that of the tube forming the other arm. The sources of pressure are connected to the reservoir and to the end of the inclined tube; the use of a large reservoir causes the liquid level therein to remain virtually stationary, practically all the motion occurring in the tube, where the position of the meniscus is observed against a graduated scale C.

The tube is held on the frame D, which is pivoted at E and can be raised or lowered at the other end by means of the screw F, so that the inclination of the tube can be altered and the sensitivity and range thereby varied. The whole instrument is mounted on a baseboard G, fitted with three levelling screws; and provision is made for rotating the reservoir support about the fulcrum H by means of the screw K, in order to adjust the zero level of the liquid in the inclined tube. For observing the position of the meniscus, it is convenient to have a sliding frame L, which moves along the scale and carries a cross-wire above the tube and a small strip of white ivorine M below.

It is usual to provide two spirit levels NN, which are adjusted and locked when the manometer is initially calibrated, so that the tube may be set to two known inclinations. The instrument then has two ranges for indicating low and high pressures respectively. The advantage of this will be realised when it is remembered that the velocity head is proportional to the square of the wind speed, and that it is usually necessary to allow for the measurement of fairly high speeds. If, therefore, the tube is set to such an inclination that the full scale reading corresponds to the highest wind speed liable to be encountered, the readings for low wind speeds will be small, and very little accuracy will be obtained in this range.

Alcohol is found to be the liquid giving the best meniscus in this type of manometer. As regards the bore of the inclined tube, a diameter of about 0.2 inch gives the best results⁴; a convenient length of tube is one with which a full scale reading of 12 inches can be obtained. With such a tube set at an

inclination of eleven degrees, the highest pressure indicated, if alcohol is used as the manometric liquid, will correspond to a wind speed of about 80 feet per second. If this is adopted as the higher range, a convenient lower range will probably be found to be that in which the second level is set at two and a half degrees, when the maximum pressure will correspond to an air speed of about 40 feet per second. The scale can be read to about 0.02 inch, and an accuracy of 3 per cent. or so will be obtained at a pressure corresponding to 10 feet per second on the lower range. If this degree of accuracy at low speeds is not considered sufficient, the levels can be adjusted to smaller angles; at higher speeds the precision is, of course, much better.

The manometer is not a fundamental standard; for accurate work, it requires calibration against a primary manometer, since various factors, such as lack of straightness of the inclined tube and surface tension effects due to want of uniformity in the bore, preclude the possibility of calculating true pressures sufficiently accurately from the observed readings and the known inclination of the tube. In order, however, to estimate initially the angle to which the tube should be set for any desired range, the following formula will give sufficiently exact results; it is the formula that would be valid if the instrument were not subject to the errors mentioned above:—

$$\sin \theta = \frac{h}{l\sigma}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where h is the maximum water column to be observed,

l is the total length of the scale,

σ is the specific gravity of the liquid,

and θ is the required slope of the tube to the horizontal.

Manometers of this type are to be recommended for general use: they are consistent in their readings, accurate when calibrated, and comparatively inexpensive when their degree of performance is taken into consideration. They are, moreover, convenient and robust, and by their aid observations of pressure can be rapidly performed. The only disadvantage is the fact that each manometer requires calibration initially against a standard instrument. A certificate giving the results of calibration can be obtained by submitting the instrument to the National Physical Laboratory for test; as a rule, the manufac-

turers will obtain an N.P.L. certificate and supply it with the gauge. If an absolute standard, such as the tilting micromanometer, is available, the calibration can easily be carried out in the manner described at the end of this chapter.

A modification of the inclined-tube manometer has been designed by Hodgson, in which the manometer tube, instead of being inclined at a constant angle along its entire length, is curved in such a manner that a uniform scale of velocity is obtained. A form of curved-tube manometer fitted with a pressure scale is shown in Fig. 77, which is reproduced by the courtesy of George Kent Ltd., Luton.

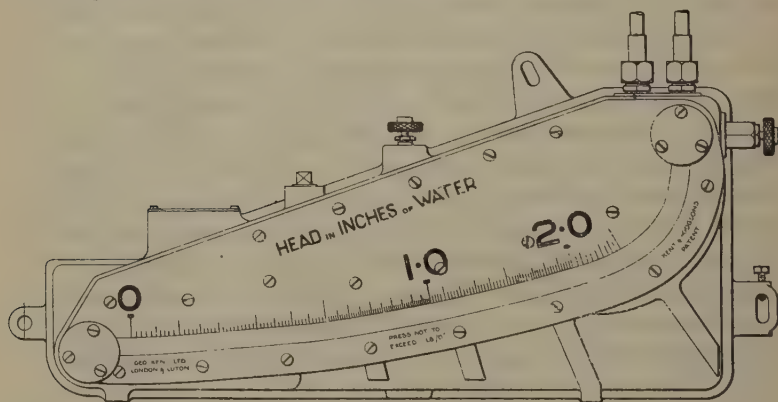


FIG. 77.—Curved-tube manometer.

The instrument is in effect an inclined-tube manometer having a tube of continuously varying slope, such that the inclination is least at the lowest speeds, i.e., when the velocity heads are least. In this way, one retains the main advantage of the inclined-tube manometer, i.e., adequate sensitivity at low velocities, without either an unduly restricted velocity range, an impracticably long tube, or the inconvenience of having to adjust the inclination of the tube to suit the velocity range.

Curved-tube manometers can be supplied either with pressure, velocity, or quantity scales. The pressure scale is to be preferred for general purposes, since an instrument so equipped can be used to measure static pressure or velocity head in any pipe or duct into which the pitot and static tubes can be inserted, and in any conditions of temperature and pressure. With a

velocity scale, the instrument can be used in the same way to measure velocity directly, but static pressure can only be obtained by calculation. With a quantity scale, the instrument can be used only for the particular pipe for which it is designed. Moreover, both velocity and quantity scales are accurate only at one particular value of the air density; as a rule, therefore, corrections have to be applied to the readings to allow for departures of the temperature and pressure from the specified conditions. Attention is drawn to this point in the discussion of the theory of the instrument given at the end of the present chapter, where it is shown that the corrections are proportional to the square root of the density.

Large-Range Micromanometers.—A useful and robust type of manometer consists in effect of a flexible U-tube, the lower ends of the two vertical limbs being connected by a length of rubber tubing. One of the limbs remains stationary, the other being raised or lowered, by an amount corresponding to the applied pressure difference, by means of a micrometer screw to which it is attached. Fig. 78 illustrates a manometer that utilises this principle and incorporates some useful features introduced at the University of Toronto.⁵ The construction of the instrument is clear from the figure, and needs little comment. Rotation of the nut supporting the movable limb of the U-tube is prevented by means of a keyway, which engages with a vertical key cut on the right of the vertical supporting standard carrying the head of the micrometer screw. In place of a cup for the moving limb, an inclined glass tube of variable slope is employed, having a horizontal mark etched on it. The zero reading is first established with the liquid meniscus in this tube in coincidence with the fixed mark; when a pressure difference is applied, the inclined tube is raised on its nut until the liquid is again in equilibrium at the mark. Complete turns of the micrometer screw are indicated by the pointer moving over the vertical scale, and fractions of a turn are observed on the graduated head. The fixed limb of the U-tube consists of a reservoir of large cross-sectional area in comparison with that of the inclined tube, so that the change in level in the reservoir can be neglected. A convenient feature of this manometer is the vertical glass tube fixed alongside the scale as shown: this

tube is connected to the reservoir in parallel with the inclined tube by means of a T-piece ; and if its open end is connected to the same source of pressure as the open end of the inclined tube, the level of the liquid in it indicates the approximate height to which the nut should be raised when the order of magnitude of

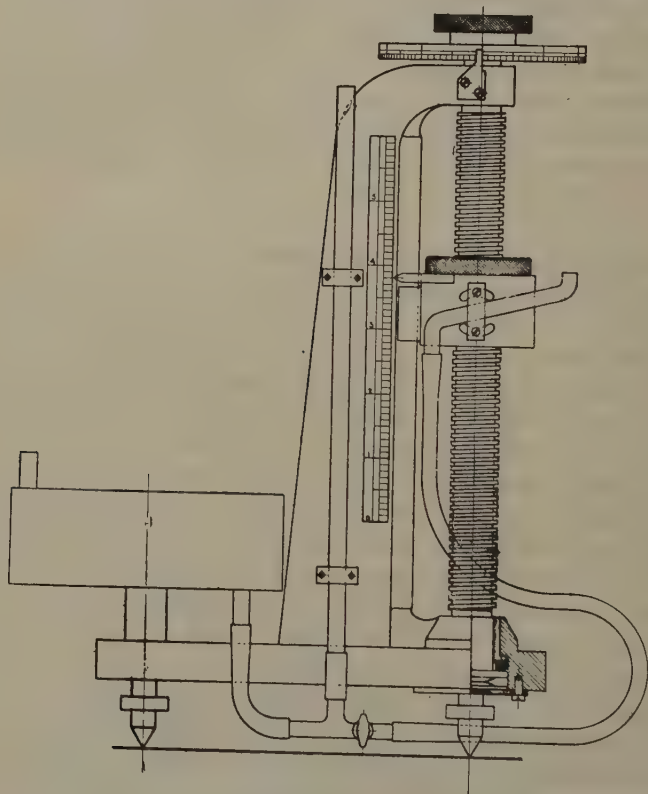


FIG. 78.—Large-range micromanometer.

the applied pressure difference is unknown beforehand. The final adjustment is made by the aid of the meniscus in the inclined tube.

As in all types of manometer in which the meniscus in an inclined tube is observed, alcohol is found to be the most suitable liquid ; and the best sensitivity for general use is said to be obtained by the use of an inclined tube of about 0.2-inch bore set at a slope of three degrees or so, corresponding to a

magnification of roughly 20 to 1. No appreciable increase of sensitivity can be gained by reducing this slope without making the instrument unduly sluggish ; tubes of smaller bore reduce the sluggishness to some extent, but the meniscus is not so good.

This instrument, though not as simple or rapid in use as the plain inclined-tube manometer, is certainly more accurate, and has the advantage that it does not require calibration ; the readings need correction only for the specific gravity of the alcohol used, in order to convert them into inches of water. Once the accuracy of the micrometer screw thread has been checked, the manometer can therefore be used as an absolute standard. It is stated ⁵ that the movement of the liquid column can be read to within about 0.0002 inch on the average, with an extreme possible error of 0.0005 inch. A further important advantage of this instrument is the fact that the range is not as restricted as it is in other sensitive manometers, since the micrometer screw may be made some 10 inches long, or more.

A manometer similar in principle and construction to that just described has been designed for use at the National Physical Laboratory,⁶ but the position of equilibrium is determined in a different manner. The two limbs of the U-tube in this case consist of two glass cups, one of which is similar to those used in the tilting manometer and can be raised or lowered by means of the micrometer screw, as in the instrument just described. The other cup is fixed, and is constructed as shown in Fig. 79 ; it will be seen that the arrangement is precisely similar to that of the central vessel in the Chattock manometer and, as in that instrument, use is made of a water-paraffin meniscus on the mouth of the central tube to indicate equilibrium. The meniscus is suitably illuminated from behind and is viewed through a microscope. For the zero setting, the meniscus is adjusted so that its image coincides with the intersection of the cross-wires in the eyepiece of the microscope ; an applied pressure difference is then balanced by adjusting the level of

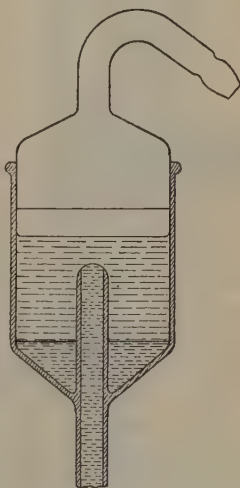


FIG. 79.

the movable cup until the image of the bubble once more coincides with the intersection of the cross-wires. In this instrument, the cross-sectional area of the rubber connecting tube is not small compared with that of the cups. Hence, in order to avoid appreciable errors due to changes in volume of the rubber tubing, it is advisable to have a long length of tubing, so that it hangs approximately vertically from each cup

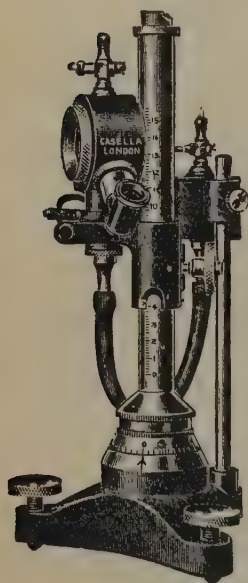


FIG. 80.—Large-range micromanometer.

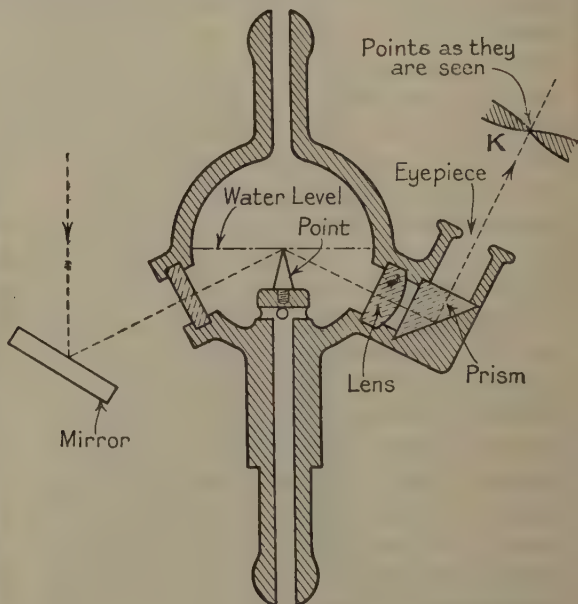


FIG. 81.

over the entire range of the movable cup. The changes of shape that then occur in the tubing will only produce negligibly small volume changes.

One further example of this class of manometer may be mentioned on account of the ingenuity of the method of indicating balance. This instrument is a water manometer that has been put on the market by Messrs. C. F. Casella & Co. Ltd., and can be used to measure heads of 3 feet of water or more. Fig. 80 shows an example having a range of 150 mm. of water. Little comment is needed on the general working, which will be apparent from the illustration. The fixed cup, on

the right, can be clamped at any convenient position (according to the pressure to be measured) on the vertical rod ; and the movable cup, which is also the measuring chamber, is raised or lowered on the main vertical column by means of a micrometer arrangement.

Details of the measuring chamber are shown in Fig. 81, which is reproduced from *The Engineer*.⁷ The height of the chamber is adjusted until the water level in it is just flush with the point fixed in the centre. The point is illuminated by light from a suitable source, reflected from the mirror through the glass window on the left of the chamber ; and it is viewed, together with its image reflected in the water surface, by means of the magnifying optical system on the right. The water level is adjusted, by raising or lowering the chamber, until the point appears just to touch its reflected image. The height of the chamber when the point and its image touch is read off the column and the micrometer, firstly when both sides of the instrument are open to atmosphere and again when they are connected to the two sources of pressure whose difference is to be measured. The difference between the two readings gives the differential pressure in terms of water column ; and it is stated that the system is capable of an accuracy of within 0.0004 inch head of water.

An instrument on similar lines was made by the Askania Company in Germany before the war of 1939-45.

The Two-Liquid Differential Manometer.—A convenient instrument for work where high accuracy is not required is shown in Fig. 82. It is, in reality, a modified form of U-tube, magnification of the readings being secured by the use in two non-miscible and mutually insoluble liquids of nearly equal densities. A serious fault of the plain U-tube—namely, zero errors due to tilt of the instrument—is overcome in the manometer under consideration by arranging the two limbs concentrically as shown.* Each limb is surmounted by a

* This type of two-liquid manometer was originally designed at the Royal Aircraft Establishment for use on aeroplanes, see *Reports and Memoranda of the Advisory Committee for Aeronautics*, No. 144 (1913). The instrument mentioned in that Report had the lighter liquid in the outer reservoir, the surface of separation of the two liquids being in the outer tube. The form here described is preferable, since a more definite meniscus is obtained.

reservoir, external connection to the reservoir of the inner tube being made by means of a short glass tube passing through the top of the outer reservoir, to which it is sealed on the outside as shown. The two liquids are arranged with the light one in the inner tube, the surface of separation or meniscus being near the upper end; and pressure readings are obtained in terms of the movement of the meniscus down the central tube. The pressure difference should be applied so that the higher positive pressure acts on the inner reservoir, thus giving a downward movement to the meniscus.

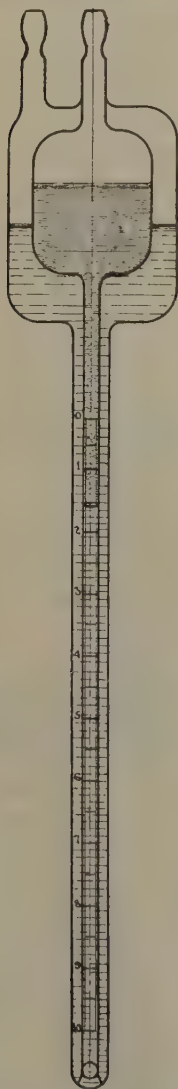


FIG. 82. — Two-liquid differential manometer (concentric tubes).

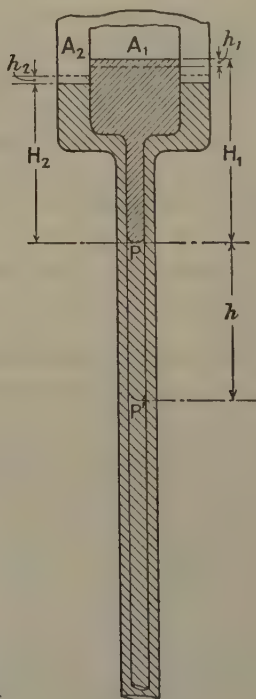


FIG. 83.

The relation connecting the applied pressure difference in inches of water with the movement of the meniscus can easily be deduced as follows :—

In Fig. 83, let A_1 and A_2 be the areas of the inner and outer reservoirs respectively, and a the area of the central tube. Also, in the zero equilibrium position before a pressure difference is applied, let the boundary between the lighter and heavier liquids be at a point P in the central tube, such that the free levels

in the two reservoirs are at heights H_1 and H_2 above P as

shown. Also, let σ_1 and σ_2 be the specific gravities of the lighter and heavier liquids respectively.

From a consideration of the equilibrium of the liquids in the inner and outer tubes at the level of the point P, we have the equation

$$H_1\sigma_1 = H_2\sigma_2. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Now let a pressure p inches of water be applied to the inner reservoir. The meniscus will move down a distance h to the point P', and at the same time the level in the inner reservoir will fall and that in the outer reservoir rise, by amounts h_1 and h_2 respectively.

The equilibrium condition for the level P' now becomes

$$p + (H_1 - h_1 + h)\sigma_1 = (H_2 + h_2 + h)\sigma_2. \quad . \quad (5)$$

The volume of liquid displaced in each reservoir must be equal to the volume of liquid displaced in the inner tube. That is

$$ah = A_1h_1 = A_2h_2.$$

Hence we may write

$$\left. \begin{aligned} h_1 &= \frac{a}{A_1} h, \\ h_2 &= \frac{a}{A_2} h. \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Substituting the relationships of equations (4) and (6) in (5), we obtain

$$p + h\sigma_1 \left(1 - \frac{a}{A_1}\right) = h\sigma_2 \left(1 + \frac{a}{A_2}\right),$$

$$\text{or} \quad p = h \left[(\sigma_2 - \sigma_1) + \frac{a(A_1\sigma_2 + A_2\sigma_1)}{A_1A_2} \right], \quad . \quad (7)$$

which is the relation between the applied pressure difference p in inches of water and the corresponding displacement h of the meniscus.

The magnification is obviously equal to the reciprocal of the term in square brackets on the right-hand side of (7); and from this it is evident that, for a manometer of given dimensions, the more nearly equal the specific gravities of the two liquids used, the greater will be the magnification. Unfortunately, the

magnification that can be usefully employed in practice cannot be increased indefinitely without the introduction of serious disadvantages and causes of inaccuracy. If σ_1 and σ_2 differ only by a small amount (5 per cent. or less) the instrument must have long tubes in order to give readings over a reasonable range of pressure, and the movement will be sluggish. Moreover, when the densities are too nearly equal, trouble is experienced with the meniscus, which becomes irregular in form and tends to break up. It is not advisable to aim at a magnification greater than 10 or 12, under which conditions the gauge should prove fairly satisfactory. A further factor that sensibly influences the performance of the instrument is the bore of the tubing, which should be neither so large that the shape of the meniscus is under insufficient capillary control, nor so small that the capillarity forces are excessive, in which case the liquid column will break if the pressure difference applied to the instrument is subject to rapid changes or fluctuations.

The choice of suitable liquids is also a matter of some importance. As stated above, the primary requirements are that they should neither mix nor be soluble in one another. It is desirable also that the meniscus should be of good shape and that the liquids should not adhere strongly to the walls of the tube when the surface of separation moves. Moreover, since the indications of the instrument are dependent upon the densities of the two liquids, the latter should as far as possible have low and approximately equal coefficients of thermal expansion, so that the calibration does not alter appreciably with temperature changes. Finally, volatile liquids should preferably not be employed, since they cause progressive zero changes as evaporation of one or both occurs.

Various liquids have been tried in gauges of this type, but none has been found to give entire satisfaction. The author has obtained good results with a gauge having a magnification of 10, in which benzyl alcohol and an aqueous solution of calcium chloride were used; the former liquid was coloured by the addition of a small quantity of aniline black, soluble in oil, in order to throw into greater prominence the surface of separation of the two otherwise colourless liquids. A number of instruments of this type using these liquids have been employed by various workers, and have been found satisfactory. The

advantage of using a liquid such as calcium chloride solution is that its specific gravity can be varied by the amount of the solid salt added, so that, when used in conjunction with a liquid of fixed specific gravity (in this case benzyl alcohol), different degrees of magnification can be secured. In the gauge under consideration, the specific gravity of the alcohol was 1.048, and that of the heavier calcium chloride solution was adjusted to give a magnification of 10. The variation in the calibration of the instrument with temperature was found to be about 3 per cent. for a temperature range of 12° C., the magnification being higher by this amount at the lower temperature. Errors due to this cause can to a large extent be eliminated by immersing the gauge in an outer vessel containing water, whose temperature is always adjusted to a convenient constant value before readings are taken. An alternative and perhaps easier course is to apply corrections determined by calibrating the gauge at different temperatures. Since, to a first order, the coefficients of thermal expansion of the liquids will be linear, it is sufficiently accurate to assume a linear variation of calibration with temperature. Adequate information for the determination of the correction factor for any temperature will therefore be obtained by calibrating the gauge at two temperatures, which should preferably be chosen at the extreme ends of the range to which the instrument is likely to be subjected.

Before the liquids are introduced into the manometer, it is advisable to shake them well together in a large flask and to allow them to separate out. Each should then be filtered in order to remove small bubbles of the other, and poured separately into its appropriate reservoir. In this manner, each of the two liquids is saturated with the other before being introduced into the gauge glass, so that no subsequent changes of density (and hence of calibration) are likely to occur from diffusion and interchanges of water content at the common surface. In addition to this precaution, it is essential—and the importance of this point cannot be over-emphasised—for the glass-work to be thoroughly clean chemically before the liquids are introduced. The slightest trace of grease on the interior of the inner tube will spoil the definition of the meniscus, and cause difficulty in reading its exact height. The improvement in the working of the manometer and in the appearance of the meniscus after a dirty

instrument has been thoroughly cleaned is remarkable. Cleaning is best effected by means of strong nitric acid and alcohol, or strong sulphuric acid and potassium bichromate.* The reagents should then be removed by washing with distilled water and pure alcohol; the final drying may be accelerated by blowing warm air through the glass-work.

The following internal dimensions are suggested for the various elements of a gauge of this type :

Diameter of outer reservoir	.	.	3.5 inches.
Diameter of inner reservoir	.	.	2.5 „
Diameter of outer tube	.	.	0.5 inch.
Diameter of central tube	.	.	0.2 „

The length of the tubes will naturally be determined by the range of pressure that the gauge is intended to cover. With a magnification of 10, a length of about 24 inches should be sufficient, corresponding to a maximum pressure reading of about 2.3 inches of water, and allowing for the necessity of obtaining a zero reading slightly below the top of the inner tube. For pressures greater than this, sufficient accuracy will usually be obtained by the use of an ordinary U-tube water manometer. For strength, it is advisable to fuse the bottom of the inner tube to the outer tube, and to make provision for the flow of liquid by piercing about four holes in the walls of the inner tube at its lower end; the total area of the holes should be at least equal to the area of the inner tube. A scale of inches and tenths, or preferably millimetres, should be engraved on the inner tube for observation of the position of the meniscus.

In a more simply constructed but somewhat less reliable form of this gauge, illustrated in Fig. 84, the two reservoirs and the tubes leading from them are arranged side by side instead of concentrically. Care must be taken not to tilt this instrument whilst observations are in progress, as appreciable movements of the meniscus will be caused by comparatively small inclinations. The formulæ derived for the type with concentric tubes apply, without alteration, to this gauge also.

* Nitric acid and alcohol are probably the best cleansing agents, but they must be applied cautiously to avoid damage to the glass-work due to too vigorous reaction.

Other Types of Manometers.—From the gauges described above, it will be possible to select, at a moderate cost, an instrument suitable to most of the conditions that will have to be met in practice. It must not, however, be thought that the types mentioned represent all the instruments worthy of consideration. For further information, reference may with advantage be made to a paper by Hodgson⁸ which contains much interesting and useful matter.

Very Sensitive Manometers.—Various types of manometer having a sensitivity ten or more times that of the most sensitive of those already described—the Chattock—have been designed for special purposes. Such instruments are more suitable for laboratory than for general industrial use, since they are delicate scientific instruments that require careful handling, setting up, and manipulation, and have not the speed in use of the less sensitive types. Brief descriptions follow of a few of these extra-sensitive manometers.

The first, which is due to Hodgson,⁹ is an inclined-tube gauge having a large reservoir fitted with a cylindrical displacer, partly submerged and partly above the liquid in the reservoir, actuated by means of a vertical micrometer screw. The liquid meniscus in the inclined tube is maintained at a fixed mark, any departure therefrom, due to an applied pressure difference, being restored by an appropriate vertical motion of the displacer in the reservoir. The meniscus is specially illuminated, and is viewed through a microscope having a magnification of 60. The calibration is determined by the relation between the internal diameter of the reservoir and the external diameter of the displacer, so that the instrument may be used as a fundamental standard. The manometer is relatively

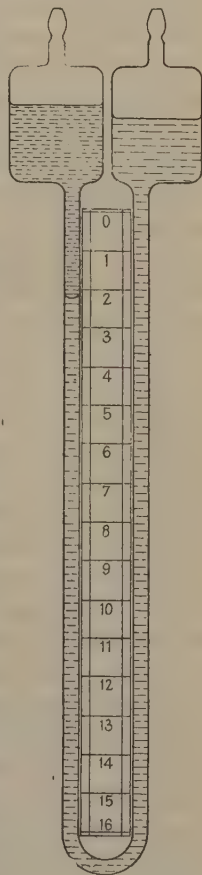


FIG. 84. — Two-liquid differential manometer (parallel tubes).

expensive, but it is robust and portable, and the sensitivity claimed for it is about 0.00001 inch of water. This high precision is attained by the use of a displacer having a small diameter in relation to that of the reservoir.

Another type of sensitive manometer is that designed by the author for the calibration of the standard pitot-static tube at low speeds.¹⁰ Essentially, this is a Chattock manometer with the central cup arrangement replaced by a piece of glass capillary tubing. The two end-cups and the connecting tube are filled with the manometric liquid (xylol was found to be the most suitable), and a small air bubble is introduced into the capillary. When the pressure in one cup exceeds that in the other, the air bubble, which fills the bore of the tube, tends to travel along the capillary, but the operator prevents it from doing so by tilting the cups by means of the usual micrometer arrangement. The applied tilt is a measure of the difference of pressure.

This description is merely a bare outline of the principle of the instrument. For full details of the various expedients adopted to overcome the serious difficulties that occur when very low differential pressures have to be measured, the reader is referred to the original paper. It must suffice here to state that this instrument can be made to respond to a pressure difference equivalent to a head of about 0.000005 inch of water.

Falkner,¹¹ by swinging the glassware of a Chattock gauge through a large angle, so that it was at an angle of almost ninety degrees to the length of the tilting frame, i.e., almost parallel to the axis of tilt, obtained a sensitivity about equal to that of the type last mentioned. A further ingenious innovation introduced by Falkner was to fill the cups, the horizontal limbs, and the lower part of the central vessel (see Fig. 74) with a lighter liquid than that used in the upper part of the latter, so that the controlling force on the surface of separation, or the bubble, which forms the indicator in this type of gauge, was considerably less than in the conventional Chattock instrument. In this way, a much more mobile bubble was obtained, which responded with greater movements to very small pressure differences. The liquids used by Falkner were xylol for the main filling, and water.

A novel type of instrument, of much the same sensitivity,

was shown at the Physical Society's Thirtieth Exhibition of Scientific Instruments, in January, 1946. No description has yet been published beyond a note in the catalogue of exhibits. Briefly, the principle is as follows: The pressure difference is applied to a liquid-sealed bell, or to a diaphragm, carried by a balance arm, on which is also mounted a pair of differential light shutters. When the arm moves under a change of pressure, the shutters alter the relative illumination of two photo-cells. These form part of an electrical circuit such that the current caused by the motion of the arm is used to restore the equilibrium of the latter, and, at the same time, is observed and serves as a measure of the differential pressure.

Other types of very sensitive manometer are described in References 12 and 13.

Calibration of Manometers.—In calibrating any type of manometer against a standard, it is convenient to take simultaneous readings of the two instruments when one side of each is connected to the same source of pressure, which can be varied at will, the other sides being open to atmosphere or to any constant pressure. For the standard, it is necessary to use an instrument, such as the tilting micromanometer, from whose indications, which depend solely upon its mechanical construction and dimensions, the applied pressure differences can be accurately calculated.

The simplest method of conducting the calibration is to open one side of each gauge to atmosphere, and to connect the other sides together through a T-piece and tap, by means of which the pressure can be applied to both gauges simultaneously. This method is subject to three serious disadvantages. In the first place, when the tap on the variable pressure side is shut, the connecting tubes will enclose a small volume of air; and the small local atmospheric temperature fluctuations which are continually occurring will produce serious changes in the readings of both gauges, so that simultaneous observation of both under steady conditions will not be possible. A similar effect arises from the alteration in the enclosed volume while the standard instrument is being adjusted: this volume should be sufficiently large to render negligible the pressure changes to which it is subjected by the movement of either gauge. Finally,

when very small pressure differences are being measured, the atmosphere cannot be considered as a source of constant pressure ; the small local variations of pressure due to draughts and similar causes would make the attainment of steady conditions impossible if this crude method of calibration were adopted.

A logical extension of the above method in the light of these considerations leads to the system of connections for manometer calibrations shown in Fig. 85, by means of which a sufficient degree of steadiness is secured. Corresponding sides of both gauges are connected in parallel to two reservoirs, which are completely immersed in a water bath to reduce the effect of

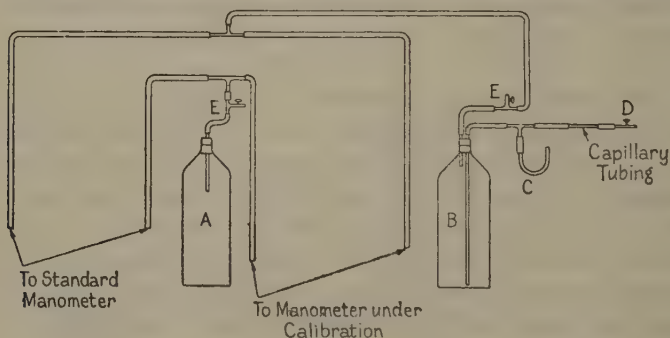


FIG. 85.—System for calibration of manometers.

temperature variations. One of these reservoirs, A, forms the constant-pressure side, and has no other connections; the pressure in the other, B, can be varied through a second tube leading into it. It is advisable, if possible, to arrange the gauges so that a movement in the positive direction is obtained by decreasing the pressure in B below atmospheric ; the pressure can then be varied by sucking some of the air out of the reservoir. In this way, the enclosed air is not warmed, as it would be if the pressure were raised by blowing. In order to avoid sudden large variations of pressure while air is being sucked out of B, and to provide a means of controlling the applied pressure difference with some degree of precision, it is advisable to have a length of about 4 inches of capillary tubing in the circuit. A further useful addition is the U-tube water manometer C, from which the approximate pressure in B can be observed, so that the tap

D can be closed when the desired pressure difference is obtained. It is convenient also to provide by-passes EE and taps, so that the two reservoirs and both sides of the gauges can be opened to atmosphere to enable zero readings to be taken when required.

For the reasons stated, the two reservoirs must be of considerable volume: quart Winchester flasks are about the minimum size that should be employed. Some difficulty is usually experienced in making a flask of this type leak-tight. The writer has found that an effective method is to insert the cork or rubber bung with the necessary glass connecting tubes, and then to tie a cylinder of stout paper tightly round the neck and to pour in sufficient molten paraffin wax to cover the bung and holes through which the glass tubes enter. When solid, the wax forms an efficient seal.

By means of this arrangement, calibrations can be rapidly and accurately performed, preferably by two observers taking simultaneous observations on the two gauges.

Testing for Leaks.—In all air-flow measurements involving observations of pressure, care should be taken to ensure the absence of leaks in the various instruments, connections, and lengths of rubber tubing employed. Glass joints, such as taps and the caps of the two vessels of the tilting manometer, should be greased with a thin coating of vaseline, and all parts of the pressure circuit should be tested for leaks. Particular attention should be paid to the concentric type of pitot-static tube, which is liable to develop internal leaks from one tube to the other, and to all rubber tubing, which is apt to perish and crack, especially at the ends where it is stretched over metal or glass connecting pieces.* Even new rubber tubing should not be regarded as above suspicion. The author has found appreciable porosity in rubber tubing taken from stock. It is advisable therefore to test separately each length of rubber tube used.

A simple leak tester is shown in Fig. 86. It consists of a small U-tube water gauge with two connecting tubes to one of its limbs. One end of the tube or connection to be tested is connected to the tube A by means of a piece of rubber tubing, which has previously been ascertained to be itself free from

* This tendency can be greatly reduced if a touch of glycerine is applied to the outer surface of the glass or metal before the joint is made.

leaks; the other end is closed, and a small pressure (about $1\frac{1}{2}$ inches to 2 inches of water) is applied by blowing gently through the tube B and closing the tap before the pressure is released. Any small leaks will now be indicated by a gradual creep of the water levels in the two limbs. In order better to observe the motion, it is convenient to have a number of horizontal lines, about $\frac{1}{4}$ inch apart, ruled on the board to which the U-tube is fixed. The function of the two bulbs in the leak tester is to reduce the risk of ejecting water from the limbs if excessive pressure is inadvertently applied.

In tests of concentric pitot-static tubes for leaks, each tube should be dealt with separately: no difficulty will be found in the case of the impact tube, since its end can be effectively closed by pressing the ball of the thumb against it. In order to test the static tube, the static holes are best closed by winding tightly round the portion of the tube in which the holes are situated three or four thicknesses of the thin rubber tape used for electrical insulation.

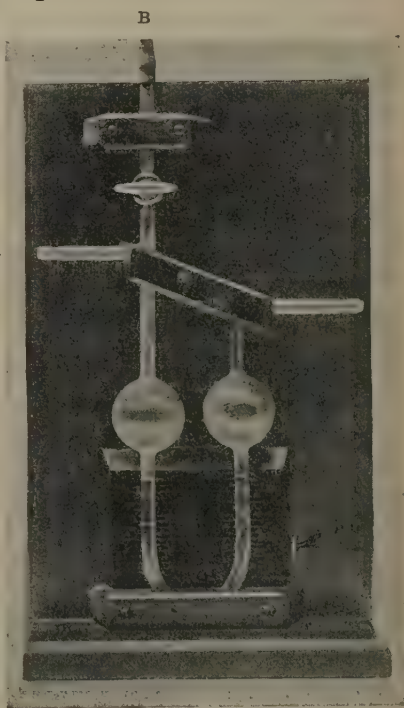


FIG. 86.—Leak tester.

As regards choice of rubber tubing, it will be found that for general use fairly thick-walled tube of red rubber, having a bore of $\frac{3}{16}$ inch and an external diameter of about $\frac{3}{8}$ inch, will be suitable. Such tubing will usually be found to be non-porous and considerably more permanent than thin black tubing, which is not to be recommended for this type of work. It is probably also to be preferred to thick-walled pressure tubing, on account of the greater ease of making connections.

Theory of the Curved-Tube Manometer.—We shall assume that the manometer is intended to be used with a standard pitot-static tube. If, then, p is the pressure difference in pounds per square foot that this instrument transmits to the manometer when used in an air stream of v feet per second, we have the equation

$$p = \frac{1}{2}\rho v^2.$$

If h is the corresponding height in inches through which the level of the manometric liquid rises above its zero level (we shall take the reservoir to be sufficiently large to warrant the assumption that all the change in level occurs in the curved tube),

$$5.2 \frac{h}{\sigma} = \frac{1}{2}\rho v^2,$$

where σ is the specific gravity of the manometric liquid.

Hence
$$v\sqrt{\rho} = \sqrt{\frac{10.4h}{\sigma}}. \quad \dots \dots (8)$$

In Fig. 87, let O be the zero position of the liquid meniscus against the upper side of the curved tube, and take O as the

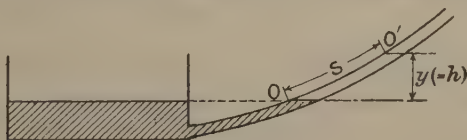


FIG. 87.

origin of co-ordinates. The condition to be fulfilled is that, for a given increment in $v\sqrt{\rho}$, the meniscus is to move a given distance along the tube to O' . Hence the equation of the curve of the tube is

$$s = Av\sqrt{\rho}, \quad \dots \dots (9)$$

where s is the distance moved measured along the tube from O and A is a convenient constant.

Hence, from (8),

$$s = A\sqrt{\frac{10.4h}{\sigma}} = B\sqrt{h}.$$

If x and y are the co-ordinates of O' , measured from the origin O ,

$$s = B\sqrt{h} = B\sqrt{y}.$$

Differentiating, we have $ds = \frac{B}{2\sqrt{y}} dy$,

$$\text{i.e.,} \quad \sqrt{dx^2 + dy^2} = \frac{B}{2\sqrt{y}} dy,$$

$$\text{or} \quad \frac{dx^2}{dy^2} = \frac{B^2}{4y} - 1.$$

$$\text{Hence} \quad dx = \sqrt{\frac{B^2}{4y} - 1} dy,$$

and the curve to which the tube is to be bent is obtained by integrating this equation. This gives

$$x = \int \sqrt{\frac{B^2}{4y} - 1} dy + C.$$

The right-hand side is easily evaluated by the substitution $y = \frac{B^2}{4} \sin^2 \theta$, and the final expression will be found to be

$$x = \frac{B^2}{4} \sin^{-1} \sqrt{\frac{4y}{B^2}} + \frac{B}{2} \sqrt{y \left(1 - \frac{4y}{B^2} \right)}. \quad (10)$$

The constant of integration C is found to be zero by putting x and y equal to 0 simultaneously at the origin.

If, therefore, the axis of the tube is bent to the form represented by equation (10), the gauge will show equal movements of the liquid meniscus along the tube for equal changes in $v\sqrt{\rho}$. An instrument constructed on these lines requires calibration against a standard manometer, just as the ordinary inclined-tube gauge does.

It is clear from the above analysis that, if the instrument is furnished with a velocity scale, this can only be accurate at one value of the air density ρ , which must be specified. For any other density ρ' , the readings must be multiplied by $\sqrt{(\rho/\rho')}$; that is, if the scale is said to be accurate for air (assumed dry) at a temperature T absolute and at a pressure p , then the readings observed at a temperature T' and a pressure p' must be

multiplied by $\sqrt{\frac{T'p}{Tp'}}$ in order to obtain the true velocity.

REFERENCES

1. Chattock, *Phil. Mag.* (Series 6), **1** (1901), p. 83.
2. Duncan, *Journ. Sci. Insts.*, **4** (1927), p. 376.
3. Pannell, *Engineering*, **96** (1913), p. 343.
4. Hunsaker, *Smithsonian Misc. Coll.*, **62** (1916), p. 27.
5. Parkin, *University of Toronto School of Engineering Research*, Bulletin No. 2, Paper No. 1 (1921).
6. Report of the National Physical Laboratory (1921), p. 170.
7. *Engineer*, **151** (1931), p. 248.
8. Hodgson, *Proc. Inst. Marine Engineers*, **36** (1924), p. 487.
9. Hodgson, *Journ. Sci. Insts.*, **6** (1929), p. 153.
10. Ower, *Phil. Mag.* (Series 7), **10** (1930), p. 544 ; also *Reports and Memoranda of the Aeronautical Research Committee*, No. 1308 (1930).
11. Falkner, *Reports and Memoranda of the Aeronautical Research Committee*, No. 1589 (1934).
12. Fry, *Phil. Mag.* (Series 6), **25** (1913), p. 494.
13. Reichardt, *Zeitschrift fur Instrumentenkunde*, **55** (1935), p. 23.

CHAPTER X

METHODS OF FLOW MEASUREMENT BASED UPON THE RATES OF COOLING OF HOT BODIES

THE relation between the rate of loss of heat of a heated body and the speed of flow of a fluid in which it is immersed has formed the subject of a considerable amount of research. Attention has mainly been concentrated on problems connected with the use of electrically heated wires as anemometers ; but there have also been investigations of the behaviour of a specially constructed alcohol thermometer when used for the same purpose.

The laws governing the convective cooling of hot cylindrical wires by streams of fluids are now, empirically at all events, fairly well understood ; and measurements of air flow have successfully been made in large numbers of cases by methods based on these laws. Such methods possess distinct advantages, chief among which is the possibility of obtaining high sensitivity for low rates of flow ; but certain manipulative difficulties have hitherto prevented them from coming within the domain of general practical anemometry. Nevertheless, for special work, particularly under laboratory conditions, instruments using these principles may be made to yield excellent results ; and for this reason it has been thought desirable to include a comparatively brief discussion of the subject here. Further information may be obtained by reference to the original papers quoted in the bibliography at the end of this chapter, and to other sources cited in those papers themselves.

The Cooling of Heated Wires.—The most extensive researches into the laws of cooling of electrically heated wires are probably those of L. V. King.¹ Developing the theoretical treatment initiated by Boussinesq,² King showed that the relation between the heat loss H per unit length of a cylinder of diameter d , maintained at a temperature T above that of the fluid in which it is immersed, could be represented very nearly

by the equation

$$H = KT + \sqrt{(2\pi KS\sigma d)} V^{\frac{1}{2}}T, \quad . \quad . \quad . \quad (1)$$

where K is the thermal conductivity of the fluid, S its specific heat at constant volume, σ its density, and V its velocity.*

King showed that this expression might be expected to hold down to a value of Vd of 0.0187, V being expressed in centimetres per second and d in centimetres; which is equivalent to a limiting speed of about $\frac{1}{4}$ foot per second for a wire of diameter 0.001 inch. It should be noted—and attention is drawn to this point by King—that the theoretical relationship (1) is derived on the assumption that V is the total speed of the fluid past the wire. Now a heated wire placed in air will give rise to convection currents which will assist in the cooling of the wire; and at very low velocities these currents may have an appreciable effect in modifying the true speed of the air past the wire. It is probable, therefore, that in practice the lower limit of the speed range over which (1) is applicable will be somewhat higher than its value as given by the above-mentioned criterion.

Equation (1) may be written in the form

$$H = B\sqrt{V} + C, \quad . \quad . \quad . \quad . \quad (2)$$

where B and C are functions of temperature and the properties of the fluid and the wire mentioned above. For a wire of given dimensions, maintained at a constant excess temperature and always used in the same fluid (air, for example), B and C are constants, and may be calculated for the prescribed conditions. In order to verify his theoretical deductions, King undertook a comprehensive experimental investigation of the heat losses from electrically heated platinum wires ranging in diameter from about 0.001 inch to 0.006 inch, when moved on a rotating arm through still air at speeds between about 0.5 and 29 feet per second. He found that, within these limits, the results could be accurately expressed by an equation of the form of (2); and further work carried out by him and other investigators has shown that this equation holds also for speeds up to at least 100 feet per second. The experimental values of B and C determined by King were in moderately good agreement with those calculated from theory.

* The fluid is assumed to be flowing in a direction perpendicular to the axis of the cylinder. Also, note that $\sigma = g\rho$, where ρ is the density as defined and used in previous chapters.

The Use of Electrically Heated Wires in Anemometry.

—If a current i ampères is flowing in a wire of resistance R ohms, the heat generated per second is equal to i^2R/J calories, where J is the mechanical equivalent of heat in joules per calorie. Hence, for an electrically heated wire equation (2) becomes

$$\frac{i^2R}{J} = B\sqrt{V} + C. \quad . \quad . \quad . \quad (3)$$

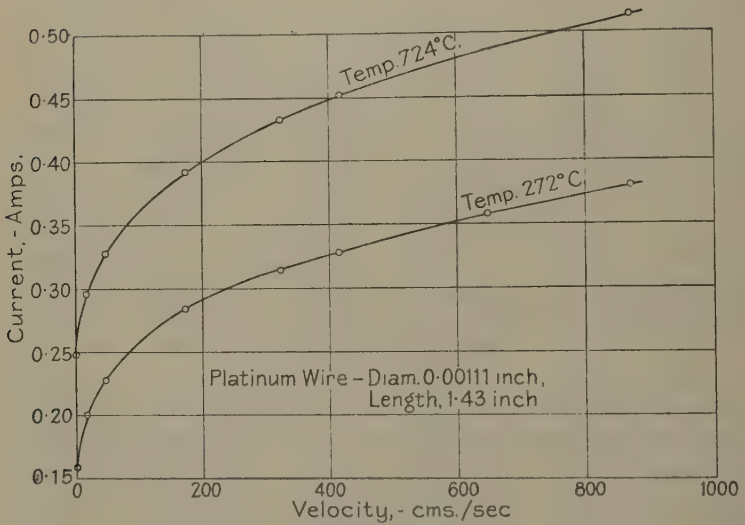


FIG. 88.

Now if the temperature of the wire is maintained constant, its resistance R will also be constant and, if we confine our attention to a particular fluid, (3) becomes

$$i^2 = K_1\sqrt{V} + K_2, \quad . \quad . \quad . \quad (4)$$

where K_1 and K_2 are constants for a given wire.

This relation between current and air speed for a wire maintained at a constant temperature in air is shown graphically in Fig. 88, which is taken from King's results. The upper curve was obtained with the wire at a temperature of 724° C. ; for the lower curve the temperature was 272° C. These curves show well the special characteristic of the hot-wire anemometer, namely, the high degree of sensitivity at low air speeds. It will

be seen that the rate of change of current with air speed is much more rapid at the lower speeds than at the higher, and also that the higher the temperature of the wire the higher is the sensitivity over the whole range.

Turning once more to equation (4), we see that, if i_0 is the value of the current necessary to maintain the wire at a prescribed constant temperature in still air (i.e., when $V = 0$), $K_2 = i_0^2$, and we may write

$$i^2 = i_0^2 + K\sqrt{V} \quad . \quad . \quad . \quad . \quad (5)$$

as the relation between current and air speed in its most useful practical form. The heat loss represented by $i_0^2 R$ is the total loss due to radiation and convection in still air. For the reasons stated above, when V is so low that it becomes comparable with the speed of the free convection currents due to the hot wire in still air, equation (5) requires modification; but for ordinary purposes this will not be necessary. Some calculations made by King indicate that for a wire of 0.003-inch diameter the free convection current has a velocity of the order of $\frac{1}{2}$ foot per second when the wire is at 1000°C. , and about $\frac{1}{4}$ foot per second at 200°C. If, therefore, a wire is to be used to measure low speeds of this order, the calibration should be determined from tests at such speeds.

Effect of Variation of Atmospheric Conditions.—The variable most affecting the relation between the current in a given wire and the air speed is atmospheric temperature. From what has been stated above, it will be clear that equation (5) only holds when the wire is maintained at a constant temperature, and that the values of i_0 and K , once determined, only relate to a specified *excess* temperature of the wire over that of the surrounding air. Any change in the air temperature will therefore produce a change in the calibration curve of a given wire. Changes in air density due to variations of atmospheric temperature also need consideration. If we assume that the other quantities occurring in equation (1), namely, the specific heat and conductivity of the air, are not affected by variations in atmospheric temperature, and if we retain the atmospheric temperature as a variable, equation (1) may be reduced to the form

$$i^2 R = C_1(T_w - T_a) + C_2(T_w - T_a)\sqrt{V}\sigma, \quad . \quad (6)$$

where $T = T_w - T_a =$ wire temperature — atmospheric temperature,

$$C_1 = \text{constant} = i_0^2 R / (T_w - T_a),$$

$$\text{and } C_2 = \text{constant}.$$

Equation (6) may be written

$$\sqrt{V\sigma} = \frac{i^2 R - C_1(T_w - T_a)}{C_2(T_w - T_a)} \quad \dots \quad (7)$$

Treating T_w as constant (hence R is also constant) and $(V\sigma)$ as a single variable, and differentiating with respect to T_a , we have

$$\frac{1}{2\sqrt{V\sigma}} \frac{d(V\sigma)}{dT_a} = \frac{i^2 R}{C_2(T_w - T_a)^2} \quad \dots \quad (8)$$

Equations (7) and (8) may be combined to give, when $i_0^2 R / (T_w - T_a)$ is written for C_1 ,

$$\frac{d(V\sigma)}{(V\sigma)} = \frac{2dT_a}{T_w - T_a} \frac{i^2}{(i^2 - i_0^2)}, \quad \dots \quad (9)$$

which shows the fractional change in $V\sigma$ for a change in air temperature given by dT_a .

For a platinum wire 0.0025 inch in diameter used at 1000° C., King found $i_0^2 = 0.5$ (amps.)² approximately, whilst for a range of velocities from 16.4 to 92 feet per second i varied between 1 and 2 ampères. If these values are substituted in (9), it will be seen that a change of air temperature of $\pm 2^\circ$ C. produces a change in $(V\sigma)$ of less than 1 per cent. over this range of air speed. If the air temperature changes by more than this amount a correction can easily be applied by the use of (9).

The other effect of a variation in T_a , to which reference has already been made, is the accompanying change in the density σ . In this it is associated also with changes of barometric pressure and humidity. The equations already given show that the hot wire really measures the product of velocity and density. If, therefore, a given wire is calibrated in terms of velocity, this calibration will only hold for a given value of σ . It will readily be seen from equation (6) that, if V is the velocity corresponding, according to the calibration curve for density σ_0 , to a current i , the same current will correspond, when the density is σ_1 , to an

air speed $V\sigma_0/\sigma_1$. The densities σ_0 and σ_1 are easily calculable from observations of air temperature and barometric pressure. Relative humidity may also be taken into account if desired, but its effect will rarely be important.

In the above analysis it has been assumed that changes of atmospheric conditions do not affect the other properties of the air appearing in equation (1), namely, the specific heat and the thermal conductivity. This assumption will be true for all practical purposes, even under extreme variations in natural conditions; but for special work, such, for example, as the measurement of the flow of hot air, the possibility of variations in these properties should not be overlooked.

Methods of Measurement in Hot-Wire Anemometry.—

It has been shown above that, if a wire is maintained at a constant temperature (and consequently also at a constant resistance), the current varies with the air speed according to the law expressed by equation (5). Alternatively, if the current passing through a wire is kept constant, its temperature and resistance will vary with the air speed in a definite manner. There are, accordingly, two possible methods of measurement, which we may term respectively the constant-resistance

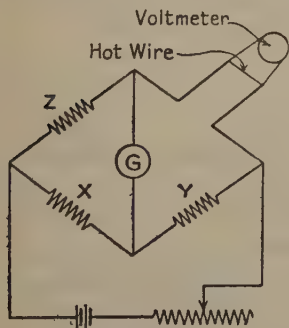


FIG. 89.

method and the constant-current method. Experiments by Simmons and Bailey³ show that, except at low wire temperatures (less than about 150°C.), the constant-current method leads to higher sensitivity. In practice, the constant-resistance method is, however, probably to be preferred; partly on account of the fact that it is somewhat easier accurately to maintain a constant resistance than a constant current, and partly because the constant K in equation (5) can then be determined by a calibration at a single air speed, i_0 being the current necessary to bring the wire to the desired resistance in still air. On the other hand, the constant-current method, as we shall see, allows a direct reading of air speed to be obtained.

(a) *Constant-Resistance Methods.*—In the simplest case, the

hot wire forms one arm of a Wheatstone bridge (see Fig. 89), the opposite arm Z consisting of a manganin resistance having practically zero temperature coefficient. The hot wire is of some pure metal, such as nickel or platinum, whose electrical constants are accurately known; the resistances forming the other three arms of the bridge can accordingly be arranged so that, when the bridge is in balance, the resistance, and consequently the temperature, of the hot wire is at its desired value. Any change in the speed of the air stream past the hot wire will then necessitate a change in the heating current in order to restore the temperature of the wire to its former value. When the wire is again at this temperature, the bridge will once more be in balance; and the change in current will be a measure of the air speed.

The current may be measured in a number of ways. An ammeter of the requisite sensitivity and range may be connected in series with the hot wire, so that the current passing through it can be measured directly; or a high-resistance voltmeter connected across the wire may be used as a measure of the current, and hence of the wind speed, since the wire always has

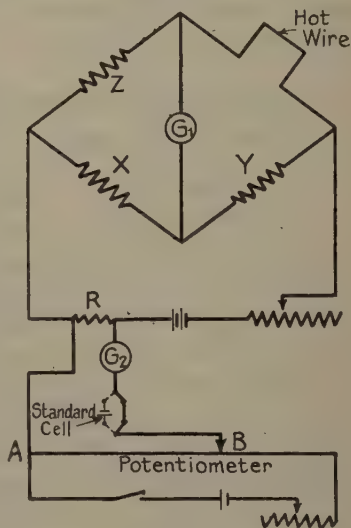


FIG. 90.

the same resistance. Alternatively, the current in the external circuit may be measured by an ammeter. For the highest accuracy, the scheme illustrated in Fig. 90, which was used by Simmons and Bailey,³ is recommended. Here the current in the external circuit is measured by a potentiometer method; the voltage drop across a standard known resistance R , having a negligibly small temperature coefficient, is determined, and the current flowing can easily be calculated.* The standard

* An equally good alternative is to measure the voltage drop across the wire itself by taking the potentiometer leads to its two ends instead of to the standard resistance R . The latter is then not required.

cell shown in the diagram is used to calibrate the potentiometer directly in terms of voltage. Since no current flows when balance of the galvanometer G_2 is obtained during standardisation of the potentiometer, the voltage across the portion AB is equal to the known voltage of the standard cell. A small variable rheostat, as shown, is connected in series with the potentiometer cell in order that balance of the standard cell may always be obtained at the same potentiometer reading, in spite of fluctuations in the voltage of the potentiometer cell.

The usual arrangement of the arms of the bridge is to make the two ratio coils X and Y of equal resistance, and the manganin resistance Z equal to that of the hot wire when the latter is at its desired temperature. This value can be estimated beforehand from a knowledge of the temperature-resistance coefficient of the material of which the wire is composed. The ratio coils should be of the same material, preferably manganin.

As regards the temperature of the hot wire, it follows from equation (9) that the higher the temperature excess $T_w - T_a$, the smaller will be the errors due to changes of atmospheric temperature. If, therefore, it is desired to avoid the necessity for applying temperature corrections, the wire must be at a high temperature. King¹ advocated a working temperature of 1000°C ., for which, as we have seen, the errors in the measurements of V will not often exceed 1 per cent., if the initial calibration is performed at, say, 17°C . In order to maintain a high working temperature such as this, fairly large currents are required at the higher air speeds; and in these circumstances it may be desirable to use, as King did, a Kelvin double bridge in place of the ordinary Wheatstone net. A description of this form of bridge, which allows large currents to be passed through the anemometer wire without overloading the ratio coils, will be found in any textbook dealing with electrical measurements. It has the additional advantage of high sensitivity; and its readings are, to the order of accuracy required for hot-wire anemometry, independent of contact and lead resistances. The latter feature is of importance for some types of work, as it allows the hot wire to be mounted on a moving body, the current being led to it through brush contacts or slip-rings.

Although the use of a high working temperature has the

merits of increased sensitivity and the reduction of errors due to small changes of air temperature, a temperature as high as 1000°C . is not to be recommended for general purposes. In the first place, there is a risk of fusing the anemometer wire by the accidental passage through it of a comparatively small excess of current, especially when working at low air speeds. Secondly, there is an "ageing" effect, mentioned by King¹ and found by Simmons and Bailey³ to be quite appreciable with fine wires (0.001-inch diameter). This effect causes progressive changes in the calibration of a given wire, and there is evidence to show that it is more important the higher the working temperature. It seems, therefore, preferable to work with the anemometer wire at some lower temperature, say 400°C . to 500°C ., and either to correct if necessary for atmospheric temperature variations by the use of equation (9), or to adopt some form of circuit that automatically compensates for such variations.

King described a method by which the readings obtained from the Kelvin double bridge can be made to indicate velocity independently of the air temperature, to the degree of accuracy required in practice. Certain of the ratio coils, instead of being made of manganin, are made of the same material, in this case platinum, as the hot wire, so that they have the same temperature coefficient of resistance. These coils are exposed to the air stream so that they can readily attain its temperature.

A method of temperature compensation suggested by Davis⁴ is illustrated in Fig. 91. The three resistances X, Y, and Z are of manganin; and in the arm of the bridge containing the resistance Z there is a second coil P, which can be short-circuited by means of the key K. The connections are so arranged that when this coil is short-circuited the current through the bridge is automatically decreased about one hundredfold, so that only a very small current passes through the anemometer wire. The latter now acts as a platinum resistance thermometer; and, by movement of the contact C along the slide-wire SS, the bridge is balanced at a position depending on the temperature of the air flowing past the anemometer wire. If required, a scale of air temperature may be marked along SS. The contact is fixed at the balance-point on SS, and the key is now raised. This process increases the resistance of that arm of the bridge by the resistance P, which is constructed of

manganin and is therefore unaffected by ordinary temperature changes, and simultaneously sends a much larger current through the whole bridge, so that the hot wire is heated. This current is then adjusted by the rheostat until balance is again obtained. It will be clear that the resistance of the anemometer wire is then increased by an amount equal to the resistance of P . If the resistance-temperature law for the anemometer wire is linear, it follows that the wire is always heated to a constant temperature excess above the air temperature; and, actually, the departure from this condition is not sufficient to cause appreciable errors. Davis used a platinum wire of 0.006 inch diameter, heated to 108°C . above atmospheric temperature.

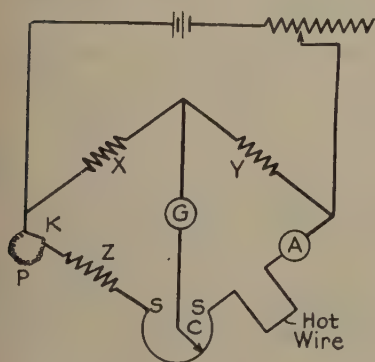


FIG. 91.

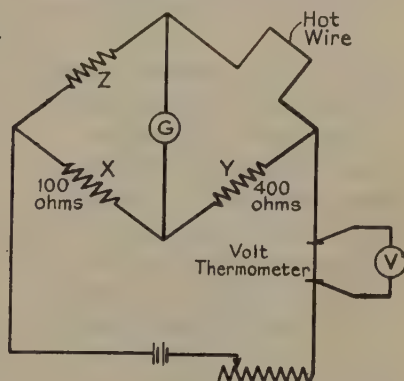


FIG. 92.

A valuable method of measurement, due to Callendar, has been described by R. O. King.⁵ In series with the external circuit of the Wheatstone bridge is a device called a volt-thermometer (Fig. 92), which consists simply of a length of fine platinum wire whose function is described below. In the bridge itself, the two ratio coils X and Y are of manganin, having resistances of 100 ohms and 400 ohms respectively; the standard resistance Z is 1 ohm, and the anemometer wire has a resistance of 4 ohms when hot. It will be clear therefore that no appreciable error is made in assuming that the current through the volt-thermometer is equal to that through the anemometer wire. Instead of observations of this current, measurements of the voltage drop across the volt-thermometer are made. As the external current is altered to maintain

balance of the bridge, so the heating effect of this current through the volt-thermometer wire changes its temperature, and consequently its resistance and the voltage drop across it. Callendar has shown experimentally that, over a considerable range, there is a linear relation between the resistance R of the wire and the voltage drop V ; and that, for a 3-inch length of 0.004-inch diameter platinum wire, this relation can be written

$$R = 0.607V + 1 \text{ ohms.} \quad (10)$$

Hence, if i is the current through the anemometer wire, since this current may be taken as flowing also through the volt-thermometer, we have

$$i = \frac{V}{R} = \frac{V}{0.607V + 1}. \quad (11)$$

Plotting this relation between i and V (Fig. 93), we see that V increases rapidly with i , the rate of increase being, in fact, somewhat faster than the square of the current. The physical significance of this is that as the current in the wire is increased, its temperature rises rapidly. Its resistance therefore increases at a proportionate rate, so that a rapidly rising voltage drop occurs across the wire. This point is well illustrated by the curve of Fig. 94, given by Simmons and Bailey,³ which shows the rapid rise of temperature with heating current in still air of a platinum wire 0.00105 inch in diameter and 3.07 inches long.

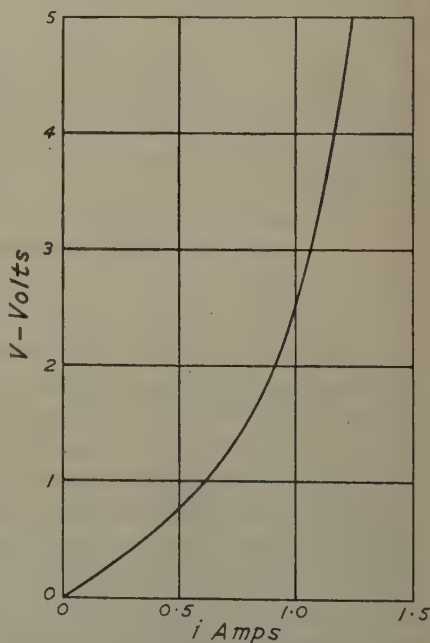


FIG. 93.

It will be seen therefore that, whereas in the ordinary method of measurement the air speed varies, according to

equation (5), approximately as the fourth power of the quantity observed, namely, the current in the anemometer wire, it can be made to vary as some power between 1 and 2 of the observed reading, if the latter is the voltage drop across the volt-thermometer wire instead of the heating current. A valuable increase in accuracy is thus afforded by this method. A further important feature is the possibility of extending considerably the range of air speed that can be observed with one measuring instrument—in this case, a voltmeter. For if two similar volt-thermometer wires are used in parallel in place of the single wire, only half the total current will flow through each; and consequently a full scale deflection of the voltmeter connected across these wires will correspond to twice the total current passing when one wire is used. That is, since the wind speed is approximately proportional to the

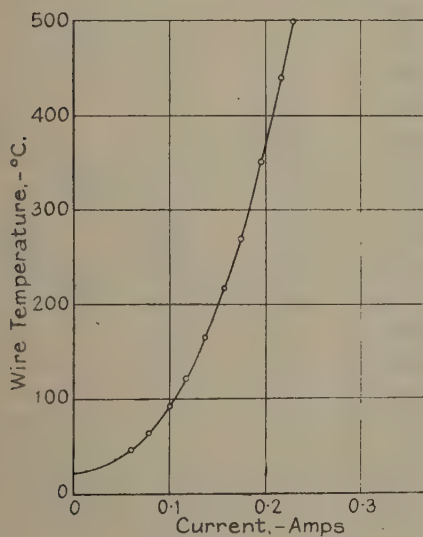


FIG. 94.

anemometer wire was of 0.004-inch diameter platinum and had a total length of 6 inches, arranged in four sections. The volt-thermometer consisted of four platinum wires, each 0.004 inch in diameter and 3 inches long, with the necessary connections so that one wire alone, or two, three, or four in parallel could be used, as required, to give the maximum accuracy for any

fourth power of the current, a given voltmeter deflection will correspond to roughly 2^4 or 16 times the air speed that it previously indicated. R. O. King⁵ claims that the accuracy attainable is well within 1 per cent. for all speeds up to about 300 feet per second. The factor limiting the indefinite extension of the scale in this way will be the maximum current that can safely be passed through the circuit.

R. O. King⁵ has described an anemometer working on these lines. The

particular air speed being measured. It was found that a single platinum wire of these dimensions would safely take a current producing a 5-volt drop across it. In order to prevent the loss of heat by convection from the volt-thermometer and to protect it from draughts, its four wires were enclosed in a fairly air-tight box. This arrangement was found to be satisfactory up to values of the total current of about 2 or $2\frac{1}{2}$ ampères. For measurements of high rates of flow, when the current attained the order of 5 ampères, it was found necessary to surround the box containing the volt-thermometer wires by a water-jacket, in order to maintain sufficiently constant temperature conditions at the wires. A five-range voltmeter, reading up to 5 volts, was used with this anemometer.

(b) *Constant-Current Method*.—A hot-wire instrument based upon this method of measurement has been designed by MacGregor Morris, and has been put on the market for the measurement of low air speeds. Fig. 95 shows the essential features of the circuit used. The anemometer wire A is of nickel, 0.005 inch in diameter and about 6 inches long, having a cold resistance of about 0.9 ohm. B is a similar nickel wire, and C and D, which form the other arms of the bridge proper, are manganin resistances of 1 ohm each. The galvanometer used in the constant-resistance circuits as a null-reading instrument to indicate balance of the

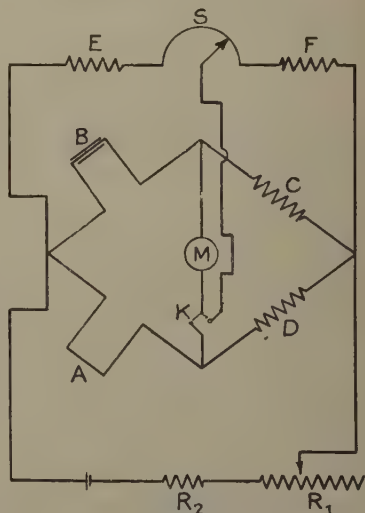


FIG. 95.

bridge is now replaced by the reading milliammeter M. If we assume for the moment that the wires A and B are exactly similar, as are also the resistances C and D, we see that the current in the external circuit will always divide into two equal currents through the arms BC and AD. The bridge will be in balance in still air when the resistances of A and B are made equal to those of D and C, that is, when the

current in each branch heats the nickel wires sufficiently to raise their resistances from 0.9 to 1.0 ohm : no current will then flow through the milliammeter. If now the wire A is exposed to the air current, its resistance will drop and the bridge will no longer be balanced. The milliammeter then registers an " out-of-balance " current, which is used as a measure of the air speed.

A special feature of this instrument is the method of compensation against variations of atmospheric temperature. All four wires forming the arms of the bridge ABCD are carried in a frame which is exposed to the air current to be measured. The anemometer wire A is, of course, bare, but the other nickel wire B is shielded by a thin metal tube from the convective cooling of the stream. By means of a simple change-over switch, the shielded wire B and the manganin resistance C can be made part of an auxiliary bridge circuit, whose other two arms are the manganin resistances E and F of about 20 ohms each. The junction on the bridge between E and F can be varied over the range of the slide-wire S, which has a total resistance of about 5 ohms ; so that the ratio of the total resistances of the arms E and F can be accordingly varied. The process of switching over to the auxiliary bridge connections automatically connects the milliammeter between the junctions of E and F and of B and C. At the same time, a large additional resistance is put into the external circuit, so that the current through the wire B is reduced to a sufficiently small value to ensure that it produces no appreciable rise of temperature of this wire over that of its surroundings. In this condition, B acts as a resistance thermometer ; and the auxiliary bridge is balanced at zero on the milliammeter scale by moving the adjustable contact over the slide-wire S. The balance-point will depend on the temperature of the wire B, that is, in effect, on the prevailing air temperature, since the tube enclosing this wire is exposed to the air current.

A scale of temperature is marked along the slide wire ; and the temperature reading at which balance is obtained is noted. The bridge is then thrown out of balance by moving the contact on the slide wire through a definite temperature range of about 40° F., which varies slightly with each instrument. This alters the ratio of E to F, so that balance of the auxiliary bridge can only be restored by sending an increased current through the

nickel wire B, sufficient to raise its resistance by an amount equivalent to a definite temperature excess (about 40° F.) over that of the air. With the contact on S in its new position, therefore, the high series resistance in the external circuit is cut out; and the current is adjusted by the rheostat R, until balance is again obtained. The instrument is now adjusted ready for use, and the change-over switch is put into the position that brings the main bridge ABCD into the circuit. If the wires are accurately adjusted, so that the pairs A and B, C and D are exactly similar, the bridge will be in balance when the air speed past A is zero,* but when the air is moving, A will be cooled, whereas B, being shielded, will not; further, C and D, being of manganin, will not alter in resistance. An out-of-balance current will therefore flow through the milliammeter, which can be calibrated in terms of air speed.

If the wires A and B are not exactly similar, there will be a slight zero error on the instrument, i.e., a small milliammeter deflection at zero air speed. This, however, can easily be eliminated by arranging the temperature excess to which B is initially set to be such that, when the main bridge is in the circuit, the resistances of A and B will be equal, so that the current through A will be equal to that through B. Thus, by a simple alteration of the balance-point on S, which will constitute a constant for the instrument, the main bridge can always be made to balance in still air. The wires A and B will then be at slightly different temperatures, but the difference between their temperatures in still air will be constant. It is for this reason, namely, that the wires A and B are not always exactly similar, that the temperature excess to which the auxiliary bridge is to be balanced varies somewhat from one instrument to another. Although there is less difficulty in making the manganin resistances C and D equal, any slight dissimilarity in these can be allowed for by the same means.

An instrument having resistances and dimensions of wires of the magnitudes mentioned above is suitable for measuring velocities up to 200 feet per minute. A photograph of such an anemometer is reproduced, by the courtesy of H. Tinsley & Co. Ltd., the manufacturers, in Fig. 96. The frame carry-

* Apart from very small errors due to natural convection currents from A.

ing the bare and shielded nickel wires and the two manganin resistances forming the other arms of the bridge will be seen to the left of the diagram, the milliammeter and the controlling switch and resistances being conveniently arranged in the box on the right. The anemometer wire is always held in a vertical position when in use, so that natural convection currents always act in the same direction relatively to the wire. Calibration of the instrument is also performed with the wire vertical. A two-volt accumulator, fitting into a special com-

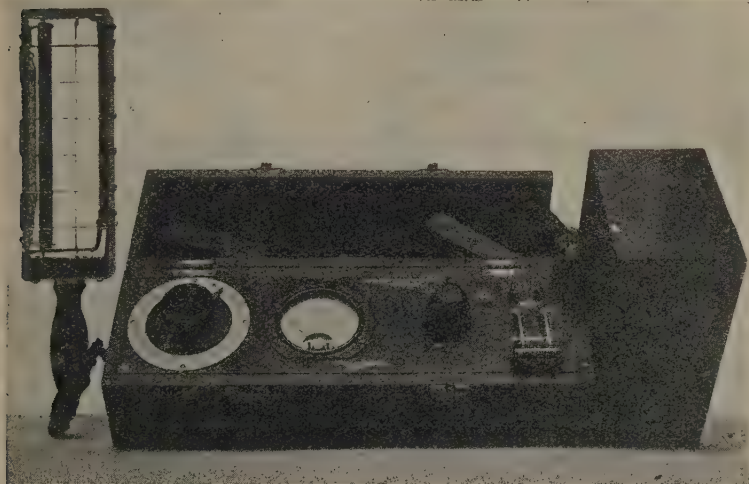


FIG. 96.—The MacGregor Morris hot wire anemometer.

partment in the box, is supplied with the instrument. A small fixed resistance (R_2 in Fig. 95), permanently in series with the battery circuit, is provided, to prevent accidental short-circuiting of the cell.

Notes on Anemometer Wires.—Platinum has been more extensively used for anemometer wires than any other metal, chiefly because of its chemical inertness to atmospheric conditions even at high temperatures. For low temperature wires, such as those used by MacGregor Morris, nickel has been found to be suitable, provided that it is in a high state of purity; electrolytically prepared nickel gives satisfactory results.

The dimensions of the wire are of some importance. Fine wires seem to be more prone than thick wires to the ageing effect to which reference has been made previously. From this point of view, therefore, it is desirable to use wires of large diameter. Other factors, however, operate in favour of fine wires: the sensitivity, that is, the rate of change of current with air speed, is higher the smaller the diameter, and the current necessary to maintain a wire at a given temperature is lower. The higher sensitivity is not of great practical significance, for modern electrical measuring instruments have reached such a state of excellence that robust instruments, sensitive enough for any case of hot-wire anemometry, will almost always be procurable. The lower current necessary with finer wires is a more important consideration; few experimenters seem to have used wires exceeding 0.006 inch in diameter.

As regards the length of the wire, the sensitivity is greater the longer the wire; but here again, other factors are more important in determining the length. There is firstly the question of the voltage necessary to raise a wire to a given temperature. This voltage is proportional to the length; so that if, as is usual, accumulators are used to heat the wire, more cells will be needed for longer wires. Hence, from this point of view, too long a wire is undesirable. The length will also depend to some extent on the use to which the instrument is to be put. If there are marked differences in velocity from point to point in the field in which measurements must be made, obviously errors will result from the use of a long wire which has been calibrated in a uniform air current; whereas if the field is reasonably uniform a long wire is not objectionable. In general, wires 1 or 2 centimetres in length will be found convenient. They will not usually lead to appreciable errors due to this cause, and are not too short for ease of handling during their construction.

A typical form of hot-wire anemometer is shown in Fig. 97. A is the platinum wire forming the heated element; its two ends are hard-soldered, or preferably welded, to two short lengths BB of stouter platinum wire which, in turn, are soldered to two manganin supports CC. The latter are fixed to an ebonite rod D, which can be held in any convenient manner so as to bring the wire A into the region at which the velocity is to

be measured. The manganin wires CC should be bent slightly outward in order to put a slight tension in the wire A. It will then not sag or bend owing to thermal expansion when hot.

In order to ensure good electrical contact between the ends of the wire A and the two pieces BB, ordinary solder should never be used to make the joint. Hard solder is often satisfactory, but it is preferable in all cases to make welded joints. This is

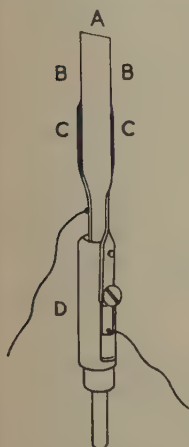


FIG. 97.

particularly desirable if the wire is used at high temperatures. A welded joint can be made as follows: One end of the wire A is laid in position just below the extreme end of one of the stouter platinum wires B, and a pointed carbon rod is placed loosely in contact with the same end of B. The other end of the carbon rod is connected to one terminal of a 2-volt cell, whose second terminal is in electrical connection with B through the ordinary hot-wire lead C. A tapping key included in the circuit is then depressed for a fraction of a second, just long enough for B to be heated to bright redness. It will then usually be found that the end of A has fused to B, making a good and permanent electrical connection. The process * sounds complicated and uncertain; but the author can assert from his own experience that, with a little practice, it is not difficult to make satisfactory joints in this way without undue expenditure of time. It facilitates matters to view and adjust the ends of A and B under a low-power microscope, one having a magnification of, say, 15 to 20 diameters.

There are one or two other points to note in the construction and use of hot-wire instruments. Thus, the method of support should be designed to introduce as little disturbance as possible into the air stream, especially if the instrument is to be used in relatively small pipes. It should also be rigid and rigidly mounted. Care should be taken that the wire is not subjected to vibration, either mechanical or aerodynamic in its origin; for if there is any appreciable oscillatory motion of the wire

* Due to the late H. C. H. Townend.

relative to the air, the rate of cooling will be increased and the velocity readings of the instrument will be high.

Before use, a new wire should always be annealed by passing a current through it sufficient to raise its temperature to red heat for some time; for platinum wires, King advocated a period of annealing of some hours' duration. Finally, the calibration should be checked at frequent intervals, particularly if the wire is used at a high temperature. For calibrating hot-wire instruments, the author has attached them to the end of a rotating arm of about 4 feet radius, and has thus moved them at measured speeds through still air. The path of the wire was in every case totally enclosed by an annular pipe or tunnel, through an annular slit in the roof of which the hot-wire support from the arm protruded vertically downwards into the interior. Such protection from draughts is essential for accurate low-speed calibration; for there is sufficient air movement in a room under ordinary conditions to make calibration impossible at speeds of 1 or 2 feet per second or less, unless such precautions are taken.

Reference has already been made to the progressive changes that often occur in the calibration of a wire. Thus, suppose for the system of measurement illustrated in Fig. 90 we were to determine a calibration curve on a certain day by moving the wire at a series of known speeds through still air, and observing at each speed the potentiometer reading p when the Wheatstone bridge was balanced. We should then obtain a curve between p and the air speed v . If we were to repeat this process on another occasion, it is more than probable that a different calibration curve would be obtained—the value of p at a given speed and also at zero speed, i.e., in still air, would probably be different. This variation has been noticed and discussed by several observers, and some possible causes have been suggested by Simmons⁷ (see below). Some wires, possibly as a result of better electrical connection between their ends and the supports, are considerably more stable than others; but however great the care taken in construction, it is largely a matter of chance whether a given wire will preserve its calibration unchanged for any length of time.

Fortunately, however, there is a practical artifice by means of which this difficulty can to a large extent be overcome. It

consists in making allowance for the current necessary to heat the wire to the desired temperature in still air. Thus, still keeping to the system of measurement of Fig. 90, let us suppose that, when the wire is in stationary air, the potentiometer reading, which is a measure of the current, is p_0 , and at a speed v it is p . Then if we plot, instead of p against v , $(p - p_0)/p_0$ against v , we usually find that, provided the change in p_0 from time to time is not too large, we obtain a unique calibration curve in circumstances in which quite distinct curves would have been obtained if simply p had been plotted against v . It seems, in fact, that for a small change in p_0 , the reading p at a given speed changes in the same ratio as p_0 . When the calibration curve of a wire is plotted in this form, it will often be found to remain constant over periods of some days, so that it need not be checked at frequent intervals. A good indication of when it is desirable to perform a check calibration is afforded by the value of p_0 ; when a comparatively large change in this quantity is noted, some points on the calibration curve should be checked.

Wire Temperatures for the Measurement of Very Low Speeds.—Some general remarks have already been made on the question of the wire temperature for ordinary use. For the measurement of very low air speeds—speeds of the order of a few inches a second—other factors arise. If the wire is too cold, there is insufficient sensitivity over the low-speed range; and also, possibly because atmospheric temperature changes begin to make themselves felt, the value of p_0 seems to be more variable than with a hotter wire. Further, when the sensitivity is low, as it is with a cold wire, the calibration curve, even of the type $(p - p_0)/p_0$ against v , is more susceptible to changes in p_0 . On the other hand, a peculiar effect* is found when the wire is too hot: in these circumstances, at very low values of v the value of p is definitely, but slightly, less than the still-air value p_0 . This means that the wire is actually hotter when moving at very low speeds than when at rest. A typical calibration curve of the type $(p - p_0)/p$ is sketched in Fig. 98; and a range of negative values of $(p - p_0)/p$ resulting from this effect is seen

* First noted by Ower and Johansen,⁶ and subsequently confirmed by Cooper and Linton.⁸

near the origin. It is evident that this part of the calibration curve cannot be used in practice, because any negative value of $(p - p_0)/p$ might relate to either of two values of v .

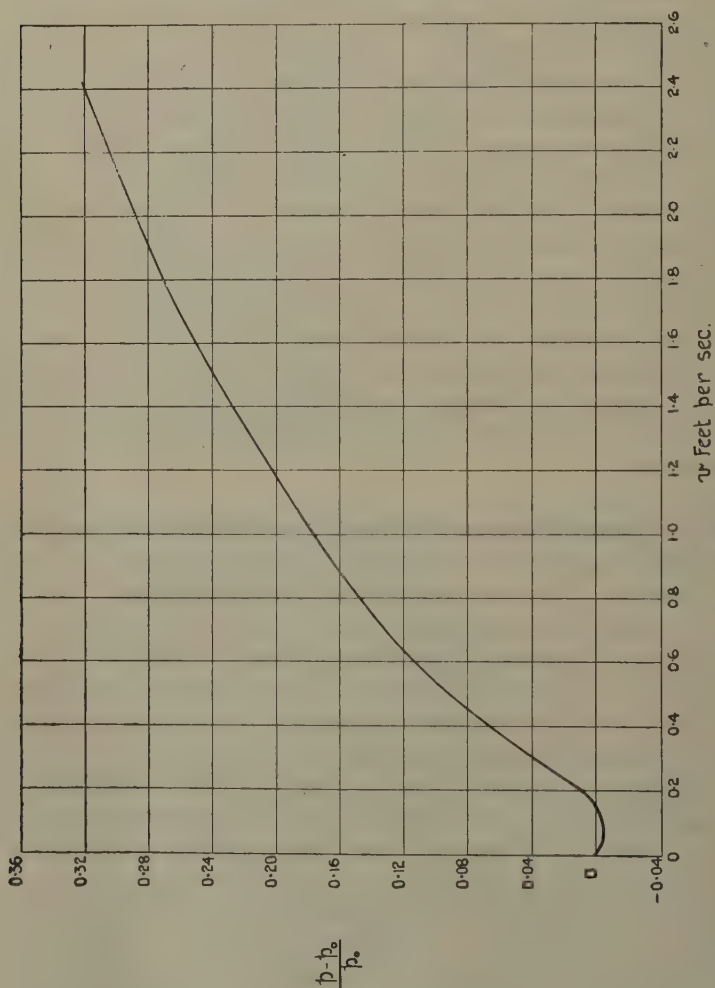


FIG. 98.

It is probable that this effect occurs whatever the temperature of the wire, an increase of temperature causing positive values of p to begin at a higher speed. The phenomenon may be explained by the supposition that, when the wire is at rest

relatively to the air, a stable system of convection currents is set up, which cannot become established when there is relative translational motion. Hence at very low translational speeds, the resultant air speed, which determines the cooling, may be less than the speed due to convection only past the heated stationary wire. The hotter the wire the faster the natural convection currents, and hence the higher the translational speed at which cooling begins in excess of the purely convective cooling when the wire is at rest.

Cooper and Linton⁸ found that the range of speed within which negative readings were obtained was considerably less for a vertical than for a horizontal wire. Thus, with a wire operated at the same temperature (about 200° C.) in both cases, the range of negative readings extended from 0 to about 6 cm. per second when the wire was horizontal, and from 0 to about 1 cm. per second when it was vertical. Moreover, the calibration curves for the two cases differed significantly at all speeds below 10 cm. per second.

For the measurement of very low air speeds it is therefore important that the wire should be used in the attitude in which it is calibrated; and it seems that, if possible, the wire should always be vertical for choice (see also remarks at the end of the next section). Moreover, it is desirable that the temperature of the wire should be adjusted to give the maximum sensitivity consistent with a positive value of p at the lowest speed to be measured. The author has found that a horizontal platinum wire, 0.001 inch diameter and about 0.5 inch long, run at a temperature of about 60° to 70° C., will measure satisfactorily air speeds down to about 0.1 feet per second. In an actual case, a wire of these dimensions and at this temperature gave a calibration curve that intersected the axis of v at about 0.075 feet per second. The method of ensuring that the wire is adjusted to the desired temperature is described on pages 263 and 264.

The Shielded Hot-Wire Anemometer.—Simmons⁷ has developed a modified form of hot-wire anemometer, which has a much more permanent calibration than any of the types so far described. His general experience with hot wires led him to the conclusion that the deposition of dust on the

wire is largely responsible for the wandering calibration, and that another cause may be the change of specific electrical resistance with the strain caused by the air forces acting on the wire. The latter effect may be appreciable with the fine, mechanically weak wires used in ordinary hot-wire anemometry; and obviously the cleaning of these wires to remove dust is hardly practicable. Simmons therefore decided to try the effect of shielding the wire both from dust and from the air forces by enclosing it in a fine tube.

The wire of the new instrument is of nichrome, 0.004 inch in diameter and about $\frac{3}{4}$ inch long; it is threaded into one of the tubes of a short length of silica twin-bore tubing such as is commonly used to insulate thermo-couple leads. Thus shielded from the full cooling effect of the air-stream, this hot wire would constitute too insensitive an anemometer if it were used in the ordinary way. Simmons overcomes the difficulty by inserting into the other bore of the silica tubing the hot junction of a nichrome-constantan thermo-couple, the two free ends forming the cold junction being fused to two copper supports outside the silica tube, one at each end, where they are maintained at the temperature of the air stream. The thermo-couple thus produces an E.M.F. appropriate to the excess of the temperature of its hot junction over the temperature of the air stream, and by using the change in this E.M.F. produced by the motion of the air as a measure of the air-speed, Simmons obtains an instrument of adequate sensitivity. The hot wire is heated to about 120° C. in still air by a current of about 0.6 amperes, which is maintained constant throughout.

The experience so far gained with this instrument is very satisfactory. With periodic cleaning of the silica tube to remove dust, the calibration remains constant for indefinite periods, as compared with the days, or often hours, of the conventional types using fine wires. Cleaning is easily carried out by the application with a camel-hair brush of a solvent such as carbon tetrachloride or acetone. The instrument is much more robust than the ordinary type with a wire of the same diameter; and the effect of variations of air temperature of as much as 30 or 40° C. is very small. Finally, the sensitivity is ample: a three-range electrical circuit is used, with which useful ranges of 0 to 0.4, 0 to 2.0, and 0 to 5.0 ft. per sec. can

be obtained, the maximum speed in each range being measurable to an accuracy of 0.35 per cent. Simmons recommends that the instrument be used with the wire vertical, as this gives the highest sensitivity at low speeds, below about $\frac{1}{4}$ ft. per sec. At higher speeds, the performance is the same whether the wire is vertical or horizontal, provided that, in the latter case, it is at right angles to the stream; if it lies along the stream, the sensitivity is much less.

For further particulars of this interesting development in hot-wire anemometry, reference should be made to Simmons's original paper.⁷

Relation between the Temperature and the Resistance of a Wire, and the Adjustment of a Wire to a desired Temperature.—The equation connecting the resistance of a wire with its temperature $T^{\circ}\text{C.}$, is, sufficiently closely for practical purposes,

$$R_T = R_0 (1 + \alpha T), \quad . \quad . \quad . \quad (12)$$

where R_T is the resistance in ohms at $T^{\circ}\text{C.}$, R_0 is the resistance at 0°C. , and α is the temperature-resistance coefficient, which is equal to 38×10^{-4} for platinum and 62×10^{-4} for nickel.

If the dimensions of the wire are known, R_0 can be calculated, from the equation

$$R_0 = \frac{4lK}{\pi d^2} \text{ ohms}, \quad . \quad . \quad . \quad (13)$$

where l is the length of the wire and d its diameter, both in inches, and K , the specific resistance, may be taken as 4.2×10^{-6} for platinum and 4.6×10^{-6} for nickel. If, then, l and d are known, the Wheatstone bridge can be designed to run the wire at any desired temperature. All that is necessary is to make the balancing resistance (Z in Fig. 89) equal to the resistance of the hot wire at that temperature, as calculated from (12) and (13).

It is often convenient, however, not to trouble about the actual value of the temperature of the wire in the early stages. The resistance Z may, for example, be adjusted by trial so that the bridge is balanced when the hot-wire is just on the point of glowing in still air. This temperature is convenient for the measurement of all except very low speeds (for which, see

above) ; the wire is then hot enough to be amply sensitive for most purposes ; and, at the same time, the danger that the wire will be burnt out, if the wind speed should suddenly be reduced before the current in the wire is broken, is not as great as if the wire is run at a higher temperature. The approximate temperature of the wire can then be easily determined by measuring firstly the resistance of the balancing resistance Z —for this is equal to that of the wire—and secondly the resistance of the wire itself at air temperature (T_a). For the latter measurement, a very small current should be passed through the wire in order to avoid appreciable heating, since it is assumed that the temperature of the wire is equal to that of the surrounding air. If R_T is the resistance of the wire at temperature $T^\circ \text{C.}$, and R_a is its resistance at air temperature T_a , we have, from (12),

$$\frac{R_T}{R_a} = \frac{1 + \alpha T}{1 + \alpha T_a}, \quad . \quad . \quad . \quad . \quad (14)$$

an equation from which T can be calculated, since all the other quantities are either known or can be measured.

It should be added that small errors are incurred in applying equation (12) ; for there is a slight loss of heat by conduction from the wire to its supports, for which no allowance is made, and also, the assumption that α is constant is not strictly accurate. However, these errors are not serious for practical purposes.

Electric Flow Meters.—Thomas⁹ and Callendar¹⁰ have designed electrical meters which indicate the rate of flow of air from measurements of the heat energy that must be supplied to the moving stream, at a specified position, in order to maintain a specified difference of temperature between two given sections on either side of that at which the heat is supplied. If $t^\circ \text{C.}$ is the constant temperature difference between these sections, the weight of air W flowing per second is given by

$$W = \frac{H}{st} \text{ grams per second,} \quad . \quad . \quad . \quad (15)$$

where H is the heat supplied, in calories per second, and s is the specific heat of air.

If this heat is supplied by an electric current of i ampères

flowing through a resistance R ohms placed in the air stream, equation (15) becomes

$$W = \frac{i^2 R}{Jst}, \quad (16)$$

where J is the mechanical equivalent of heat in Joules per calorie. Alternatively, if V is the voltage drop across the resistance, we may write

$$W = \frac{V^2}{RJst}. \quad (17)$$

In Callendar's instrument, use is made of the relationship in the form of (17). In brief, the apparatus consists of a resistance coil JK (Fig. 99) of material having a constant resistance, made in the form of a mat extending over the cross-section of the air stream to be measured, so that the latter is uniformly heated.

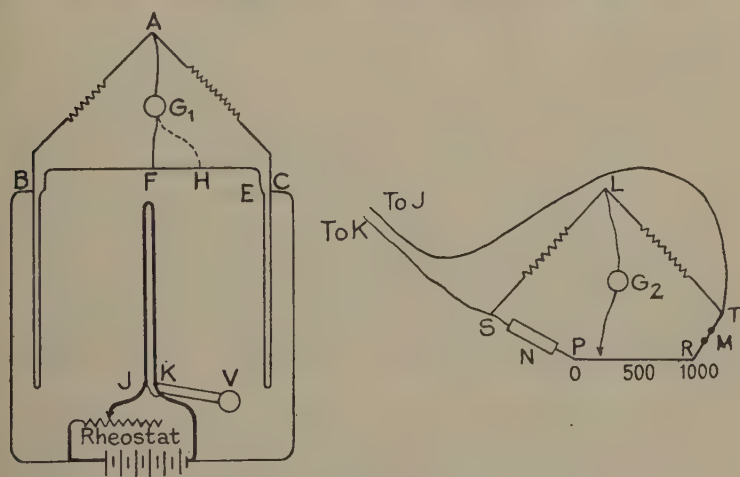


FIG. 99.

The temperature rise is measured by means of two resistance thermometers BD and EC , consisting of grids of fine platinum wire placed at equal distances on either side of the heating mat. These thermometers form part of the Wheatstone bridge circuit $BACEFD$, in which F is the balance point when no current is passing through the heating grid, i.e., when both thermometers are at the same temperature. When the air current is flowing from left to right and the grid is heated, EC is hotter than BD

and its resistance is higher ; a new balance-point can then be obtained at H on a slide-wire resistance between D and E. The resistance FH is made equal to the difference in resistance of the two thermometers when their temperatures differ by a fixed amount— $2\frac{1}{2}^{\circ}$ C. was used in an actual case—and the heating current through the grid JK is adjusted to maintain this temperature difference constant, i.e., to keep the bridge BACEFD balanced. In equation (17) we then have the numerator constant, so that W varies as V^2 .

The voltage drop across the heating grid, which is thus a measure of the rate of flow, is determined by an arrangement similar to the volt-thermometer previously described. The leads from the terminals JK of the grid are connected to a second Wheatstone bridge SLTRP, of which the arms SL and LT are equal resistances, having zero temperature coefficient. The arm TR contains the volt-thermometer M, which is, as before, simply a fine platinum wire enclosed in a box, and N is a balancing resistance. PR is a potentiometer wire, at any point of which the connection to the galvanometer G_2 may be made. Instead of calibrating the bridge in terms of the distance of the balance-point from P when known voltages are applied, we may correlate the rate of air flow by direct calibration with the position of the balance-point. The method of operation in taking an observation is to adjust the rheostat in the heating circuit of JK until the galvanometer G_1 indicates balance of the thermometer bridge with the connection at H, and then to find the balance-point for the galvanometer G_2 on the potentiometer PR.

Complete descriptions of the instrument and of measurements conducted with it are given in References 10, 11 and 12, from which further details may be obtained. That it is an instrument of more than laboratory interest is clear from the fact that it has been used to measure the air consumption of aero-engines on the bench and in the air.¹²

In the Thomas meter,⁹ a similar arrangement of heating and thermometer coils is used ; but in this case the power supplied to the heating circuit is measured directly on a wattmeter. In an automatic form of this instrument, which has been put on the market for industrial use, an ingenious device is introduced to avoid the necessity for adjusting the current through

the heating coil by hand, in order to maintain the thermometers at a constant temperature difference. By means of an electric motor drive, the needle of the galvanometer in the thermometer bridge is clamped between contacts at regular intervals of a few seconds. If the temperature difference is not at its prescribed value when one of these periodic contacts is made, the galvanometer needle is not at its zero position; and a relay circuit is made, which automatically varies a rheostat in the heater circuit, and so increases or decreases the current in this circuit according to whether the temperature difference is too low or too high. The amount by which the resistance is varied is made to depend upon the deflection of the galvanometer in the bridge circuit, that is, upon the amount by which the temperature difference between the two thermometers has deviated from its standard value.

The Kata-Thermometer.—Similar in principle to the hot-wire anemometer, but different in that the supply of heat whose rate of loss is measured is discontinuous and non-electrical, is the kata-thermometer. This instrument was first designed by Leonard Hill,¹³ for use in his researches on the effect of atmospheric conditions on health and industrial efficiency, to measure the rate of loss of heat from a surface at approximately body temperature. The kata-thermometer is essentially an alcohol thermometer of special type; in its original form it had a bulb about $1\frac{1}{2}$ inches long and $\frac{3}{4}$ inch in diameter, and a stem leading from the top of the bulb with the temperature range 100°F to 95°F . marked on it. The bore of the stem was enlarged at its upper end, so that the instrument could be heated considerably above 100°F . without the risk of fracture.

If an instrument of this kind is heated and then allowed to cool, the same amount of heat will always be lost while the top of the alcohol column falls from the 100° to the 95° mark, but the rate of loss of heat will depend on the atmospheric conditions. Although the instrument was not designed primarily as an anemometer, experience with it has shown that, provided that the bulb and stem are quite dry, the mean rate of cooling over the marked temperature range is related to the air speed V by the equation

$$H = (a + b\sqrt{V})\theta, \quad . \quad . \quad . \quad (18)$$

where a and b are constants, θ is the mean excess temperature of the kata-thermometer over that of the surrounding air, and H is the total heat lost in cooling from 100° F. to 95° F. divided by the area of the cooling surface and by the cooling time.

It is noteworthy that this law is of the same form as that found by King for hot wires.

The total heat lost in cooling through the given temperature range, although constant for a given instrument, varies somewhat from one kata-thermometer to another. Each instrument is calibrated in still air by the makers, and is given a "factor" F , which is the total heat lost per unit area of cooling surface in cooling through the specified range of temperature. Therefore, in order to calculate H in equation (18), it is necessary only to observe the time T in seconds taken by the alcohol column to fall from the higher to the lower temperature mark, and to divide the factor F by this time.

The first form of kata-thermometer was subject to two major objections: firstly, that it was liable to be affected by radiation from its surroundings, and secondly, that the cooling times in hot atmospheres were very long. Various modifications were introduced at different times,¹⁴ without, however, any change in the main dimensions: all modern kata-thermometers have brightly silvered bulbs in order to reflect incident heat radiation as much as possible, and the working temperature range is either from 130° F. to 125° F. or from 150° F. to 145° F. in order to reduce the time of operation in hot spaces. The working forms of equations (18) for these instruments are as follows:—

For the kata-thermometer working between 130° F. and 125° F.:

$$(a) F/T = (127.5 - t) (0.061 + 0.0139\sqrt{V}).$$

$$(b) F/T = (127.5 - t) (0.011 + 0.017\sqrt{V}).$$

For the kata-thermometer working between 150° F. and 145° F.:

$$(a) F/T = (147.5 - t) (0.074 + 0.0184\sqrt{V}).$$

$$(b) F/T = (147.5 - t) (0.018 + 0.0223\sqrt{V}).$$

In these equations, t is the temperature of the air in degrees

Fahrenheit ; equations (a) are used for velocities below about 200 feet per minute, and equations (b) for velocities above this. V is given in feet per minute.

The use of the kata-thermometer as an anemometer is restricted mainly to the measurement of the speed of moving air currents in the study of ventilation problems in rooms and enclosed spaces. For that purpose, it has the advantage that it is largely non-directional, giving values of the resultant air speed past the bulb practically independent of the direction of flow ; and it is this resultant velocity that is one of the factors associated with the cooling effect of the air movement on human beings and with comfort, which are the primary concerns of ventilation. Thus the kata-thermometer measures directly the velocity that one needs to know in such work, and there is no difficulty, as there would be with other types of anemometer, in aligning the instrument correctly with an air current whose local direction is probably unknown. Another advantage of the instrument is that it can measure low speeds.

Disadvantages of the kata-thermometer are, firstly, the length of time necessary to take readings and, secondly, the comparatively low accuracy. To take a reading, the instrument is first of all heated to well above the upper temperature, generally by immersion in hot water, then thoroughly dried and exposed to the air current. The top of the alcohol column should then be well above the upper temperature mark in order that, by the time it reaches the latter, the rate of cooling shall have become steady. The time taken for the top of the column to fall from the upper to the lower mark is then observed by means of a stop watch. The instrument must not be allowed to move during an observation, and it should be held at arm's length or, better, held in a clamp or suspended well clear of the observer. At least three readings should be taken for each velocity determination. The error of a determination may be about 5 to 8 per cent.

Further particulars and working instructions will be found in Reference 15.

REFERENCES

1. King, L. V., *Phil. Trans. Roy. Soc., A*, **214** (1914), p. 373. See also *Phil. Mag.* (Series 6), **29** (1915), p. 556 ; *Journ. Franklin Inst.*, **181** (1916), p. 1, and **183** (1917), p. 783.
2. Boussinesq, *Comptes Rendus*, **133** (1901), p. 257.

3. Simmons and Bailey, *Phil Mag.* (Series 7), **3** (1927), p. 81.
4. Davis, *Proc. Phys. Soc.*, **33** (1921), p. 152.
5. King, R. O., *Engineering*, **117** (1924), pp. 136 and 249.
6. Ower and Johansen, *Reports and Memoranda of the Aeronautical Research Committee*, No. 1437 (1931), Appendix IV.
7. Simmons, *Journ. Sci. Insts.* In the press.
8. Cooper and Linton, *Proc. Nova Scotian Inst. of Sci.*, **19**, Pt. I (1934-35), p. 119.
9. Thomas, *Journ. Franklin Inst.*, **172** (1911), p. 411.
10. Moss and Stern, *Aeronautical Research Committee: Engine Subcommittee Reports*, No. 55 (1920).
11. Moss and Stern, *Reports and Memoranda of the Aeronautical Research Committee*, No. 725 (1921).
12. Moss, *Reports and Memoranda of the Aeronautical Research Committee*, No. 798 (1922).
13. Hill, Griffith, and Flack, *Phil. Trans. Roy. Soc., B*, **207** (1916), p. 186.
See also Hill, Vernon, and Hargood-Ash, *Proc. Roy. Soc., B*, **93** (1922), p. 198; Hill, *Medical Research Committee, Special Report Series*, No. 32 (1919); Hargood-Ash and Hill, *ibidem*, No. 73 (1923).
14. Angus, Hill, and Soper, *Journ. Industrial Hygiene*, **12** (1930), p. 66; Angus, *Engineering*, **83** (1932), p. 281. See also Yaglou and Dokoff, *Journ. Industrial Hygiene*, **11** (1929), p. 278; Cockerell, *Journ. of Hygiene*, **35** (1935), p. 255.
15. Bedford, *Medical Research Council War Memorandum*, No. 17 (1946).

CHAPTER XI

EXAMPLES FROM PRACTICE

IN this chapter, a few typical examples will be given from practice of some of the methods of measuring air flow described in the foregoing pages, the methods of calculation being shown in detail.

(1) Measurement with a Pitot-Static Tube of Quantity of Air Flowing along a Pipe.—The pipe is of 12-inch internal diameter ; the pitot-static combination is of standard type, the differential pressure being equal to the velocity head without correction. The manometer used is of the Krell inclined-tube type, graduated in inches. It is assumed to have been previously calibrated, with the result that a mean factor of 0.138 has been found applicable over the whole scale to convert the readings to inches of water.

According to the rules given in Chapter V, we have to divide the pipe into five zones of equal area and take velocity readings at the centre of area of each zone along two perpendicular diameters. In all, twenty readings will be taken, ten along each diameter ; and the position of the axis of the pitot-static head along each diameter, measured from one end of the diameter, are as follows (applying the rules given in Chapter V) : 0.31, 0.98, 1.76, 2.71, 4.10, 7.90, 9.29, 10.24, 11.02, and 11.69 inches. The velocity readings obtained on the manometer are 6.72, 7.75, 8.74, 7.90, 7.66, 7.12, 6.55, 5.66, 4.40, and 2.56 inches on the horizontal diameter of the pipe, and 4.28, 5.85, 6.96, 7.13, 7.42, 7.38, 7.02, 6.88, 5.82, and 4.15 inches on the vertical diameter. In order to convert these readings of velocity head into inches of water, so that we can use the formulæ of Chapter V, we must multiply them by 0.138, the calibration factor of the manometer. Equation (11) of Chapter V gives the velocity head h_m corresponding to the mean velocity in the pipe, and in this case it

becomes

$$\begin{aligned}\sqrt{h_m} &= \frac{1}{20} (\sqrt{6.72} \times \sqrt{0.138} + \sqrt{7.75} \times \sqrt{0.138} + \dots \\ &\quad + \sqrt{4.28} \times \sqrt{0.138} + \sqrt{5.85} \times \sqrt{0.138} + \dots) \\ &= \frac{\sqrt{0.138}}{20} (\sqrt{6.72} + \sqrt{7.75} + \dots + \sqrt{4.28} + \sqrt{5.85} + \dots).\end{aligned}$$

Thus, instead of multiplying each reading by the calibration factor and extracting the square root of the product, we save work by summing the roots of the readings and multiplying the sum by one-twentieth of the root of the calibration factor.

Thus $\sqrt{h_m} = 0.0186 \times 50.18 = 0.934$.

The temperature of the air in the pipe is 50°C . and the barometer stands at 764 mm. The static pressure in the pipe is less than 2 inches of water. We use equations (12) and (13) of Chapter V to get V , the volume, or Q , the weight of air flowing in cubic feet or pounds per minute respectively. A , the area of the pipe, is 0.785 square feet.

We have

$$V = 0.785 \times 6450 \sqrt{\frac{273 + 50}{764}} \times 0.934 \text{ cubic feet per minute,}$$

$$Q = 0.785 \times 186.6 \sqrt{\frac{764}{273 + 50}} \times 0.934 \text{ pounds per minute ;}$$

i.e., $V = 3080$ cubic feet per minute,

$Q = 211$ pounds per minute.

If the mean static pressure were, say, 11.5 inches of water below atmospheric pressure, we should have to replace the figure 764 by $764 - 11.5 \times (25.4/13.56)$, i.e., by 742. It is easy to see that neglect of this correction would lead to an error of about $1\frac{1}{2}$ per cent. in both V and Q . As the static pressure is actually less than 2 inches of water, however, the error is about 0.2 per cent. ; and it is not necessary to correct the barometric pressure, since the accuracy of the measurements is not of so high an order.

(2) Calibration of a Small Plate Orifice.—This example affords a useful illustration of more than one of the points dis-

cussed in earlier chapters, and will consequently be treated at some length. The conditions to be met were as follows : For certain purposes, which are not material to this discussion, a stream of air was to be supplied to an inch diameter pipe by means of a small fan ; and it was required to provide a ready method of measuring the rate of flow, and of maintaining it at any steady value between limits of 5 and 10 feet per second. Further, the scheme adopted was to be such that it would permit the rate of flow to be determined instantly.

The low rate of flow, coupled with the condition last mentioned, imposed restrictions on the choice of method ; and it was decided that the use of some constricting device permanently connected to a sensitive, but not too delicate, differential manometer was the most suitable scheme to employ. Space was too limited to allow sufficient length of pipe upstream and downstream of the constriction for a standard constriction to be used without calibration ; for, as pointed out in Chapter VI, the coefficients that have been established for standard orifices or nozzles cannot safely be applied in such cases. It was evident, therefore, that whatever form of constriction was used would have to be calibrated *in situ*. Hence it was decided to use a plate orifice, as being the type of constriction occupying the least space and the cheapest and simplest to instal. Moreover, in view of the necessity for calibration, no attempt was made to reproduce any standard form, the orifice consisting merely of a reasonably square-edged circular hole cut in a $\frac{1}{8}$ -inch brass plate, which was clamped in flanges fixed near the outlet of the pipe. There was a length of straight pipe about 12 diameters long upstream of the plate, and a length of about 3 diameters downstream. Pressure taps for the manometer were provided fairly close to the plate, without any care as to their exact position.

An inclined-tube manometer was chosen on the score of simplicity and cheapness for indicating the orifice head ; and it was decided that a convenient length of liquid column for the lowest rate of flow (5 feet per second) would be about 2 inches. It seemed reasonable to suppose that the length of the column could easily be read to $\frac{1}{25}$ inch, so that the pressure corresponding to 5 feet per second could be read to within 2 per cent. And since the speed is proportional to the square root of the pressure,

the accuracy on speed at the lower end of the scale would be within 1 per cent., which was ample for the purpose in view. Alcohol was selected as the manometric liquid, for reasons explained in Chapter IX; it was coloured with a blue aniline dye in order to make the meniscus more prominent. The slope of the inclined tube was set to about 1 in 10.

Calculations of the necessary size of the orifice were then made as follows:

From equation (12) of Chapter VI we have

$$a_1 \rho v_1 g = q = \alpha a_2 g \sqrt{\frac{2\rho(\phi_1 - \phi_2)}{1 - m^2}}.$$

Remembering that $m = a_2/a_1$ and that $a_1 v_1 = a_2 v_2$, we may reduce this equation to the following form:

$$\frac{a_1}{a_2} = \sqrt{\frac{2\alpha^2(\phi_1 - \phi_2)}{\rho v_1^2}} + 1, \quad \dots \quad (1)$$

which enables us to calculate the area of the orifice (a_2) in terms of the area of the pipe (a_1) if we know v_1 , the velocity in the pipe, and the pressure drop $\phi_1 - \phi_2$.

In the case under consideration, $\phi_1 - \phi_2$ had been fixed at 2 inches of alcohol in the inclined tube for a speed v_1 of 5 feet per second. In the above equation, $\phi_1 - \phi_2$ must be expressed in pounds per square foot if the ft.-lb.-sec. system of units is used. The slope of the manometer tube was 1 in 10 and the density of alcohol could be taken as 0.8 approximately.

$$\begin{aligned} \text{Hence } \phi_1 - \phi_2 &= 2 \times 1/10 \times 0.8 \text{ inch of water} \\ &= 2 \times 1/10 \times 0.8 \times 5.2 \text{ lb./sq. ft.} \\ &= 0.833 \text{ lb./sq. ft.} \end{aligned}$$

For the approximate calculation of a_1/a_2 it was sufficiently accurate to assume α for the orifice to be 0.6.

Hence, equation (1) above became

$$\frac{a_1}{a_2} = \sqrt{\frac{2 \times 0.36 \times 0.833}{0.00237 \times 25}} + 1 = 3.33,$$

so that d_2 , the diameter of the orifice $= 1/\sqrt{3.33}$ inch $= 0.55$ inch, since the diameter of the pipe was 1 inch. Actually, the orifice diameter was made $\frac{1}{2}$ inch. Before this dimension could be finally accepted, however, it was necessary to see whether the irrecoverable loss of head that occurs with an orifice was

beyond the capacity of the fan available. At the maximum delivery, 10 feet per second, the pressure drop through the orifice will be four times that used in the calculation of a_1/a_2 , since the calculation was made for a speed of 5 feet per second, and $(p_1 - p_2)$ varies as the square of the speed. Thus we obtain at 10 feet per second $p_1 - p_2 = 4 \times 0.2 \times 0.8 = 0.64$ inch of water. This value requires correction, however, since it involves the assumption that $a_1/a_2 = 3.33$, whereas by making the orifice $\frac{1}{2}$ -inch diameter the ratio actually became 4. It is easy to see from (1) that $p_1 - p_2$ varies as $(a_1/a_2)^2$; hence the true pressure drop at 10 feet per second became $0.64 \times (4/3.33)^2$ inch of water = 0.92 inch of water. As we have seen in Chapter VI, the head recovered may be taken as m times the drop through the orifice, i.e., in this case as $\frac{1}{4} \times 0.92$, so that the head lost will be $\frac{3}{4} \times 0.92$ or 0.69 inch of water. The maximum head to be supplied by the fan consists of this lost head through the orifice, the frictional resistance of the total length of the pipe, and the velocity head equivalent to the speed of the air issuing from the open end of the pipe remote from the fan. It is easy to see that this head must be included; for the energy in the issuing jet is dissipated, and is only recovered when the piping forms a closed circuit, i.e., when the far end of the pipe is connected back to the fan intake.

The velocity head to be supplied by the fan is $\frac{1}{2}\rho v^2$ lb. per square foot, which, in the case under consideration, comes to $\frac{1}{2} \times 0.00237 \times 10^2$ lb. per square foot, or $\frac{1}{2} \times 0.00237 \times 100 \times 1/5.2$ inch of water, i.e., 0.023 inch of water at the highest speed. The head necessary to overcome the friction at the pipe walls is obtained thus: The value of vd/ν for the conditions of maximum delivery is $(10 \times \frac{1}{1.2})/0.00016$ approx., or about 5,000. From Fig. 6 we see that, at this value of vd/ν , $F/\rho v^2$ for smooth pipes is about 0.0050. Also, from equation (3) of Chapter III, $p_A - p_B = 4Fl/d$, where p_A and p_B are the static pressures at the two ends of the pipe, l is the length of the pipe, and d is its diameter. In the case under consideration, l/d is about 15, so that the pressure drop due to friction becomes

$$\begin{aligned} p_A - p_B &= 4 \times 0.005 \times 15 \text{ lb. per square foot} \\ &= (4 \times 0.005 \times 15)/5.2 \text{ inch of water} \\ &= 0.058 \text{ inch of water.} \end{aligned}$$

It should be remarked that this calculation is only approximate, since the data assumed relate to long smooth pipes. If we double the result to allow for possible errors due to departures from these conditions, we obtain as the requisite total head to be supplied by the fan $0.69 + 0.02 + 0.12$ or 0.83 inch of water. In the problem under discussion, this was well within the capacity of the fan available, which was a small fan of the type used in hairdressing establishments for hair-drying. The proposed orifice size was therefore suitable; and an orifice plate was accordingly made to the specified dimensions.

The next step was to calibrate the orifice over the required speed range, in order to determine the relation between the pressure drop across the orifice and the rate of flow. For this purpose, a $\frac{1}{2}$ -inch diameter smooth brass calibrating pipe (see Chapter VIII) was used, in which there was provision for traversing a section with a small hypodermic total-head tube actuated by a micrometer screw (see Fig. 65). A diagrammatic sketch of the arrangement as used for the orifice calibration is given in Fig. 100. AB is the 1-inch diameter pipe, with the orifice at C, and DE is the $\frac{1}{2}$ -inch diameter calibrating pipe, connected to the end B of the inch pipe by means of a reduction cone and rubber tubing. P is the traversing pitot, and S_1 , S_2 , and S_3 represent static-pressure taps leading to side holes flush with the inside of the brass tube. The hole S_3 was opposite the mouth of the total-head tube P, and the distances $S_1 S_2$ and $S_2 S_3$ were 20.86 and 2.20 inches respectively. The object of these three static holes has already been explained (Chapter VIII).

The calibration resolved itself into two stages. Firstly, a velocity traverse was made at one rate of flow by means of the micrometer pitot P, in order to determine whether the ratio of the mean velocity v to the velocity v_a at the axis of the pipe at P could be taken from the curve of Stanton and Pannell for smooth pipes given in Fig. 9, in spite of the possible effect of the large pipe attached to the mouth of the calibrating pipe. As will be seen later, this was found to be the case. Hence there was no need to traverse the pipe at P in order to measure the velocity of flow at each speed at which the orifice was subsequently calibrated, as would have been necessary had conditions been such that Fig. 9 could not be used. All that was necessary was to set the pitot with its mouth at the axis of the tube, and

to observe the velocity head corresponding to v_a at different rates of flow, simultaneous readings being taken of the pressure drop across the orifice C. This, then, constituted the second stage of the calibration. Details of the procedure and calculations for each stage are given below.

(a) *Determination of v/v_a for the $\frac{1}{2}$ -inch Pipe at One Rate of Flow.*—Referring to Fig. 11, we see that the total volume of air flowing through the pipe is given by

$$Q = 2\pi \int_0^R v r dr.$$

Also, v by definition is equal to $\pi R^2/Q$.

$$\text{Hence} \quad v = \frac{2}{R^2} \int_0^R v r dr. \quad \dots \quad (2)$$

In order, therefore, to obtain v we evaluate

graphically the integral $\int_0^R v r dr$ by reading v at

a number of radii, multiplying each v by its corresponding r , and plotting the product vr against r . The area of the resulting curve gives the integral.

In the actual experiment, the velocity heads at the various values of r were obtained by connecting the pitot P to one side of a Chattock manometer and the static hole S_2 to the other. This gave the total head at P less the static pressure at S_2 , whereas what was required was the total head at P less the static at S_3 , in the same plane as the mouth of the total-head tube. False readings would have been obtained, however, had P and S_3 been connected to the manometer; for the pressure at S_3 was affected by the presence of the pitot.

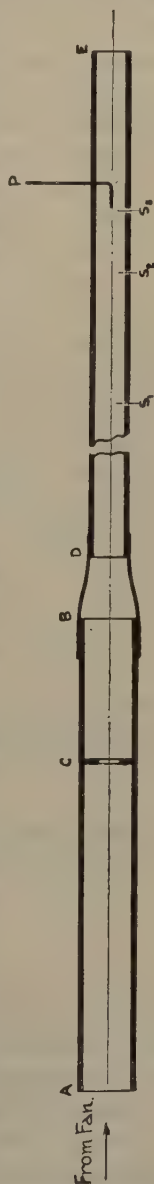


FIG. 100.

On the other hand, S_2 was far enough upstream of the pitot to be out of its zone of influence. The readings of velocity head obtained had therefore to be corrected by the amount of the static pressure drop between S_2 and S_3 . This was obtained by measuring the drop between S_1 and S_2 , and, since the pressure drop along a smooth straight pipe is proportional to the length, multiplying this drop by the ratio of the length $S_2 S_3$ to $S_1 S_2$, i.e., by $2.20/20.86$. Thus, velocity head at $P = (P - S_2) + (S_1 - S_2) \times 2.20/20.86$. The pressures $P - S_2$ and $S_1 - S_2$ were read on the same manometer, a two-way tap being provided on the one side so that it could be connected either to P or S_1 .

The Chattock gauge employed had cup centres 13.08 inches apart, and the dimensions l_2 and d of equation (1), page 212, relating to the Chattock gauge, were 10 inches and $\frac{1}{80}$ inch respectively. Hence one turn of the micrometer wheel of the gauge was equal to a head of $13.08/(10 \times 20)$ inch of water. And from the equation $p = \frac{1}{2}\rho v^2$, where p is in pounds per square foot, v in feet per second, and $\rho = 0.00237$ under standard atmospheric conditions,

$$v = \sqrt{\frac{2p}{\rho}} = \sqrt{\frac{2 \times 13.08 \times 5.2}{10 \times 20 \times 0.00237}} \times G,$$

where G is the gauge reading in complete turns necessary to balance the head due to the velocity v ,

$$\text{i.e.,} \quad v = 16.95\sqrt{G}.$$

Hence, returning to equation (2), we write this in terms of the gauge reading G and obtain

$$v = \frac{2 \times 16.95}{R^2} \int_0^R r \sqrt{G} \, dr. \quad . \quad . \quad . \quad (3)$$

The mean velocity v will be obtained in feet per second if R is in feet. But it will be noted that R^2 occurs outside the integral in the denominator, whilst the integral itself contains rdr , which is a length squared, in the numerator. Hence, there is no need to convert R or r to feet; they may both be expressed in inches.

Table 18 gives details of the traverse. The length r is easily obtained by a simple calculation from the pitot micrometer

reading when the pitot is touching the far wall of the pipe (see Chapter VIII), the radius R of the pipe (in this case 0.247 inch), and the radius of the pitot head (0.012 inch).

TABLE 18

Radius of Point of Observation r inches	Pressure Readings. Turns of Chattock Gauge Wheel				$\sqrt{G}$	$r\sqrt{G}$
	$(P - S_2)$	$(S_1 - S_2)$	$\frac{2.20}{20.86}(S_1 - S_2)$	$\frac{(P - S_2)}{= G}$		
0	8.64			9.51	3.08	0
0.072	7.95			8.82	2.97	0.214
0.122	6.98			7.85	2.80	0.342
0.172	5.70			6.57	2.56	0.441
0.197	4.95	8.21 *	0.87 *	5.82	2.41	0.475
0.222	3.86			4.73	2.17	0.483
0.235	2.63			3.50	1.87	0.439
0.228	3.71			4.58	2.14	0.488
0.203	4.79			5.66	2.38	0.483
0.178	5.63			6.50	2.55	0.454
0.128	6.84			7.71	2.78	0.355
0.078	7.82			8.69	2.95	0.230
0.050	8.15			9.02	3.00	0.150

* The rate of flow is constant, hence $(S_1 - S_2)$ is the same throughout.

Plotting the values of $r\sqrt{G}$ from this table against r , we obtain the curve shown in Fig. 101. With 1 centimetre horizontally representing 0.02 inch (scale of r), and 1 centimetre vertically representing 0.05 units of $r\sqrt{G}$, the area of the curve is found to be 73.9 square centimetres.

$$\text{Hence } \int_0^R r\sqrt{G} \cdot dr = 73.9 \times 0.02 \times 0.05 = 0.0739,$$

and therefore

$$v = \frac{2 \times 16.95}{0.247^2} \times 0.0739 = 41.0 \text{ feet per second.}$$

Now v_a is the velocity corresponding to the reading $G = 9.51$ when $r = 0$;

hence $v_a = 16.95 \times \sqrt{9.51} = 52.3$ feet per second,

and $v/v_a = 0.785$.

The value of $v_a d/\nu$ for this flow in the $\frac{1}{2}$ -inch pipe is approximately equal to $(52.3 \times 0.494)/(12 \times 0.00016)$, i.e., to 1.35×10^4 , so that $\log v_a d/\nu = 4.13$. At this value of $\log v_a d/\nu$, we find from Fig. 9 that the value of v/v_a for long straight pipes is 0.788,* which thus agrees well with the experimentally deter-

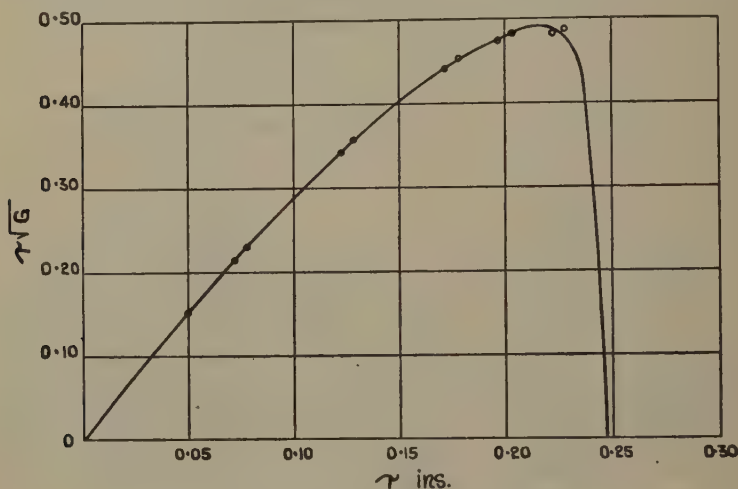


FIG. 101.

mined ratio. Hence for the subsequent orifice calibration, it was considered necessary to take only one reading of the velocity head at each rate of flow, with the pitot at the axis of the $\frac{1}{2}$ -inch pipe. The mean velocity was then taken from the lower curve of Fig. 9. Measurement of the diameter of the inch pipe showed this to be 0.998₅ inch, so that the mean velocity in the pipe was given by $(0.494/0.998_5)^2 \times v_m$, i.e., by $v_m/4.09$, where v_m is the mean velocity in the $\frac{1}{2}$ -inch pipe.

(b) *Orifice Calibration.*—The orifice pressure taps were connected to the inclined-tube manometer, which had previously been filled with alcohol and set to the desired slope. A series of simultaneous measurements was then made of the axial velocity in the $\frac{1}{2}$ -inch pipe and the differential pressure across the orifice in terms of the manometer reading. The results are shown in Table 19.

* This is the value obtained from the results of Stanton and Pannell. When the work now being described was carried out by the author, Nikuradse's results had not been published (see Chapter III).

TABLE 19

Inclined-Tube Manometer Reading inches.	Chattock Gauge Readings				Axial Velocity v_a in $\frac{1}{2}$ -inch Pipe = $16.95 \sqrt{G}$, ft./sec.	Log vd/ν	v/v_a from Fig. 9	Mean Velocity v in $\frac{1}{2}$ -inch Pipe ft./sec.	Mean Velocity in inch Pipe = $v/4.09$, ft./sec.
	$P-S_2$	S_1-S_2	$\frac{2.20}{20.86} (S_1-S_2)$	$P-S_2 = G$					
1.22	1.50	1.64	0.17	1.67	21.9	3.75	0.763	16.7	4.09
2.13	2.51	2.66	0.28	2.79	28.3	3.86	0.773	21.8	5.34
3.38	3.84	3.91	0.41	4.25	34.9	3.95	0.779	27.2	6.65
4.69	5.12	5.13	0.54	5.66	40.3	4.02	0.783	31.5	7.70
5.19	5.66	5.61	0.59	6.25	42.4	4.04	0.784	33.2	8.12
6.02	6.53	6.44	0.68	7.21	45.5	4.07	0.785	35.7	8.73
6.69	7.28	7.05	0.74	8.02	48.0	4.09	0.786	37.7	9.23
8.66	9.26	8.81	0.93	10.19	54.1	4.14	0.789	42.7	10.45
9.60	10.25	9.65	1.02	11.27	56.9	4.16	0.790	44.9	11.0
11.04	11.86	10.78	1.14	13.00	61.1	4.20	0.791	48.4	11.8

The values of the mean velocity given in the last column of this table now need correction in the light of Fage's work on the effect of turbulence and size of total-head tube (see pages 71-74). The value of vd/ν in this work was about 10,000, and the corresponding error due to turbulence is shown by the figures given on page 73 for measurements with total-head tube and side hole to be about 0.8 per cent. The ratio of the diameter of the pitot to the diameter of the measuring pipe was $0.024/0.494$, i.e., 0.05 and, by extrapolation of the figures given on page 74, we see that the appropriate error is about 1.5 per cent. It must be repeated that the work described was done before Fage's results had been published; otherwise a smaller pitot tube would have been used to reduce this correction due to size. However, taking the results as they stand—they lose nothing of their value as an illustrative example because of the large value of the correction—we see that the figures in the last column of Table 19 must be reduced by $0.8 + 1.5$ per cent., i.e., by 2.3 per cent. We thus obtain finally the results given in Table 20, from which we can plot a calibration curve of v against manometer reading, which can be used to determine the mean rate of flow in the inch pipe for any reading (within the range of the calibration) of the inclined-tube manometer.

TABLE 20

Inclined-Tube Manometer Reading inches	Corrected Mean Velocity in Inch Pipe feet per second
1.22	4.00
2.13	5.21
3.38	6.50
4.69	7.52
5.19	7.93
6.02	8.52
6.69	9.02
8.66	10.20
9.60	10.75
11.04	11.52

One further word of explanation should be added. It will be seen that all the calculations have been carried through on the assumption that the air density $\rho = 0.00237$. The calibration curve to which reference has just been made, therefore, is valid only for this value of ρ . We know, however, that the pressure drop through the orifice is proportional to ρv^2 . (This can easily be verified from equation (1) of this chapter.) Hence, if the barometric pressure and the temperature of the air in the pipe are appreciably different from standard conditions, so that ρ is not 0.00237, we have to multiply the velocity obtained from the calibration curve for any measured pressure drop by $\sqrt{(0.00237/\rho)}$. Equation (5) of Chapter V enables the value of ρ to be calculated for any observed temperature and barometric pressure.

(c) *Determination of the Orifice Coefficients.*—For the purpose for which this apparatus was required it was not necessary to determine the discharge coefficient of the orifice. But, since the results given provide all the data for the calculation, it will be useful to show how the coefficients are deduced.

The inclined-tube manometer was calibrated against the Chattock gauge by the method indicated in Chapter IX. In this way, it was found that 1 inch observed head on the manometer was equivalent to 1.727 turns of the Chattock gauge

wheel, i.e., to $\frac{13.08 \times 5.2}{10 \times 20} \times 1.727 = 0.586$ lb. per square foot.

From equation (1) of this chapter we have

$$\alpha = v_1 \sqrt{\frac{\rho \left[\left(\frac{a_1}{a_2} \right)^2 - 1 \right]}{2(p_1 - p_2)}} \quad (4)$$

The orifice diameter was measured and found to be accurately 0.5 inch, so that $a_1/a_2 = (0.9985/0.5)^2 = 3.988$, and $(a_1/a_2)^2 - 1 = 14.90$.

Hence
$$\alpha = 0.133 v_1 \sqrt{\frac{1}{(p_1 - p_2)}}, \quad (5)$$

where v_1 is the mean velocity in the inch pipe. Values of v_1 and the pressure drop (the latter in terms of inclined-tube gauge readings) across the orifice are given in the last and first columns respectively of Table 20. In order to obtain the pressure $p_1 - p_2$ in lb./sq. ft. for use in equation (5) above, we have to multiply the observations by the calibration factor 0.586 of the manometer (see above). Table 21 gives details of the calculation of the orifice coefficients for the values of v_1 taken from Table 20.

The values of the discharge coefficient in the above table afford an instance of the feature noted in Chapter VI, namely, a rise at low Reynolds numbers. At a mean speed of 4 feet per

TABLE 21

v_1 ft./sec.	Manometer Reading M inches	$(p_1 - p_2)$ lb./sq. ft. = 0.586 M	$\sqrt{\frac{1}{p_1 - p_2}}$	$\alpha (= 0.133 v_1 \sqrt{\frac{1}{p_1 - p_2}})$
4.00	1.22	0.715	1.184	0.630
5.21	2.13	1.247	0.896	0.621
6.50	3.38	1.980	0.711	0.614
7.52	4.69	2.745	0.604	0.604
7.93	5.19	3.04	0.574	0.604
8.52	6.02	3.53	0.532	0.603
9.02	6.69	3.92	0.505	0.606
10.20	8.66	5.07	0.444	0.603
10.75	9.60	5.62	0.422	0.603
11.52	11.04	6.46	0.394	0.604

second in the inch pipe, the value of vd/ν is about 2,100, and we saw in Chapter IV that this is just about the value of the

critical Reynolds number at which the flow changes from the streamline to the turbulent variety. Fig. 9 shows that the ratio of the mean velocity in the pipe to the maximum rises steeply in the transition range of vd/ν . Thus, at the lower rates of flow, the velocity at the centre is higher in relation to the mean velocity than at the higher rates. In other words, at the lower rates of flow relatively more air passes through the orifice at the centre of the pipe than at the higher rates, and so we find the discharge coefficients higher.

It will be noted that no measurements of barometric pressure or of air temperature were necessary in order to determine the values of α by this method of calibration. A standard value of ρ of 0.00237 was assumed throughout. The reason for this will be clear from consideration of equation (4) from which α was calculated. Here ρ occurs in the numerator under the radical sign, but it must be remembered that v_1 in equation (4) was determined from pitot-static pressure observations, and hence is of the form $K.P/\sqrt{\rho}$, where K is some constant and P is a pressure. Hence $\sqrt{\rho}$ cancels out of the equation. The evaluation of v_1 in feet per second would, in fact, have been an unnecessary step if the sole object of the work had been the determination of α . Since, however, v_1 had been determined for another purpose, on the assumption that ρ was equal to its standard value, the same value had to be inserted in equation (4).

(3) Effect of Humidity on Air Speed Measurements.—

It is only when great accuracy is required that account need be taken of the effect of the moisture in the air on the density ρ . The procedure then is to make use of the law of partial pressures for mixtures of gases or vapours, which states that the total pressure of the mixture is the sum of the partial pressures of its constituents. Thus, for a mixture of dry air and water vapour the total pressure, which is the observed barometric pressure H , is made up of the partial pressure of the dry air and that of the water vapour (p). Making use of this law, and of the fact that the density of water vapour is 0.622 times that of dry air at the same temperature and pressure, we obtain the following formula for the density of moist air (ρ_w),

$$\rho_w = \rho_d \times (H - 0.378p)/H, \quad \dots \quad (6)$$

where ρ_a is the density of dry air under the prevailing conditions of temperature and pressure.

This expression can be evaluated if p , the pressure of the water vapour in the air, is known; and this quantity can be determined in the usual way from humidity measurements. A convenient procedure is the following: Observations are made of the wet- and dry-bulb temperatures (t_w and t_d), and from this information the dew point t_{dp} can be determined with the aid of a table of Glaisher's factors.

$$\text{Thus} \quad t_{dp} = t_d - f(t_d - t_w), \quad \dots \quad (7)$$

where f is Glaisher's factor, as determined from tables, at the dry-bulb temperature t_d .

At the dew-point temperature, the air will be just saturated with the water vapour it contains; so that p can be obtained from a table giving the saturated vapour pressure of water at different temperatures, the temperature to be used being t_{dp} as obtained from (7).

The following numerical example will make the method clear. Let us assume that the height of the barometer H is 762.0 mm., the dry-bulb temperature t_d is 24.6°C. , and the wet-bulb temperature t_w 20.8°C. From tables, we find that Glaisher's factor at a dry-bulb temperature of 24.6°C. is 1.71.

Hence, applying (7), we have

$$\begin{aligned} t_{dp} &= 24.6 - 1.71(24.6 - 20.8) \\ &= 18.1^\circ \text{C.} \end{aligned}$$

Turning now to a table of saturated vapour pressure of water against temperature, we find the value of p at a temperature of 18.1°C. to be 15.6.

Hence, from (5) we have

$$\begin{aligned} \rho_w &= \rho_a \times \frac{762.0 - 0.378 \times 15.6}{762.0} \\ &= \rho_a \times \frac{756.1}{762.0} = \rho_a \times 0.992. \end{aligned}$$

Thus we see that the true density of the air under the assumed conditions of temperature and pressure is about 0.8 per cent. less than the value that would have been obtained had the air been taken as dry. In this example, the relative humidity

was rather high, as is seen from the fact that the wet-bulb temperature was only 3.8° C. below that of the dry bulb. Hence the correction on density for humidity was somewhat high. It will, in fact, generally be found that the correction does not exceed 1 per cent. ; and since, in the determination of air speed from pressure measurements, an equation of the type $v = k\sqrt{P/\rho}$ is usually used, the error on speed if the effect of humidity on ρ is neglected will normally not exceed 0.5 per cent.—half the error on ρ .

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